MULTIPLE MEANINGS OF MASTERY IN MATHEMATICS EDUCATION: A Q METHODOLOGY STUDY

by

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Abstract

This thesis investigates diversity in views teachers hold about mastery in mathematics. Mastery, in relation to teaching and learning of mathematics, remains poorly-defined and under-researched, yet mastery programmes attract generous government funding in England. This research employs a Q methodology to quantitatively evaluate the subjective opinions of 45 specialist mathematics teachers. Unlike conventional Q methodology research, which utilises a high degree of researcher-participant interaction within an interpretivist paradigm, this study collected anonymous data online and took a postpositivist position.

The findings identified a set of mastery pedagogies, which combine progressive educational aims with traditional educational practices, that could fulfil the aim of 'all children achieving a deep understanding of mathematics'. Adopting this approach requires changes to mathematics teacher training and development, and the practices of school mathematics departments. This approach could address retention of mathematics in England which is currently of national concern.

The research revealed four distinct viewpoints, or factors, labelled '*travel far, travel together', 'know your subject, follow the teacher', 'create a curriculum for interconnected understanding' and 'variety in teaching, learning and assessment'*. A teacher's view of mastery depends on their combined beliefs in student potential and how much a student's learning journey should be explicitly crafted by the teacher. The teachers' competence and confidence in mathematics, and their previous experiences in teaching and professional development were distinctly different across the factors.

This research realised the potential for Q methodology, which balances the rigour and precision of large-scale randomised controlled trials with the relevance and richness of smaller studies, to be utilised more in educational research, within and beyond mastery in mathematics.

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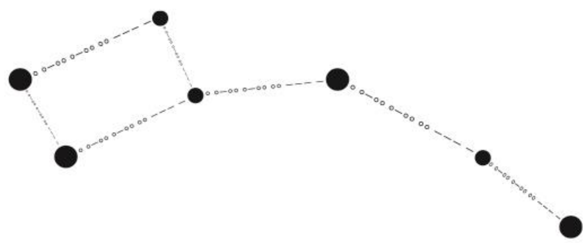
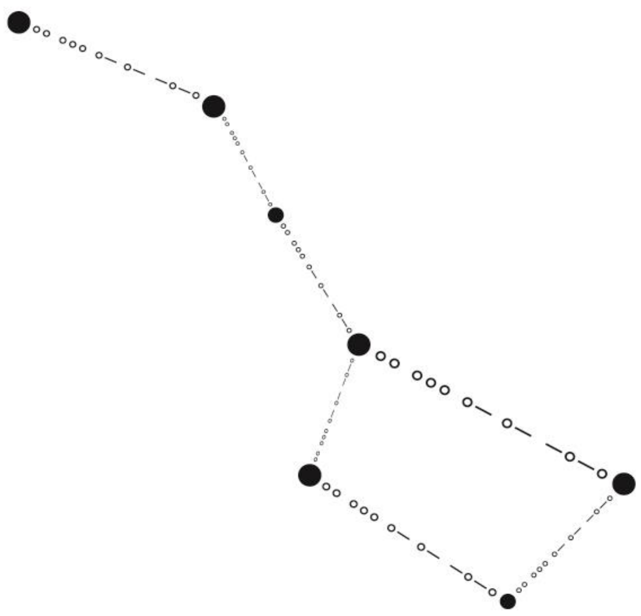
For my two real-life constellations, more inspirational than all stars in the universe.





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1 Why research mastery with mathematics teachers?

1.1 Introduction

Mastery learning, and teaching for mastery, is a policy initiative in English mathematics education that has developed over the last ten years. At the time of writing, in 2021, there were at least six organisations providing professional development for teachers related to mastery in mathematics. This scene-setting chapter introduces 'mastery' as a word, and a concept, increasingly associated with the teaching and learning of mathematics in England. The ambiguity in meaning of this single word, the core of a state-funded policy initiative to grow mastery programmes, provided the motivation and the rationale for the thesis.

The chapter opens with a real-life example of how teachers from two different countries approached a mathematics problem in two fundamentally different ways, and why only one of these approaches was considered to illustrate mastery. This provided an initial spark of motivation to research mastery in relation to approaches to mathematics. An investigation into the online popularity of the word 'mastery' associated with mathematics education in internet searches increased this curiosity.

I continue the chapter by considering the general definition of 'mastery', and why this leads to differing pedagogical decisions in relation to mathematics education. These different decisions are related to mastery in education's origins, the different mastery programmes available, and a teacher's own philosophy of mathematics.

I conclude this opening chapter summarising why multiple meanings of mastery are a problem in mathematics education and hence why this study was needed. The completed study's findings and reflections on my chosen methodology have implications for mathematics educators, school departments and policymakers.

1.2 A mathematical problem to illustrate mastery

I began to think deeply about the meaning of mastery after considering two contrasting approaches to a mathematical problem. I explain the context of this below.

The English government began to explore the feasibility of a mastery approach in 2014, funding a small group of English mathematics teachers to travel to Shanghai. Once there, the teachers spent time observing and talking to Chinese mathematics teachers about how they planned and taught mathematics. The group was accompanied by the Director of Primary Mathematics at the England National Centre for Excellence in Teaching Mathematics (NCETM), Dr Debbie Morgan CBE.

Dr Morgan recounted a situation to me: she posed a mathematical problem to the English and Chinese teachers to pass time on a long train journey. The problem, 'Make 37' (Nrich, 1997), is reproduced in Figure 1.1.

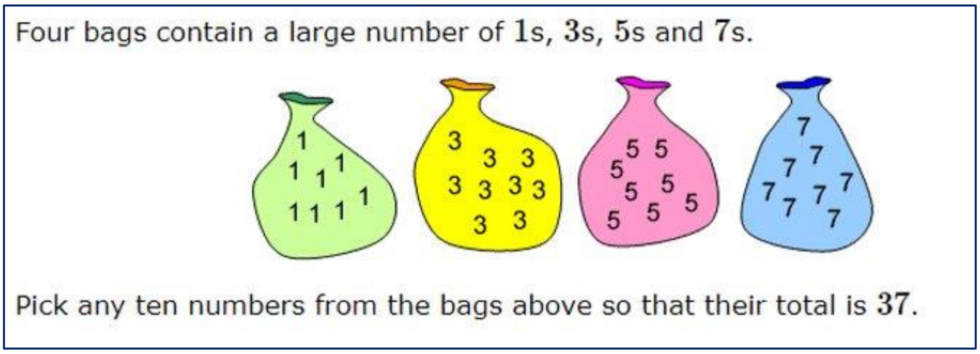


Figure 1.1: Nrich 'Make 37', available at https://nrich.maths.org/make37

Dr Morgan noticed two distinct reactions:

- The English mathematics teachers immediately got out a pencil, and paper, and started trying out different combinations of the numbers as a 'way in' to solving the problem,
- The Chinese teachers sat back in their chair, folded their arms, and smiled, instantaneously knowing the solution.

For Dr Morgan, the two approaches to the problem illustrated fundamentally different attitudes to mathematics teaching and learning. The English teachers started with the *specific* problem posed and were able to unpick the mathematical reasoning needed to solve the problem by experimentation, reflection, and refinement. The Chinese teachers went straight for the *general* solution, using their knowledge of the underlying structure of mathematics to solve the problem without ever trying specific numbers. *[The problem is impossible; the total needed (37), and the numbers given in the bags (1, 3, 5 and 7) are all odd numbers. It is impossible to sum 'an even number of odd numbers' to make an odd number total].* Dr Morgan claimed that the Chinese mathematics teachers demonstrated a deep understanding of the underlying structure of mathematics that is used to solve unfamiliar problems. The Chinese teachers' approach exemplified mastery, the English teachers' approach did not.

I reflected on Dr Morgan's story a great deal: What had really caused the two approaches to be so different? Could I attribute to differences in language, or a random observation? Are English and Chinese teachers taught differently in every subject? Can differences in the way that mathematics is taught lead to radically different approaches to problem solving, and different levels of understanding? Were English mathematics teachers exhibiting a 'student' attempt at a non-standard problem whilst Chinese teachers approached the problem like a 'mathematician' (Schoenfeld, 1992)?

My reflections inspired me to research the different components of what the mathematics teaching profession thought of as 'mastery'.

1.3 The rapid rise of mastery in mainstream mathematics education

In the last ten years, 'mastery' has become both a popular talking point amongst mathematics teachers and a well-funded government policy strategy. This increase in popularity has not yet resulted in an accepted definition or a body of research evaluating its efficacy.

I first heard the term ‘mastery’ associated with mathematics education at some point in the early 2010s. I was (and am) a university lecturer in mathematics education, responsible for training secondary mathematics teachers across a variety of Initial Teacher Training (ITT) routes. Hence, I heard about mastery slightly in advance of others in the profession. Google Trends reports United Kingdom searches for ‘mastery’ standalone or in combination with ‘mathematics’ or ‘maths’ were rare until 2013. Mastery’s subsequent rise in popularity remains to the present day (Figure 1.2 and Figure 1.3).

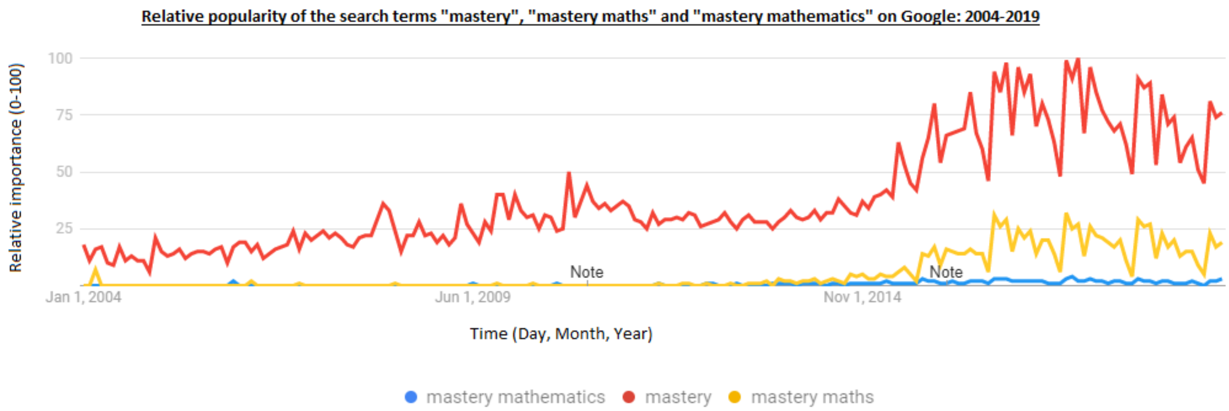


Figure 1.2: Searches for ‘mastery mathematics’, ‘mastery’ and ‘mastery maths’ on Google from the UK, 2004-2019 [source: Google Trends, accessed November 2019].

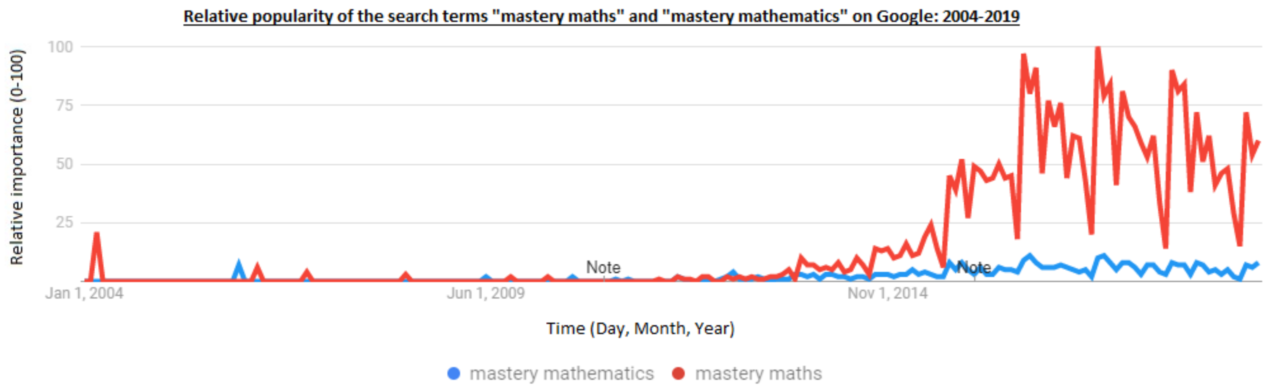


Figure 1.3: Searches for ‘mastery mathematics’ and ‘mastery maths’ in Google from the UK, 2004-2019 [source: Google Trends, accessed November 2019].

Two drivers of this increase were the attention given to the high attainment in mathematics in South East Asia, as measured by PISA (OECD, 2015) and TIMSS (Mullis *et al.,* 2012), and the popularity of Hattie’s *Visible Learning* meta-analysis, published in 2008.

Programmes promoting mastery in mathematics are well-funded in England. A 2012 Department for Education (DfE) National Curriculum review comparing the mathematics curriculum in England to countries including Hong Kong and Singapore (DfE, 2012), and a government-funded exploratory visit to observe mathematics teaching in Shanghai in 2014 (DfE, 2014) preceded a £40 million award to the NCETM to spread a 'Teaching for Mastery' approach to thousands of primary schools (DfE, 2016b). The NCETM's current aim is to co-ordinate the implementation of its Teaching for Mastery programme to 60 percent of primary schools and 50 percent of secondary schools by 2023 (NCETM, 2019). Separate from the DfE funded mastery initiative, other education providers offering mathematics mastery programmes include Ark Schools' Mathematics Mastery programme, La Salle Education's Mastery in Mathematics, Maths No Problem!'s Teaching Maths for Mastery or the self-titled programmes by Inspire Maths and White Rose Maths.

1.4 No common definition for mastery

Traditionally, to 'master' something refers to the attainment of a certain level of knowledge and expertise, following a long period of study and practice, often in the region of 10,000 hours (Ericsson, Krampe and Tesch-Römer, 1993). In the context of an educational approach that promotes 'practise for progress and attainment' there is no commonly agreed definition for mastery beyond an historic definition by Block and Burns: 'an explicit philosophy about learning and teaching' in which 'under appropriate instructional conditions, virtually all students can learn well' (1976, p. 4). Anderson and Block (1977) describe mastery as 'both a philosophy of school learning and an associated set of specific instructional practices' (p. 163).

'Mastery learning' was first associated as an educational approach the United States of America in the 1960s, and two different programmes developed simultaneously. Bloom's historic (1968) 'Learning for Mastery' focused on improving the distribution of children's attainment from '1/3 of students achieve a good standard, 1/3 of students fail, and 1/3 learn something, but not enough to be thought of as good students' (p. 1). Keller and Sherman's 'Personalised system of instruction' (Keller, 1968) emphasises mastery as a journey in student autonomy, where a pupil works at their own pace on a specific topic until such time that a certain standard, as judged by a tutor, is reached. Guskey (1987) describes a 'mastery learning' model as a series of formative assessment loops.

Disagreement about what constitutes mastery learning is evident in contemporary English mathematics programmes. The January 2019 Cambridge Mathematics 'espresso' evidence summary contrasts the NCETM director's notion of mastery as 'an approach that ensures all children can succeed in mathematics' with Mathematics Mastery's notion of mastery as 'being able to solve non-routine problems, often by using multiple representations' (Rycroft-Smith and Boylan, 2019). Neither Singapore (the influence for Mathematics Mastery) nor Shanghai (the influence for the NCETM) self-describe their approach to mathematics teaching using the word mastery (Boylan, Maxwell *et al.,* 2018). In an article to the Association of Teachers of Mathematics, Professor Boylan notes mastery as a 'slippery word' and the current mastery policy as possible 'remasters, remixes or mash-ups.' (2019).

1.5 Mastery and the philosophy of mathematics

Disagreements about mastery run deeper than pedagogical strategies. There is a lack of clarity about which mathematical knowledge should be attained as evidence of mastery. Existence of different types of mathematical knowledge lead to different opinions about what a 'mastery' level of knowledge would be, and subsequently a range of opinions about how mathematics should be taught.

Mathematicians and mathematical philosophers categorise mathematical knowledge in different ways (e.g. Ernest, 1991; Davis and Hersh, 1980). *Absolute* mathematical knowledge is certain, real, objective, and provable, with mistakes and contradictions to eliminated. Mastery of absolute knowledge is therefore a function of memorisation and practice, and thus teaching and assessment decisions should be made on that assumption. *Fallible* knowledge is limited to context and time, so mastery becomes a function of determining the probabilities of error in judgement. Teaching and assessment therefore must include the skills of critical analysis and reflection. If mathematics is a *game* of symbols and rules created to explain observations and solve human problems, then its treatment perhaps requires less sensitivity than if it is a *language* to unite (or divide) society and facilitate communication and agreement.

A multiplicity of views about mathematical knowledge has implications for teaching and how knowledge is acquired, retained, applied, and added to. Absolute and finite knowledge could potentially be mastered on one's own with adequate access to appropriate resources (such as a teacher, textbook or otherwise) and sufficient motivation. However, relative knowledge requires definitions and facts to be debated and refined through a more social and enquiring engagement within mathematics education. This determines whether mastery is taught, learnt, acquired, assessed, or aspired to.

1.6 The need to categorise and understand different notions of mastery

Those who make decisions about the mathematics that children learn, and how it is taught, need to be aware that mastery comes from multiple origins and carries multiple meanings. They need to understand the types of views that teachers have about mastery. Unless there is a greater awareness of the range of views and opinions about mastery, there is a real risk that decision makers in mathematics education will use incomplete information to make professional judgements that will have major effects on children's learning. Hence, my research study, which evaluates the different opinions of mastery held by mathematics teachers and educators is important.

A teacher's viewpoint of mastery, and their associated practices, is both derived from and influenced by an interplay between their beliefs, pedagogic content knowledge and pupil responses (Askew, *et al*, 2015). This research study identifies discrete viewpoints of mastery, which are themselves multi-faceted. The study correlates viewpoints of mastery with characteristics of Ernest's (1991) philosophies of mathematics and makes accounts *of* the teachers who held these viewpoints but stops short of accounting *for* these correlations. The mechanisms for how multiple mastery interventions and approaches have been differently recontextualised (e.g. Bernstein, 2000) by groups of teachers was not considered in the study.

In my professional role as a teacher educator who regularly gives talks about the importance of being research-informed, asymmetry of information about an educational policy, programme, or intervention troubles me a great deal. I felt compelled to understand the different meanings teachers attach to the term 'mastery' in relation to their curriculum, planning, teaching and assessment, and the learning of their students. My own teaching background was in secondary schools and I originally designed the study for secondary teachers. However, much of the literature reviewed was cross-phase in nature, and primary teachers who self-declared as mathematics specialists were welcome contributors to the research.

State-funded schools in England teach mathematics as stated in the National Curriculum (DfE, 2013), which specifies topics that should have been taught to children during each year or Key Stage of education, but not how they should be taught. The government has devolved decisions about curriculum design and delivery to bodies that maintain schools (these include local authorities or trusts), which may devolve them further to school leaders and class teachers (DfE, 2010). However, this devolution represents at best a 'supported autonomy' (DfE, 2016a, p. 4). Schools are subject to inspection against the 2019 English state schools' inspection framework, which judges the quality of education by curriculum 'intent' (is it designed for all children?), implementation (is it delivered effectively to all children?), and impact (do all children make progress?) (Ofsted, 2019). Schools must provide appropriate justification for the curriculum and policy decisions they make.

Hence, it is vital that the education community and Department for Education have transparent information and a common language about schools, departments, or teachers that 'do mastery', 'teach for mastery' or 'have a mastery curriculum'. Only then can individual mastery programmes be understood and compared, and only then can future research testing the relative effects of mastery interventions be validated.

1.7 The importance of this research study to the mathematics education community

Despite strong evidence that multiple meanings of mastery exist, and a strong likelihood that mastery will continue to feature in educational funding and policy decisions, no previous study had asked mathematics teachers or educators what mastery means to them, their practice, or the learning of their students. This research addresses this gap. The study was conceived to categorise different meanings of mastery, and the types of educators who hold these views. The implications of the findings should influence decisions made by teacher educators, practitioners, school leaders and policymakers. The findings will also be of interest to existing and new providers of mastery continuing professional development (CPD) activities, including the NCETM. In addition to the implications of the findings, this study is also an informative case study of Q methodology research. Q methodology is little-used in UK education research, and the ability of Q studies to quantify subjectivity makes the methodology a creditable addition to quantitative or qualitative approaches.

1.8 Structure of the thesis and the research question

My motivation to understand teachers' viewpoints of mastery was both the rationale for the research and a determinant of its structure. In this thesis the literature review precedes the methodology, however in practice the two chapters were written in tandem. As my understanding of mastery deepened through engagement with literature, so my choice of methodology and decisions about methods were developed and refined. Choosing Q methodology allowed me to create a framework for categorising mastery's corpus and reading about components of mastery reinforced the decision to utilise Q methodology and create the statements.

Basing a thesis topic on an area of interest and concern helped me remain motivated and engaged during challenging times (not least a global pandemic) but turning an 'itch to scratch' into an investigable research question was an iterative process of reading and reflecting on mastery and methodology literature. However, a decision was eventually made.

The specific research question for this study is:

'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their practice and the learning of their students?'

Chapter 2 is a review of mastery literature. To make sense of this wide field I devised a framework, adapted from Ernest's (1991) ideologies of mathematics education, to categorise the choice of papers and to keep track of the emerging landscape of diverse mastery views.

I introduce Q methodology in Chapter 3. I explain the origins and philosophy of Q methodology, my own chosen position within the methodology, and the applicability and generalisability of the study. Q methodology is also the focus of Chapter 4: by the end of the chapter the reader will be aware of how the study was constructed and the steps I took to ensure the data collection methods were robust and accurate.

Chapter 5 details how the individual views of mastery expressed by the research participants were collected, analysed, and categorised into groups, or Factors, that represent distinct sets of opinions. The techniques of Q factor analysis and rotation work in combination with the researcher's abductive reasoning to make sense of mastery's multiple meanings.

Chapter 6 explains how the findings are presented: colour-coded factor arrays, factor infographics and factor narratives. Whilst the use of arrays and narratives is standard practice in

Q methodology, I detail how and why I have made changes to standard presentations and how these changes aid analysis and discussion.

Chapter 7 is a full presentation of the findings using the representations describes in Chapter 6. The reader is led through the viewpoints of the four factors and the characteristics of the research participants who are associated with each viewpoint.

Chapter 8 details the implications of the findings for those concerned with mastery in mathematics education. Each Factor's set of opinions about mastery can be associated with differences in teacher ideology, and thus different decisions about curriculum and pedagogy. I discuss the importance of the findings on school departments, teacher educators and mathematics education policymakers.

In Chapter 9 I highlight the unique ability of Q methodology to have led to these findings as a 'call to arms' for the increased use of Q studies within and beyond mastery in mathematics.

Chapter 10 'closes the circle' and considers the 'make 37' problem through the lenses of Factors 1, 2, 3 and 4. I also reflect on the numerous limitations of the study, and suggestions for future research, in this final chapter.

A glossary of terms and acronyms in English mathematics education and Q methodology are included at the end of the thesis.

1.9 Conclusion

Chapter 1 introduced the context of the thesis with evidence that mastery in mathematics, despite its sudden increase in popularity and large amount of government funding, has no single origin, definition, or educational aim. This uncertainty led to necessitates an urgent need to understand what practitioners themselves understand by the term, and how it relates to their professional decisions in teaching and learning. This need will be addressed by the research question: ‘What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?’.

2 Literature review

2.1 Introduction

This literature review chapter charts the development of mastery in education and explores the range of pedagogical strategies associated with mastery teaching and learning. Reviewing literature to research teachers' opinions on a concept with multiple meanings provided multiple challenges: these were overcome using a classification of mathematics teacher ideologies as a framework. The literature review enabled me to categorise the range of opinions of mastery originating from this classification. The categories were a stepping-stone to the creation of a Q methodological research study design, detailed in Chapters 3 and 4.

Chapter 1 of this thesis introduced the motivation for this research study: to investigate English mathematics teachers' beliefs about what it means to master mathematics at secondary school, and the features of teaching they believe best facilitate the development of mastery in their students. In general terms, mastery can mean knowing a lot, doing something well compared to what can be known, or exceeding the prowess of others. The etymology of the word 'mastery' is the French word *maistrie,* and its meanings include: 'superiority' (in competition or over others), 'authority' (over others), 'a feat of strength, skill or power', 'comprehensive knowledge of a subject, art or process' or 'a field of knowledge' (OED, 2001). The existence of competing interpretations of mastery in general terms is highly relevant to education because it means that mastery could represent a norm-referenced or criterion-referenced measure of attainment or progress. Teachers' opinions of mastery are dependent upon their beliefs in the aims of education, and the practices associated with achieving these aims.

2.1.1 Methodology of this literature review

As described in Chapter 1 my research study was motivated by a need to know what teachers believe mastery is, and how this belief influences their pedagogical choices. A review of literature found multiple origins of mastery programmes, underpinned by contrasting educational aims. There was evidence that ‘mastery’ pedagogical approaches differed between programmes.

My research question, ‘What do specialist mathematics consider mastery to mean, in relation to their own practice and the learning of their students?’, necessitated a literature review to investigate the range and scope of mastery over time and across programmes. I used the findings of literature to make a ‘net’, cast over the field of mastery in mathematics, rather than a ‘funnel’ to narrow broader themes down into small areas of investigation. Reflecting on Williams’ (2020) categories, my review is partly historical (it traces the histories of the different notions of mastery and their influences on practice), partly integrative (it brings together what is known about mastery from different perspectives) and partly theoretical review (it aims to uncover relationships, or tensions between different theoretical approaches). This style of literature review is compatible with Q methodology (see Chapter 3) and was written alongside choosing and refining the methodology (see Chapter 1.8).

I started the review by using literature that described or reported on contemporary mastery programmes in England to find seminal papers and search terms. Specifically, these were: Boylan, *et al*’s, ‘Mastery innovation timeline and influences’ (2019, p. 48, Figure 1), the NCETM’s primary and secondary principles of teaching for mastery, (2016a; 2017), Drury’s ‘How to teach mathematics for mastery’ (2018) and McCourt’s ‘Teaching for mastery’ (2019). I used search engines provided by Google, Google Scholar and Canterbury Christ Church University’s library database. I searched for ‘Mastery AND mathematics’ in Google, Google Scholar and CCCU LibrarySearch. I studied the publications and websites of Ark Schools’ Mathematics Mastery programme, the NCETM, La Salle Education’s Mastery in Mathematics, Maths No Problem!’s Teaching Maths for Mastery, Inspire Maths and White Rose Maths. Sources also came from article references and recommendations from professional colleagues or social media. I also included sources referenced from articles found in my initial search. This continued until no new definitions presented themselves.

The literature review identified a wide range of pedagogies associated with mastery in mathematics and how they relate to fundamental opinions about what it means to master mathematics. It indicated that differing aims of mastery lead to contrasting pedagogical strategies. Hence, there was strong evidence that the research question 'What do specialist mathematics consider mastery to mean, in relation to their own practice and the learning of their students?' is a question that has not been fully answered and needs addressing.

2.1.2 A framework for the literature review

The initial search for relevant literature conducted using the method outlined above produced many results with a wide variety of foci. I realised I needed a framework to categorise the findings and chose Ernest's (1991) classification of teacher ideologies. I could then thematically explore the literature across different aspects of mastery belief and pedagogy. I chose Ernest's classification because he created it to group fundamental principles behind mathematics teachers' pedagogical choices; exactly what I needed to achieve for mastery.

Ernest's model segments beliefs held by different groups of mathematics teachers, how those beliefs shape their educational aims, and how they believe they should best act to achieve these aims. He categorises these beliefs in relation to the fundamental contributions of 'the philosophy of mathematics, the set of moral values, and the theory of society' (the primary level), and contributions which are derived from these (the secondary level) (ibid, p. 137). Ernest's framework has six primary and eight secondary elements: since much of the literature discussed themes that overlapped elements, I combined and reduced these to three primary elements and three secondary elements (see Table 2.1). Even when combined in this way, overlaps remained; good evidence that a research study that effectively categorises teachers' beliefs and pedagogies of mastery was needed. Ernest's categories do not include a 'teacher professional development' element and since this theme emerged as I reviewed the literature, I included it as an additional element.

Educational Ideology element (Ernest, 1991, p. 153, Table 7.1)	**Literature review category name** 'Meanings of mastery: ……'
Primary elements	
Philosophy/ view of mathematics	The philosophy of mathematics
Educational aims/ mathematical aims	The educational aims of mastery programs The US Singapore and Shanghai England
Theory of the child/ theory of ability/ theory of assessment	For all or for some? Norm referenced or criterion referenced?
Secondary elements	
Theory of learning/ theory of social diversity	A mastery curriculum
Theory of teaching mathematics	Mastery and instruction Small-steps and variation Mathematical fluency
Theory of resources	Mathematical representation
Additional element	
	Teachers as professionals

Table 2.1: Literature review framework

Ernest (ibid) identified five educational ideologies of mathematics teachers: 'old humanists', 'industrial trainers', 'technical pragmatists', 'progressivists' and 'public educators'. Whilst the literature review did not attempt to align aspects of mastery with these ideologies, I return to them in Chapter 7 when discussing the implications of the study's findings.

2.2 Meanings of mastery: Primary elements

Ernest's model assumes that mathematics teachers' fundamental (or 'primary') values about mathematics education originate from what they think mathematics is, their beliefs about morality and society, and the role of education in shaping a child's current and future role in society. Ernest segmented these values into six elements (1991, p. 134, Table 6.3). My variation on Ernest's framework identified three primary elements: philosophy of mathematics, educational aims (to include mathematics educational aims) and theory of the child (which includes their potential and how achievement is assessed) (Table 2.1).

Different philosophies of mathematics have existed since the time of Plato and Aristotle, and the extent to which mathematics always provides a 'right answer' (and who judges this) remains fiercely debated by mathematicians. The ramifications of the ongoing debate for mastery include an uncertainty about whether mastery is something to be achieved, or an ongoing developmental process. Educational aims are shaped by societal cultures and expectations, and the literature highlighted differences in countries associated with mastery in mathematics. The final primary element investigated was the belief in the potential and opportunity of all children to experience mastery in mathematics, and how that achievement is assessed. Whilst extensive literature exists in relation to these three primary elements, their interrelationship in determining beliefs about a specific aspect of mathematics education (i.e., mastery) has not been investigated, hence the need for this research.

2.3 The philosophy of mathematics

It is not known how a teacher's notion of mathematical mastery is affected by their philosophy of mathematics. Ernest's ideology framework classifies types of teachers according to their philosophy of mathematics as a 'set of truths and rules', 'unquestioned body of useful knowledge', 'body of structured pure knowledge', 'personalized maths' or 'social constructivism' (p. 138). Ernest associates the first three types with mathematical absolutism, with aspects of formalism present in the third type. The teacher with a 'personalized maths' philosophy associates mathematics with constructivism and the 'social constructivist' philosophy is most strongly associated with falliblism.

McCourt, author of *Teaching for Mastery* (2019), connects mastery directly with mathematical philosophy, claiming that 'Aristotle himself would recognise the version of mastery described in his book (p. 21)'. Aristotle, a pupil of Plato, was taught absolutism*:* that mathematics exists independent of human existence (Linnebo, 2018). Platonic logicists, who include Leibniz, Cantor, Frege, Russell and Carnap, consider mathematical truth as deduced through reduction of mathematical concepts to axioms and logic (Ernest, 1991). Logicists associate mastery of mathematics as acquiring a high level of knowledge and understanding of a predetermined structure. This is demonstrated by acquisition, memorisation and application of rules, skills, and algorithms, leading to uncovering and proof of more of the mathematics existing 'out there'.

Aristotle's own philosophy of mathematics retained absolutism but believed that there are limits to human observation and discovery. Learning mathematics is about improvement and refinement and attempting to push forward boundaries that others have not yet reached. Mathematics is a set of symbols created by people to explain our observations of the world. There is no 'out there', it is all 'in here'. This is the epistemology of formalism (e.g. Hilbert, 1926) and constructivism (e.g. Brouwer, 1913). Mathematics is a human-created game, with human-created rules to represent, explain, and predict the world. To master mathematics is to accept a model where there is evidence and invent new ones where there is not.

The scientific revolution of the 19th century precipitated the notion that 'as knowledge grows, so do the limits of what can be known' (Ernest, 1991, p. 20). The questioning of Euler's fifth postulate, firstly by Gauss in 1817 (O'Connor and Robertson, 1996), the formation of the Russell-Zermelo paradox in 1901 (Irvine and Deutsch, 1995) and Godel's incompleteness theorems in the 1930s (Raatikainen, 2018) led to non-Euclidean geometry, complex numbers and proof by machine. For the first time, mathematical knowledge was considered fallible; observations and proofs made at a point in time could be disproved in later periods or paradigms (Kuhn, 2012). Mathematical fallibility meant that mathematics could be considered a human problem-solving activity (Davis and Hersh, 1980; Tymoczko, 1986; Polanyi, 1958). Social realists such as Durkheim and Wolff (1964), Lakatos (1976) and Kitcher (1983) claimed that mathematics itself was created by society to explain, categorise and articulate observations and that we study mathematical ideas to explain our own construction of the world. For fallibilists, to master mathematics is to be able to analyse, justify and question existing mathematical ideas, and create new ones (Peck, 2018). The acceptance of fallibility facilitated the diversification of mathematics to engineering, economics, games design and other subjects that wholeheartedly accept questioning, imperfection, and practical solutions. A student who 'learns for mastery' must gain a life-long motivation for controlling their own success.

Ernest makes the claim that different philosophies of mathematics lead to different outcomes for educational practice (1991, p. 111). By considering Ernest's teacher ideologies in relation to mathematical philosophy, one could conjecture that teachers with absolutist beliefs will associate mastery with pedagogy that prioritises memorisation of facts and rules; formalists with applying those facts and rules to both familiar and unfamiliar contexts (that for constructivists would be created by the learners); and fallibilists with pedagogy that encourages learners to doubt, question and add to existing knowledge.

A research study which explores how views of mastery influence teacher pedagogy should investigate potential correlation between a particular opinion of mastery practices with a specific philosophy of mathematics.

2.4 The educational aims of mathematical mastery programmes

Boylan, *et al.*'s, (2019) evolutionary timeline linking contemporary mastery practices with their origin in culture or educational research identifies multiple influences on English mastery programmes, with different programmes utilising different strategies. Original mastery practices have varied educational aims but the rationale for adopting these practices in England is founded by a policy decision to improve the performance of English students in international mathematics performance assessments. It is not known which educational aim(s) individual teachers associate with mastery.

English mastery of mathematics developed from educational programmes and traditions from both the 'West', originally the United States of America (US), and the 'East': China, especially Shanghai, and Singapore (Boylan, *et al*, 2019). The educational aims of mastery from each tradition were different. 'Western' programmes focused on the potential of mastery teaching to ensure all students reached their individual potential, or to narrow attainment gaps. 'Eastern' curriculums, not given the term mastery (the term was adopted to include their approaches in England much later) were founded on the aim that students should acquire a deep understanding of fundamental mathematics.

2.4.1 The United States of America (US)

The educational aim of the two original US mastery programmes, created in parallel by Bloom and Keller in the 1960s, was to narrow the educational attainment distribution. There is evidence that the practices of these programmes raised attainment. A meta-analysis by Hattie (2008) and a report by the Sutton Trust (EEF, 2018a) that measured positive learning gains from US-style mastery learning practices led to the adoption of these into England.

These early pedagogies, including the Winnetka Plan pioneered by Washburne (1922) and the research outputs of the Chicago Laboratory School beginning with Dewey and developed by Morrison (1926), incorporated strategies shown to be effective in individual tutoring into group-based learning situations. Bloom was motivated by a desire to change education so that 'most students (perhaps over 90 percent) can master what we have to teach them, and it is the task of instruction to find the means which will enable our students to master the subject under consideration' (1968, p. 1). Keller wished to replicate principles of 'highly individualized' instruction, 'facilitating demand for perfection at every level of training and for every student' and 'the minimizing of the lecture as a teaching device and the maximizing of student participation' (1968, p. 79). These mastery programmes subscribed to Carroll's (1963) model of school learning, where learning could be measured as the proportion of actual time spent learning compared to the time needed to be spent. The actual time spent depended on a pupil's access to education and willingness to learn, and the time needed depended on their aptitude for the topic, the quality of instruction and the ability to understand the instruction. Bloom's later description of mastery in his 1984 paper focused on diagnostic and formative assessment.

Bloom's own research claimed a student receiving mastery teaching achieved a level one standard deviation above an equivalent student receiving conventional teaching (1984). A major meta-analysis of mastery programmes conducted by Guskey and Gates (1986) claimed a positive correlation between school effectiveness and mastery learning approaches, and between the implementation of mastery learning and an increase in student learning outcomes. Guskey (1987, p.19) acknowledges that the label 'mastery learning' in the 1980s applied to a 'broad range of educational materials and curricula, many of which bear little or no resemblance to the ideas described by Bloom' in his own paper describing elements of mastery learning. This alternative group of mastery programmes evolved from Keller and Sherman (Keller, 1968). A meta-analysis undertaken by Kulik, *et al*. (1990) which included programmes similar to both Bloom's and Keller's approaches reported wide-ranging effectiveness. There were observed learning gains in lower achieving students, and improved student attitude to the course and their teachers. Conversely, a meta-analysis by Slavin (1987) which imposed stricter criteria for mastery programmes (and thus excluded several studies with positive outcomes) found no statistical link between these programmes and higher achievements on standard tests.

Professor Hattie's (2008) meta-analysis of about 800 studies and 129 interventions for improving educational attainment, published in the book *Visible Learning*, indicated an average seven-month learning gain from students engaging in mastery learning programmes. In 2012 the Sutton Trust, a UK-based charity for educational equality, investigated the impact of mastery programmes by combining six meta-analyses from the 1980s and four single studies in the 2000s. Their reported findings of a ‘moderate impact for low cost’ was incorporated into the Educational Endowment Foundation (EEF)’s toolkit of educational interventions in 2015, and updated in a 2018 report (EEF, 2018a). Most programmes included in this study are variations or a composite of Bloom’s (1968) or Keller and Sherman’s (Keller, 1968) programmes.

2.4.2 Singapore and Shanghai

Neither Singapore nor Shanghai would describe their mathematics education policy or practices as mastery (Boylan, *et al.,* 2018). The educational aim of mathematics education in Singapore is for all children to access educational opportunities (Kaur, 2014) and prioritises ‘the intelligent and creative use of mathematics as a means for solving problems’ (Soh, 2008, p. 28). Chinese mathematics education aims for all children to work towards self-perfection and acquire mathematical knowledge that is both broad and deep and can be applied to multiple types of problems (Li, 2004). Mathematics education in Shanghai combines elements of the Singapore approach with a more traditional Chinese approach to development of teacher subject knowledge and student mindset (Boylan, *et al.,* 2018).

England adopted aspects of Singapore and Chinese mathematics for reasons connected to student attainment rather than being driven by educational aims. The principal driver for change was the 2011 Trends in International Mathematics and Science Study (TIMSS), where Singapore, Hong Kong SAR and Chinese Taipei ranked 1st, 3rd and 4th respectively (Mullis *et al.,* 2012). The ‘Maths Mastery’ programme, heavily influenced by Singapore mathematics education, was founded in 2012, and the first government-funded mathematics education visit to Shanghai took place in the same year. The 2012 Programme for International Student Assessment (PISA) ranked Shanghai, Singapore, Hong Kong, and Chinese Taipei in positions 1st-4th (OECD, 2013). Singapore and China have continued to score highly in international mathematics assessments: the TIMSS rankings for Singapore and China were maintained in 2015 (Mullis *et al.,* 2016) and the top five ranking countries in the 2018 PISA were Singapore and different regions in China (OECD, 2019). In 2016 £40 million of government funding was made available to 35 DfE-funded Maths Hubs to improve the ‘south Asian maths mastery approach’ (DfE, 2016b): this is co-ordinated by the NCETM and their Teaching for Mastery programme.

Mastery programmes in England have not yet been subject to large-scale analysis but early studies have shown some evidence that the programmes benefit younger children. Elements of the primary Maths Mastery programme, evaluated by the EEF, were reported to have a small positive effect on attainment (Jerrim and Vignoles, 2016). A DfE-commissioned longitudinal evaluation of the impact of the China-England mathematics teacher exchange (which inspired and informed the development of the Teaching for Mastery programme) reported that participation in the exchange influenced teacher practices. Of the schools that had implemented a Shanghai-informed approach to mathematics teaching, a small positive impact was found on the attainment of students at the end of Key Stage 1 (aged 7), but not Key Stage 2 (aged 11) (Boylan, *et al.,* 2018).

Ernest’s mathematics education ideologies segment educators by their views on the aims of mathematics education. The paragraphs above highlight differences in the aims that influenced the development of mastery programmes in the US, and mathematical approaches in Singapore and Shanghai. The adoption of aspects of mastery programmes in England from these countries has been motivated not by educational aims, but by the possibility of improved attainment, as measured by the EEF and international assessments. It is not known which aspects of historical mastery programmes influence the opinion of mathematics teachers in England. Hence, the need for a research study which investigates which elements of US, Singapore and Chinese pedagogy mathematics teachers consider to be ‘mastery’.

2.5 Theories of the child, ability, and assessment

The final primary element of Ernest's framework of educational ideologies segments educators by their opinion of the status and potential of a student. I have combined this with Ernest's secondary 'theory of ability' element to incorporate their opinion of a child's relative abilities and the 'theory of assessment' to consider how mastery, and progress towards it, could be assessed. A teacher's view of mastery is influenced by their view on what it means for a child to master mathematics, and whether all children have this potential. Not all aspects of mastery programmes or learning have the same assumptions about the learner, and it is not known how an individual teacher's view of their learners influences their beliefs about mastery.

2.5.1 Mastery for all or for some?

Mastery learning programmes emphasise that a given level of attainment must be achieved by all students. What is not consistent is how teachers aim to achieve this in their classroom. Two strategies to achieve this identified in the literature are keeping groups of students on a topic until all have achieved a given level of attainment and by employing teaching strategies that allow children with lower prior levels of attainment to accelerate their progress through 'growing' their potential. It is not known how different teachers perceive these strategies in relation to their opinion of mastery in mathematics.

There is a level of agreement in the literature that mastery in mathematics is facilitated by teachers 'levelling' the attainment of the group of students they are teaching. The EEF mastery report describes the need for children to remain on a particular topic until they 'demonstrate a high level of success on tests, typically about 80%, before moving on' (2018a, p. 1) and McCourt's mastery cycle aligns with this (2019, p. 26). The NCETM Teaching for Mastery programme states that 'every attempt is made to keep the class together' (2017, p. 1), emphasises specific strategies such as 'whole-class interactive teaching', guidance in lesson design and where students should be sat, and suggested key facts that should be 'learnt to automaticity to avoid cognitive overload' (2016a, p. 1). Drury separates 'success for all' into ensuring that no learner is 'left behind' and 'a higher proportion of students excelling' (2018, p. 13). Guskey (1987) describes mastery as a programme of classes of approximately 30 children and one teacher, whose practice emphasises frequent diagnostic and prescriptive (formative) feedback in relation to clearly articulated learning objectives. The teacher uses 'corrective activities' for those not meeting the required standard, and 'enrichment activities' for those who meet the standard early. Keller's (1968) programme, which he uses in a higher education setting, is much more student-focused and encourages individual study at the students' own pace, with a tutor intervening as and when required.

Mastery's notion that *all* students have the capability to achieve a given level of attainment is shared with the principles of growth mindset theory (Dweck, 2006; Kamins and Dweck, 1999). Growth mindset theory links a child's self-efficacy with their achievements: the more a child believes they are successful, the greater the rate of learning and the higher level of attainment reached. Growth mindset theory shares Carroll's (1963) model of school learning, emphasising the importance of a pupil's access to education and willingness to learn in addition to a measure of aptitude. Growth mindset theory assumes the brain will grow if it is exercised. Northern Irish curriculum guidance equates growth mindset with a 'mastery mindset' of showing resilience in the face of failure and the ability to respond positively to challenges (CCEA, 2008, p. 9). Pedagogies consistent with growth mindset theory share parallels with Bloom's mastery learning (1968): to improve motivation and develop the brain, there is an emphasis on the quality of instruction by a teacher or tutor being appropriate to, and informed by, each student's current achievement, vocabulary level and reading age. This instruction needs to be supplemented with a wide variety of appropriate learning material. Studying is a cooperative activity and students should work together in groups. Additional tutoring and family support are also important.

Dweck's growth mindset theory was extensively applied to mathematics education by Boaler (1998; 2015). Boaler's research supports the development of a growth mindset in mathematics through teaching strategies which include mixed-attainment classes, and giving children opportunities to make mistakes, discuss them, and learn from them. Teachers and children share high expectations of improvement, celebrate effort not attainment, and prioritise formative assessment, showing children what they need to do next rather than what they can (or cannot yet) do. Boaler emphasises the need for universal teacher belief in growth mindset theory, and the elimination of a persistent belief in a 'maths gene' and a narrow idea of what it is to be good at mathematics. Boaler's strategies were shown to be effective in improving mathematical learning for pupils previously labelled as low achieving (e.g. Watson and De Geest, 2005). Her claims and data have however been disputed by others (e.g. Bishop, Clopton and Milgram, 2012).

Guskey and Gates' 1986 meta-analysis of mastery programmes reported limited impact on student self-efficacy but an increase in teacher enthusiasm. Teachers were more likely to support growth mindset, felt less able to predict student achievement prior to the start of a programme and described a 'humbling effect' (p. 34) of an increased sense of responsibility for student achievement, coupled with a decrease in confidence in their ability to ensure student success.

There is limited prevalence of completely mixed-attainment classes in mathematics mastery education at secondary level. East Asian education systems do have mixed-attainment classes within school, but students attend different secondary schools according to their prior attainment level. In Singapore, 65% of students attend an 'Express' secondary school, 25% of students attend a 'Normal-Academic' school, and the lowest attaining 15% of students attend a 'Normal-Technical' school (Leong, Ho and Cheng, 2015). In Shanghai, 20% of students receive a vocational, rather than academic, high school education (OECD, 2016). The NCETM 'neither encourages nor discourages streaming, setting or mixed attainment teaching at KS3' [sic] (2018, p. 1).

Growth mindset strategies favour tailoring learning activities to the needs of the learner. Mastery pedagogy advocate much less flexibility. Boylan, *et al*.'s, 2018 report on the Shanghai Exchange programmes identified specific core components of Shanghai pedagogy. These were titled: Varied Interactive Teaching, Mathematically Meaningful and Coherent Activity, Engaging and Challenging for the Whole Class and Knowledge of Mathematical Facts and Language. Pedagogies included 'substantial whole-class teaching in multiple part lessons with varying forms of activity' and an emphasis on 'memorising facts, relationships and structures' (p. 7).

Jerrim (2015) advises caution in directly linking Chinese pedagogy with Chinese students' mathematical achievement. His study of why children of East Asian descent taught in a Westernised country (Australia) still outperformed their Australian peers, despite experiencing the same education system, identified key factors related to work ethic, extra tuition and taking part in mathematics competitions. Specific teacher pedagogies can therefore only play a limited part unless they foster a wider impact to learners and their families. This parallels Keller's programme advocating tutor and family involvement as an important aspect of learning for mastery.

Whilst mastery in mathematics is universally associated with all children succeeding in mathematics, tensions exist between pedagogies supporting growth mindset and universal attainment of fixed learning goals. Teachers demonstrating fixed mindset beliefs may consider mastery as distinguishing accurately between right or wrong answers, then selecting the correct set of rules and procedures to follow in any given situation and reflecting on and refining choices made. In contrast, teachers with growth mindset beliefs may perceive mathematics learning as fundamental to the development of the whole child and would thus allow choice in, and reflection on, the mathematical rules, methods and algorithms used to solve mathematical problems. There is a need to research how practicing teachers consider mastery in relation to student achievement and mindset, and how classroom practice facilitates this.

2.5.2 Should mastery be norm referenced or criterion referenced?

To master something means to 'know a lot' or 'do something well'. Is this measured in comparison to what can be known or the knowledge of other people? Literature favours mastery as criterion referenced but does not clarify whether teachers agree with this, nor how they measure this.

Bloom (1968) considers mastery as a 'minimum level of attainment for progression' and providing appropriate time, resources and teaching for every student to achieve this given level of attainment (Bloom, 1968). Drawing on Carroll (1963), Bloom claims mastery as able to positively skew the distribution of attainment so that 95% of students will achieve the good standard that only 1/3 reached before. Gentile and Lalley (2003) claim mastery as understanding a specific domain of knowledge relative to all that could be known rather than relative to the performance of others. The level of knowledge deemed to represent mastery therefore depends on the type of assessment given, the age of the children, and the time interval between learning and testing. A mastery level is assumed to be 'high' enough to be able to integrate the new knowledge with past topics and those yet to be taught. The EEF (2018a) specify mastery as scoring 80% or more on a test. This may have derived from Rosenshine's (2010) seventh principle of instruction, 'obtain a high success rate', though I could not find the origin of this in his references. Cross-subject research by Semb (1974) suggests that a high mastery criterion (they used 100% with a low comparison of 60%) coupled with more frequent, shorter assessments produces the highest student examination scores at the end of the module. Keller's own 1968 mastery programme describes 'readiness testing' as pass/fail, with borderline tests being decided through a short accompanying oral examination distinguishing major misconceptions from minor errors.

There is evidence that English mathematics education policy supports achievement as criterion referenced. In England, the statutory assessments for early years (age 5), year one phonics (age 6), Key Stage one (age 7), multiplication tables (age 9) and Key Stage two (age 11) now describe whether the student has 'met' or 'not met' an expected standard. Standards for initial teacher training in England require teachers to ensure 'pupils *master* foundational concepts and knowledge before moving on' (DfE, 2019a, p. 13, emphasis added). This guidance references 'cognitive load theory', a growing movement in English education advocating students commit facts to long-term memory, freeing their limited working memory to make connections, think and reason (Willingham, 2009). Cognitive load theory suggests a mastery level of understanding, which includes being able to solve complex problems, can only result if most of our knowledge is already stored in the long-term memory. (Martin, 2016; Paas, Renkl and Sweller, 2003).

2.6 Mastery and the primary elements of Ernest's ideology: a summary

The review of mastery literature in relation to Ernest's primary ideology elements identified multiple notions of mastery related to philosophy of mathematics, educational aims, or theory of the child and their potential. Dependent on one's philosophy of mathematics, mastery could mean acquiring a predetermined level of mathematical knowledge, the ability to critique and add to this knowledge, or application of knowledge to diverse subjects. Mastery programmes, or countries which England associates with mastery, encompass diverse educational aims including raised achievement, reduced inequality, and economically productive citizens.

Ernest's mathematics education ideologies segment educators by their views on a child's status and potential. The section above highlights that programmes were created on different assumptions about children's capabilities and whether they are about equity (all children reaching a similar level of attainment) or equality (each child reaching their own individual level of potential).

This review of literature confirms that different mastery programmes have different assumptions and that no previous research has asked practicing teachers about their views of mastery. The following gaps exist in the literature:

- How a teacher's philosophy of mathematics influences their opinion of mastery,
- How a teacher's opinion of mastery is influenced by international mathematics education practices, historical and contemporary mastery programmes,
- How practicing teachers consider mastery in terms of the (re)distribution of student achievement.

2.7 Meanings of mastery: Secondary elements

Ernest's secondary elements of his ideology derive from primary elements: teachers' pedagogical choices are informed by beliefs, rooted in their own mathematical philosophy, educational aims, and theories of the child (1991, p. 132). I replicated this in my framework (see Table 2.1) and so the review of literature categorised mastery with pedagogical choices relating to curriculum, instruction, small-steps and variation, fluency, and representation. Teacher professional development is an additional category.

'Western' and 'Eastern' mastery emphasise different teaching practices, for example formative assessment (US), and representation, variation and collaborative CPD (China and Singapore). English mastery adopts aspects of worldwide programmes. The government funded NCETM programme is inspired by mathematics practices in Shanghai and the White Rose mastery programme and its materials are an offshoot of this. Ark schools' Maths Mastery was founded on Singapore mathematics. The Complete Mathematics mastery programme is a development of La Salle's programme, and shares much of its cyclical nature with the assessment cycle initially described by Bloom and Guskey. Since teachers will have been exposed to different programmes, and of course have their own pedagogical beliefs, it is probable that they will form their own opinions about mastery. There was no evidence in the literature that teachers' opinions had yet been sought, hence the need for this research study.

2.8 Theory of learning and theory of social diversity: mastery and the curriculum

Ernest's theory of learning ideologies categorises teachers by the pedagogical choices they believe constitute effective teaching and learning. Their beliefs affect decisions such as whether the curriculum offered is the same to all students or differentiated according to (for instance) prior attainment, social class, or career aspirations. Since English mastery programmes have different core influences there are multiple curriculum choices that teachers could associate with mastery.

The current mathematics national curriculum in England changed substantially in 2014 to greater resemble that of South Asia. English maintained schools follow a national mathematics curriculum with prescribed content, but schools are 'only required to teach the relevant programme of study by the end of the key stage' and 'have the flexibility to introduce content earlier or later than set out in the programme of study' (DfE, 2013, p. 4). Hence, provided that schools teach specific content by the ages of 7, 11, 14 and 16 (the end of the 'Key Stages' in England) they have a degree of freedom in curriculum design. Curriculum content was heavily influenced by a 2008 report commissioned by the (then) Department for Children, Schools and Families (now the DfE) into the previous curriculum. The report summarised that although England covered the same topics in number and algebra as comparison countries, there was more geometry and data handling and less emphasis on connecting topics (Ruddock and Sainsbury, 2008). It is important to note that the current Singapore curriculum is heavily influenced by the recommendations of a seminal British 1982 report into the teaching and learning of mathematics, known as the Cockcroft report (Boylan, *et al*, 2019; Boyd and Ash, 2018; Brown, 2014).

A mastery curriculum is planned so more time is spent learning fewer topics. Bruner's (1965) 'spiral' curriculum consists of common themes repeatedly revisited at a different age-appropriate level throughout a child's education. A mastery curriculum makes this spiral 'short and fat'. The England-based Association of Schools and College Leaders (ASCL) report a mastery curriculum should encourage 'more time on fewer things', keep children on a topic until all have learnt it, and focus on fluency problem solving and demonstration of understanding (ASCL, 2015, p. 2). Whilst a spiral curriculum deliberately allows specific themes to be repeatedly explored at different levels at different times in a child's development, Mathematics Mastery's curriculum claims 'longitudinal coherence' through spending longer time on fundamental concepts so there is no need to revisit them specifically later. This approach claims a better way of making connections and building understanding of mathematical concepts. For example, instead of exploring area and perimeter during a topic on shapes, perimeter is studied during addition and subtraction, and area is studied during the topic of multiplication (Drury, 2018). McCourt describes 'a curriculum planned for progression: a curriculum that ensures everything is taught correctly, first time.' (2019, p. 115). Mastery curriculum design in mathematics prioritises the understanding of number early on in education, with additional concepts only studied once this understanding is place. English schools can download mastery schemes of work (White Rose Maths, 2019), or choose from a selection of DfE-approved mastery textbooks based on the Singapore or Shanghai mathematics curriculum (e.g. MathsNoProblem!, 2018), match-funded by the government (Maths Hubs, 2018). These resources advocate a specific order of topics in the mathematics national curriculum.

A mastery curriculum is planned on the assumption that all children access the whole curriculum in their age range, regardless of their prior attainment level. There is evidence that different historical and contemporary mastery programmes attempt to achieve this in different ways. Bloom was motivated by a desire to change education so that 'most students (perhaps over 90 percent) can master what we have to teach them' (1968, p. 1). Keller wished to replicate principles of 'highly individualized' instruction, 'facilitating demand for perfection at every level of training and for every student' (1968, p. 79). Modern mastery curriculums include opportunities for experiencing topics at greater depth. Mathematics mastery has developed a 'dimensions of depth' framework, (Drury, 2018, p. 6), McCourt's mastery cycle features 'corrective teaching' and 'topic enrichment and enhancement' (2019, p. 26), and the NCETM's Teaching for Mastery assessment materials make a distinction between demonstrating 'mastery' and 'mastery at greater depth' (Askew, *et al.,* 2015).

Mastery curriculums are designed on the principles of 'fewer topics taught for longer', 'not moving on until all children have learnt the topic' and 'all children have access to the whole curriculum'. These principles are enacted in the classroom through schools' and teachers' choices in planning, teaching and assessment. These are related to schools' and teachers' own theories of teaching mathematics.

2.9 Theory of teaching mathematics

The reviewed literature exploring the meaning of mastery broadly agrees on the educational aim of 'all children having a good understanding of mathematics'. There is much less agreement on how this is achieved. According to Ernest's ideologies the 'Industrial trainer' teacher prioritises hard work and rote memorisation, the 'Old humanist' favours understanding application of knowledge, the 'Progressive educator' emphasises exploration and the 'Public educator' characterises questioning as effective learning (1991, p. 138).

A tension appears between teaching children versus teaching mathematics which is linked to mathematical philosophy. Ernest claims that teachers who prioritise the experience of the child may do so 'at the expense of mathematics, failing to develop mathematical concepts and structures to a sufficient depth to give children confidence in their use as tools for thought' (Mellin-Olsen, 1987, quoted in Ernest, 1991, p. 194). Watson (2018) feels reassured that contemporary mastery programmes do not have 'slave-master, schoolmaster' connotations but there is conflicting evidence in this area, in particular relating to the teacher-student relationship and transmission versus sharing of knowledge. Watson herself uses 'novices and experts' in her co-authored 2006 paper on variation. Rosenshine (2012) uses the phrase 'master teacher', also used by Anderson and Burns (1987).

The themes that emerged from literature on mastery practices are outlined in subsections below.

It is not yet known how teachers' theories of teaching mathematics affect their opinion of mastery (or vice versa). A research study to elucidate this should examine specific mastery teaching practices and clarify the relative importance of each aspect to the teacher.

2.9.1 Mastery and Instruction

The two US mastery programmes of the 1960s disagree about how much teacher direction should feature in mastery learning programmes. Bloom asserted 'it is the task of *instruction* to find the means which will enable our students to master the subject under consideration' (1968, p. 1), whilst Keller advocated 'the *minimizing of the lecture* as a teaching device and the maximizing of student participation' (1968, p. 79) (emphases added). Kulik, *et al.*'s, mastery learning meta-analysis included programmes associated with a high degree of teacher instruction (Kulik, Kulik and Bangert-Drowns, 1990). Horton described mastery as 'managing learning rather than managing learners' (1979, p. 154).

The inclusion of cognitive load theory in English teacher education policy has resurrected pedagogies favouring direct instruction versus discovery learning (e.g., Kirschner, Sweller and Clark, 2006). Contemporary mastery programmes assume a high level of teacher expertise in mathematics, and a means of directly communicating this to the learner. High priority is given to the explicit teaching of mathematical knowledge. The NCETM's Teaching for Mastery emphasises the importance of teacher-led whole-class teaching and all children working on the same material at the same time. The MathsNoProblem! government-approved textbooks include 'a teaching approach for each chapter' (MathsNoProblem!, 2018). Keeping the class together is consistent with the growth mindset assumption that all can achieve and a curriculum that prioritises learning in a specific order. Drury is sceptical about child-led learning practices, 'because there are significant differences between how experts and novices think' (2018, p. 176). Directing the students' attention towards aspects of mathematics that need to be learned first supports the development of 'domain-specific knowledge' which can be embedded into long-term memory.

This set of highly structured teaching approaches, underpinned by the assumption that teachers should directly convey knowledge to students, contrasts teaching methods underpinned by child-centred constructivist educational philosophies of learning and teaching described by Piaget and Vygotsky (Fosnot and Perry, 1996). Bruner warned of the potential of the formalist school learning to the interplay between intuitive, inductive, and analytic thinking. He favoured a degree of discovery learning and celebrating the teacher who is 'willing to guess at answers' (1965, p.62).

There is also debate regarding the explicit teaching of formal mathematical vocabulary. Mathematical communication is one of Mathematics Mastery's 'dimensions of depth' and NCETM's primary Teaching for Mastery report found that both teachers and children were using precise mathematical language. Not all educationalists think this is necessary: Bruner asserts, 'whether the student knows the formal names (of these operations) is less important for transfer than whether he is able to use them' (1965, p. 8).

It is not clear whether individual teachers interpret 'teacher-led whole-class teaching' in the same way, or how much they associate this with mastery learning. The greater emphasis on teacher knowledge of mathematics that inevitably goes with this is important. Boyd's and Ash's investigation into how a Singapore-style mastery approach changed primary teachers' beliefs found that the structure of textbooks freed up time for them to develop greater subject knowledge, and it was this that 'revolutionised' their teaching, more than the structure itself (2018, p. 218). The teachers perceived that instead of instructing the students they were facilitating the learning, giving the children the best opportunity to reason and think mathematically. The NCETM report on its Teaching for Mastery programme found that teachers gave increased importance to their subject knowledge of mathematics and were using concept mapping and sequencing far more in their planning. They also acknowledged that this took additional time (NCETM, 2019). A research study investigating teachers' opinions of mastery learning should include their views on the components and efficacy of a whole class teaching approach.

2.9.2 Small-steps and variation

Learning, directed by teacher or students, is a process. Vygotsky (1978) described learning as a journey between what is currently known and what is not yet known. Willingham (2009) draws on neuroscience to describe learning as a connection made between currently studied content with something in the long-term memory, and Kirschner, *et al.*, (2006) defines learning as an alteration in the long-term memory. They reinforce this with the statement: 'If nothing has been altered in the long-term memory, nothing has been learned.' (p. 75). If mastery learning is ensuring all children can gain a deep understanding of the whole curriculum, the teacher must craft a learning journey so all students can embed new material into their long-term memory and thus learn the concept.

The NCETM, based upon teaching in Shanghai, articulates a learning journey for mastery as 'coherence' (2016). Lessons should be planned and taught to build a concept in small steps, highlighting the connection between them. This leads to children being able to generalise. The Chinese pedagogy that influenced coherence is the construction by the teacher of a set of steps supporting the learner to reach new learning, known in Chinese as a 'Pu Dian' (Huang and Li, 2017). The Pu Dian is used by learners to solve the 'next step' without being explicitly taught a strategy, thus eliminating the need for further instruction. Chinese mathematics lessons incorporate a question which enables student to demonstrate this, called 'Don Nao Jin'.

Coherence is a distinct aspect of NCETM teaching for mastery, but in Chinese pedagogy it is articulated within strategies appearing under the umbrella term of 'teaching with variation'. The NCETM highlights variation as a core component of coherence (2016). Variation as a pedagogy can be traced back to Chinese philosophy. The 'Zhoubi Suanjing' text of mathematics and astronomy circa 100BC describes teaching as follows: 'Similar methods are studied comparatively, and similar problems are comparatively considered. This is what sorts the stupid scholar from the clever one, and the worthy from the worthless.' (Cullen, 1996, p. 178, cited in Huang and Li, 2017, p. 15).

Variation theory unpicks the 'core connection' between a learner's starting point and the new knowledge being presented and can be split into 'conceptual variation' and 'procedural variation' (Huang and Li, 2017). Conceptual variation explores the limits of the topic being studied and is described by Leung (2013) by 'difference, similarity, and sieving'. Understanding is built as a series of simultaneously existing situations using standard and non-standard examples, for instance, when considering the question, 'what is a triangle?' (Gu, Hunag and Marton, 2004). Procedural variation builds understanding through exploration of a series of interconnected examples and exercises (ibid). The learner 'proceeds' (as opposed to following a procedure) with their knowledge and understanding by engaging with a series of examples that keeps all but one variable the same.

Planning and teaching for coherence and variation is compatible with teacher-directed pedagogy. Teachers must have good knowledge of both mathematics and education, or 'Pedagogical Content Knowledge' (Shulman, 1986). Watson and Mason (2006) describe the teacher who uses variation effectively as an expert guide who leads the student novices to uncover the 'essence of mathematics' through micro-modelling, and promoting the teacher's approach to mathematical thinking (p. 106). Teachers must create or utilise an 'instructional example space' (Goldenberg and Mason, 2008) including a variety of examples and non-examples for each concept. They must also be precise in their structure of explanations and exercises so learning emerges from the whole lesson (including connection between episodes) rather than individual items (Watson and Mason, 2006).

Conceptual and procedural variation can be utilised to develop mastery in teaching episodes and pupil exercises. Hewitt (1996) describes variation as 'learning through subordination'. By practising a skill through doing something different, a student comprehends and attempts a task without first being a master in all aspects. Lai and Murray (2012) claim using variation in teaching and practise episodes allows children to continually experience a phenomenon in a new light. Variation can 'avoid mechanical repetition and create an appropriate path for practising the thinking process with increasing creativity' (Gu, Hunag and Marton, 2004). The NCETM encapsulates variation in student work with the term 'intelligent practice' (NCETM, 2017), Maths Mastery uses variation as a lens to ensure exercises both deepen understanding and embed fluency (Drury, 2018), and Barton recommends the use of 'minimally different examples' (2018, p. 249).

Teachers need to create appropriate opportunities for pupils to construct a concept through noticing similarities and differences (Askew, 2012). This invites engagement, conceptualisation, generalisation, and abstraction, leading to a deep learning (Watson and Mason, 2006). Whilst complex problems are reduced so students can solve them (Polya, 1957), Foster (2013) warns that excessive reductionism by the teacher leads to a persistent inability of students to problem-solve. Foster argues that a teacher with deep pedagogical content knowledge will resist providing small steps for all learners, instead allowing them first the opportunity to engage with problems in their own way and responding to specific questions that learners should be asking, as part of them constructing their own steps. Watson (2000) uses a piece of wood as an analogy to illustrate this. An exercise that 'goes with the grain' has an identifiable pattern allowing students to complete future terms, whilst an exercise that 'goes across the grain' is crafted in such a way that students can see the generalisation of the concept in question.

Teachers engaging with the NCETM Teaching for Mastery find incorporating variation theory into their practice challenging, and how big or small to make the 'steps' (NCETM, 2019). A research study investigating teachers' opinions of mastery learning will aim to understand what coherence, small-steps and variation mean to teachers in the context of mastery learning.

2.9.3 Mathematical fluency and reasoning

The NCETM lists fluency as one of its five big ideas for teaching for mastery (2016). Fluency is defined as being able to solve mathematical problems efficiently, accurately, and flexibly: choosing the best method for a given situation and minimising the number of intermediate steps needed to arrive at an answer (Russell, 2000). Fluency was highlighted as important in the Cockcroft report and has been an aim in the English mathematics national curriculum since its inception. The USA National Mathematics Advisory Panel report deconstructs mathematical learning into acquisition of procedural and conceptual understanding with the addition of factual knowledge, defined as that which can be directly retrieved from memory without further thought (National Mathematics Advisory Panel, 2008). Cognitive load theory claims to commit knowledge to long-term memory requires instruction and practice, and fluency has developed once the relevant facts can be retrieved from the long-term memory without excessive time and effort (Kirschner, Sweller, and Clark, 2006). It is not known how teachers associate fluency with mastery, and how learning something to automaticity (for instance memorising multiplication tables) enhances deep learning beyond the recall associated with rote learning.

Mathematics mastery programmes stress that developing fluency is different to rote learning, where facts are committed to memory without the relevant understanding. Teaching for mastery aims to develop fluency and conceptual understanding in tandem, utilising variation theory and intelligent practice to do so. Key facts and formulae are learnt to automaticity to avoid cognitive overload and to help students move between different contexts and representations of mathematics (Drury, 2018; NCETM, 2017; NCETM, 2016). The combination of developing fluency in tandem with conceptual understanding is known as 'the paradox of the Chinese Learner', and an explanation for the continued success of students whose education appears to be dominated by rote-learning and repeated practice (Lai and Murray, 2012).

Learning of key facts alone is not sufficient. A 2020 study of year 9 low attainers reported that they were relatively stronger in arithmetic recall compared to other tested areas of mathematics (Hodgen, *et al.*), and an EEF-commissioned report into effective Key Stage 2 and 3 mathematics teaching in England found that effective fluency practice included comparison of strategies and reasoning alongside recall (Hodgen, Foster and Kuchemann, 2018). Merttens (2012) claims that the Singapore curriculum encourages rote-learning and procedural fluency at the expense of conceptual understanding.

A research study investigating teachers' opinions of mastery learning should investigate what fluency means to teachers in the context of mastery learning, and its contribution to the development of mathematical understanding.

2.9.4 Mathematical representations

Pedagogy associated with mastery includes the representation of mathematics in multiple ways to clarify and embed pupil understanding. The NCETM has 'representation and structure' as one of its big ideas, and advocates using models, images, and diagrams at secondary level (NCETM, 2017). Mathematics Mastery claims that understanding deepens from 'representing concepts using objects, pictures, words and symbols' (Drury, 2018, p. 8).

The use of multiple representations was highlighted in Bruner's (1966) *Theories of Instruction*. Bruner claimed children make sense of the world by building both physical and mental representations, categorised as 'enactive' (physical movement), 'iconic' (use of imagery relatable to the physical) and 'symbolic' (use of a letter, number or symbol unrepresentative of the observed situation). Learning is not a sequential exploration: children move 'in and out' of different representations, and true understanding is reached when someone is able to work in all three categories with confidence.

Bruner's research into multiple representations is most clearly seen in the explicit 'Concrete-Pictorial-Abstract' (or CPA) approach endorsed in the Singapore mathematics curriculum (Leong, Ho and Cheng, 2015). CPA claims a one-to-one correspondence with Bruner's representations, with 'Concrete' including concrete experiences as well as physical manipulatives (for instance 'real-life' mathematics). CPA does however imply an order for teaching and representing different 'levels' of understanding. Davydov's (1982) analysis of understanding is more circular. Understanding develops as repeated cycles of observation, representation and analysis allowing modification and refining of past learning.

The Singapore approach to CPA approach follows a pre-designed textbook scheme and teacher-led modelling of a problem a specific choice of representations (e.g. MathsNoProblem!, 2018). Bruner and Davydov situate their models of representation within child-led discovery learning. Davydov (1982) warns against too much inappropriate teacher intervention at the observation stage since there is a danger of a 'master' teacher telling children what they *should* be observing, thus disrupting the cycle. The EEF report into effective Key Stage 2 and Key Stage 3 teaching found that teacher prompts, or questions about a child's choice of representation, developed their independence and motivation (Hodgen, Foster and Kuchemann, 2017).

Whilst the use of different representations is advocated in mastery learning programmes, it is not clear whether the choice of representations should be made by the teacher or child. A research study exploring teachers' opinions of mastery should investigate their views on this.

2.10 Mastery and the secondary elements of Ernest's ideology: a summary

The review of mastery literature in relation to Ernest's secondary ideology elements identified multiple notions of mastery related to theories of learning and teaching. Central to this is the degree to which classroom learning is directed by the student compared with the teacher. Mastery programmes, or countries which England associates with mastery in the teaching of mathematics, favour a uniform curriculum and teacher-directed approach. This contrasts with much of teaching in England which has historically differentiated the curriculum experience and been more student-directed.

Literature confirms that mastery programmes may promote specific teaching ideas, and ambiguity in how the individual teacher enacts these ideas in their practice. A research study is needed to ask teachers which mastery principles they consider to be important, and how these principles affect their practice. The categories of uncertainty are:

- How much mastery is associated with teacher-led instruction,
- How much mastery is associated with planning and teaching for coherence,
- The importance and use of variation theory in mastery,
- The role of fluency in leading to a deep understanding in mastery learning,
- The role of mathematical representations in mastery, and whether these are selected by the teacher or student.

2.11 Meanings of mastery: teachers as professionals

The aims, values, and practices of teaching for mastery programmes can be embedded in teachers' beliefs and practice through professional development activities. Therefore, an important aspect of mastery learning programmes is the professional development of teachers. Multiple models of teacher professional learning exist, and all have strengths and limitations. Boylan, *et al*.'s (2018) review of professional learning programmes advocate choosing a model based on a number of factors, including the philosophical basis of the professional development activity. This links professional development models with Ernest's ideology: teachers' opinions on the principles and practices of mastery will be a factor in the effectiveness of mastery professional development.

Contemporary mastery programmes in England have specific programmes to develop mastery within schools or mathematics departments over time, rather than enhancing the practice of individual teachers, in line with DfE guidance on school professional development (DfE, 2016a). English programmes have incorporated aspects of Asian professional development, within national constraints. In Shanghai and Singapore teacher development is a continuous journey, rather than a specific programme with an end point, such as England's Qualified Teacher Status. Mathematics teachers in Shanghai and Singapore are allocated between three and ten times as many hours for professional development activities per year than mathematics teachers in England (Jain and Hyde, 2020).

Subject-specific professional development is simultaneously recognised as important and under-provided in England (Cordingley *et al.,* 2018). The longitudinal report on the impact of Shanghai teacher exchange reported an aspect of mastery pedagogy adopted by all the English schools that took part was school-based mastery professional development. This activity was characterised as focused on understanding of mathematics concepts and how to teach them. Collaboration between all teachers in the school was important (Boylan *et al.,* 2019).

Professional development, focused on teacher subject knowledge, is an important aspect of mastery programmes. In Shanghai, entering teaching is highly competitive and teacher subject knowledge is high. Primary and secondary teachers have at least an undergraduate degree in their teaching subject, and many have Masters degrees (ibid). This means that teacher education and development begins assuming all teachers have a deep understanding of fundamental mathematics (Ma, 2010). In England, primary and secondary teachers might have stopped learning mathematics at age 16 or 18 respectively, thus within a school or department there will be teachers who are insecure and unconfident in their subject knowledge. The NCETM report into the early impact of their Teaching for Mastery programme found participating teachers highly rated the importance of subject knowledge in being a good teacher of mathematics. Teachers worked with colleagues to critically reflect on their own and each other's knowledge and what they needed to work on (NCETM, 2019). Boyd and Ash (2018) found that teachers spent more planning time on subject knowledge consideration, so they could present problems and respond to student questions more effectively.

Mastery professional development activities are focused on making changes in schools or departments. In Shanghai, teachers are not considered 'senior' until after about ten years. Important aspects for teachers wanting to progress up the career ladder include experience in both teaching and supporting other teachers in disadvantaged areas, and undergoing the required in-service training (Zhang, Ding and Xu, 2016). There is an expectation that departments and schools will work together to develop teachers' expertise over time. In England, senior leaders can be unable or unwilling to allocate time and resources to in-school development, and this can be compounded by finite in-school expertise (Cordingley *et al.,* 2018). English mastery programmes are therefore a combination of external and internal activity. The NCETM Teaching for Mastery programme has a three-year input before schools are self-sustaining in relation to mastery (NCETM, 2019). The Mathematics programme includes a 'moving to mastery' year (Jerrim and Vignoles, 2016, p. 31).

In Shanghai, in-school professional development consists of lesson planning groups, grading groups, and teacher research groups, or TRGs (Zhang, Ding and Xu, 2016). The NCETM introduces schools to mastery through a TRG-style Work Group model. Components of these Work Groups include joint planning, observation, and discussion activities. The NCETM's 2019 report summarising the development of the Teaching for Mastery programme associated these groups with positive changes to subject and professional knowledge, planning, teaching and assessment practices and whole-school policy and curriculum. The impact on students was harder to measure (NCETM, 2019).

There is limited evidence of how teachers in schools consider which professional development activities they most associate with developing mastery practices. Within the structure of the professional development programme, activities to develop mastery may focus on one of the characteristics of mastery or on making changes to school or developmental practice. A research study investigating teachers' opinions of mastery practices should include teachers' views on their mastery professional development.

2.12 Categories of mastery

The completed literature review explored published articles about aspects of mastery that are listed in the second column of Table 2.1: mathematical philosophy, mastery programs in the US, Singapore, Shanghai and England, assessment of mastery, mastery curricula, instruction, small-steps and variation, fluency, representation, and teacher professional development. There were of course many areas of overlap in these categories (for example the emphasis on assessment within US mastery programmes, and the link between Chinese teachers' greater emphasis on teacher subject knowledge and professional development, and 'small-steps' planning and teaching). These overlaps needed to be disentangled before the research study design process could continue. Chapter 4 describes in more detail how these literature review findings informed the creation of a set of statements, used in the research study, to investigate teachers' individual views of mastery.

2.13 Conclusion

The literature review undertaken for this research indicated that mastery has multiple meanings, which are categorised in a framework based on Ernest's classification of mathematics teacher ideologies. The reviewed literature for mastery in each section of the framework outlined the diversity of views held by teachers within that category and confirmed a lack of previous studies that investigate what teachers think.

The research question, ‘What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?’, was addressed by using these categories as a ‘starting point’ for constructing a Q methodology research study. Study participants revealed their individual views on mastery through their relative placement of a set of appropriate statements against each other. The study’s methodology, and how the literature review categories of mastery were used to construct the study, is explained in Chapters 3 and 4.

3 Q methodology

3.1 Introduction

This chapter explains Q methodology as the chosen methodology for my research study. Although this chapter follows the thesis' rationale (Chapter 1) and the literature review (Chapter 2), I discovered Q methodology very shortly after deciding to investigate mastery. As mastery became an 'itch to scratch' (see Chapter 1.8), so Q methodology became the sharpened fingernail. My research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?' is 'an exploration of specifics', and thus exactly the type of question that Q methodology addresses (Watts and Stenner, 2012, p. 54). Choosing Q methodology also appealed to my background in STEM: like Williams (2020) I still favour research that is more empiricist whilst understanding the need for other approaches.

The chapter begins with a short introduction to Q methodology including its origins and how it mixes quantitative and qualitative elements to understand categories of views 'at a point in time'. The chapter discusses positionality and my claim to a postpositive position in this research project, through unpicking how my choice of Q methodology and the design of my study aligned with a realist ontology and modified objective epistemology. The chapter concludes by highlighting that this study showcases the potential of Q methodology in educational research, and that the study's design led to findings that could only have been discovered using Q.

3.2 Q methodology

As hinted already in Chapters 1 and 2, I designed a Q methodology study to address my research question. Q-methodology is a scientific framework for studying subjectivity, using factor analysis to identify correlations in opinions between different people, and thus identify significant groups of people with shared points of view (Stephenson, 1953). Q methodology reverses the view of the sample and the variables, thus allowing statistical analysis of studies of modest numbers of human participants (Watts and Stenner, 2012; Stephenson, 1953). From a participant perspective, completing a Q methodology study resembles a ‘card sort’ or ‘diamond nine’, two popular activities within English education. Research participants sort a set of statements, the ‘Q sample’, to indicate their relative agreement or disagreement with each statement. The ranking of each card, in relation to the others, by each research participant becomes the sample, and the participants are the variables. Factor analysis techniques are then employed to identify distinct groups of people that share similar views (Watts and Stenner, 2012).

Q methodology allows research participants to uncover and express their opinions of mathematics mastery through their completed ranking of the statements, or Q sort. The literature review in Chapter 2 was used to create the ‘concourse’, a large set of statements which encapsulate the universe of views that teachers could hold about mastery. This concourse was then reduced to a Q sample of 48 statements. The process of creating the concourse and Q sample is detailed in Chapter 4. A single mathematics teacher’s attitudes and beliefs in relation to learning is varied and complex but can be captured in a ‘card sort’ because the number of unique sorting combinations is in the order of at least 10^{13} (Brown, 1980, p. 266). Using the sorting combinations of the cards as the sample, and the research participants as the variables means that the number of teachers recruited for the study needs only to be in the range of 30-50. Each teacher sorts the Q sample according to the degree of agreement they have with the statements in relation to each other, regardless of the absolute level of the agreement, and not in comparison to any other teacher’s answers.

3.2.1 Q methodology and the ‘factor’

The word ‘factor’ appears frequently in Q methodology literature. Consequently, readers of this thesis will be confronted with the word in different contexts. The thesis glossary contains concise definitions for ‘factor’, ‘factor array’, and ‘factor rotation’. A more comprehensive explanation for the different ways that ‘factor’ is used in the thesis is below.

In a statistical sense, a ‘factor’ is the outcome of a statistical iteration process which identifies a configuration of statements that match the opinion of a sub group of a study’s participants. Once one factor is found, the process is repeated to find another configuration of statements that matches a different sub group, and so on. A perfect statistical solution would result in all participants’ sorts closely matching one factor. In this study, the iteration process generated a solution of four specific configurations of statements, or four ‘factors’.

Once the statistical analysis is completed, a ‘factor’ therefore represents a specific idealised viewpoint about (in this case) mastery teaching and learning. A sub group of study participants hold individual viewpoints of mastery which correlate with this factor (though each participant’s viewpoint is of course unique). At this point the researcher can generate a ‘factor array’ (card sort) as a visual display of the relative placement of each statement, and can identify the characteristics of each sub group of participants.

After a factor array has been generated, and participant characteristics identified, a factor is thought of less as a statistical outcome, and more of a personification of an opinion. Researchers then choose how far to take this personification: factors can be given neutral identifiers (A, B, C), human names or comparative descriptors (hawks and doves). Q researchers need to balance the need for objectivity with effective communication to readers (Mauldin, 2020). I chose to navigate the transition of a factor from ‘a math vector to a real person’ (ibid) by creating a descriptive ‘one liner’ for each factor but referring to Factors 1-4 in the thesis text. In addition to the conventional ‘factor array’, I have created additional representations of the factors to aid readers (see Chapters 6 and 7).

3.3 Positionality

I claim the design of the research study as a postpositivist application of a mixed-method methodology.

Unlike other methodologies built with fixed positions of ontology and epistemology, the Q researcher must take, and defend, their own position when designing their study, because Q methodology researchers are yet to agree on the philosophical position of Q methodology. William Stephenson, a psychologist with an additional PhD in Physics, created Q methodology to systematically study subjectivity as a methodology in its own right (Stephenson, 1953). Stephenson applied the (then) emerging field of quantum mechanics to human decision-making and argued that although human behaviour, like the path of individual electrons, was ultimately unpredictable, statistical analysis could be used to generalise types of views held by a group of people in relation to a particular subject. Brown (1980) maintains that Q stands by itself as a unique methodology with aspects of psychology, statistics, and philosophy of science principles. Stenner and Stainton Rogers (2004, p. 166) term Q a 'qualiquantology'. Contemporary Q methodology literature describes Q as inherently mixed method (e.g., Lundberg, de Leeuw and Aliani, 2020; Ramlo, 2020).

A lack of agreement about the positionality of Q studies leads to ongoing, unresolved debates about Q study design, application, analysis, and generalisability. Professor Susan Ramlo, a prominent Q methodology researcher, undertook her own study of Q methodology researchers and reported three views: an 'inherently mixed-focus' group who celebrated the qualitative aspects of Q, the 'quantitative-focus' group who celebrated quantity and objectivity within Q, and (a single) 'skeptical novice' position (Ramlo, 2019). My position puts me in the 'quantitative-focus' group.

3.3.1 Definition of postpositivism as applied to the research study

'Seekers after enlightenment in any field do the best that they can; they honestly seek evidence, they critically scrutinize it, they are open to alternative viewpoints, they take criticism seriously and try to profit from it, they play their hunches, they stick to their guns, but they also have a sense of when it is time to quit. It may be a dirty, hard and uncertain game, but it is also the only game in town'.

(Phillips, 1990, p. 38-39)

A postpositivist position assumes that reality exists (a realist ontology) and acknowledges a limited potential of a person, laden with values and ideologies, to observe this reality (a modified objective epistemology). Thus, the limitations of both the researcher and the participants to perceive and interpret the world as it externally exists must be, and are, acknowledged and woven into the research methodology, analysis, and interpretation.

Alvesson and Sköldberg (2017) describe postpositivist research as observing and explaining patterns emerging from a deeper reality. Instead of defining postpositivism, researchers favour a comparison to positivism (the belief in an evidence-based reality that can be mathematically interpreted). Postpostivism is described by Guba (1990) as a *modified* version of positivism and by Fox (2008) as a *successor* to positivism. Both authors claim that postpositivism favours a scientific method to research, whilst rejecting the classical positivist claim that scientific facts can totally be separated from values, particularly in social science research. A range of methodological positions lie within the paradigm of postpositivism: by using the word 'modified', Guba claims postpositivism as a contemporary application of the principles of positivism within a wider notion of scientific disciplines, whilst Fox, in choosing the term 'successor', dismisses these principles and sees postpositivism as an acknowledgement of the impossibility of explaining the world through scientific methods.

This research project is educational research, sitting within the discipline of social sciences. Whilst Fox (2008) completely rejects reality and objectivity in the social sciences, Phillips (1990) does not, claiming although different people and societies may claim different views of reality, at most only one of them can be correct. The classical positivist, Comte, refrained from dismissing sociology from the sciences, maintaining that to understand society, one must use the tools afforded by the fundamental sciences, with mathematics being at the foundation. He thus placed sociology at the 'particular and complex' terminus of his classification (with mathematics at the 'general and simple' end). (Bourdeau, 2020). The science philosophers, Popper and Kuhn, who both questioned the ability of the scientific method to uncover the complete truth whilst advocating that it could be modified to explore the limits of observable truth, were both postpositivists. I acknowledge that within social science falsification as proof is impossible, as it is impossible to test all people in the past, present or future in order to find one counter-example to totally disprove a theory, preferring the possibility of a 'best opinion' outcome (Pritchard, 2013).

Postpositivism is valid within the explanatory educational research that I am undertaking. Durning (1999) identifies postpositivist methodologies as appropriate means to categorise preferences amongst a diverse body of people to obtain policy insights, consistent with my own research aims. Durning also claims that Q methodology studies are an important addition to 'toolkit' summaries of randomised controlled trials and rich, narrow conclusions of practitioner action research. Baker, *et al*, (2010a) segments Q studies into analysis (the quantitative work up to and including the description of the identified groups) and interpretation (the qualitative work that informs theory, policy, or practice). Baker, *et al*, (2014) addressed this with a 'two-stage' research project incorporating a follow-up quantitative survey to determine the distribution of Factor viewpoints in a wider population. My approach maintains transparency within the interpretation phase of the project, with bias acknowledged.

A postpositivist position remains uncommon within Q methodology, but I argue it has a valid place in contemporary Q studies. Multivariate analysis requires large sample sizes to compensate for sampling errors, standardisation difficulties, the presence of another unknown variable and to minimise data 'noise' caused by random behaviour (Stephenson, 1953; Watts and Stenner, 2012). The 'inverted factor analysis' of Q methodology statistically analyses the degree to which the sample observations (the placement of the Q sample cards) are accounted for by the variables (the 'Factors', which account for groups of participants). Q methodologists acknowledge the quantitative and qualitative aspects of their studies. Ramlo and Newman (2011) claim Q methodology is 'in tune with the more modern conceptions of mixed-methods research' (p. 173).

3.3.2 Claim to postpositivism

Ramlo's and Newman's (2011) paper discusses the methodological position of Q in relation to two models: a qualitative-quantitative continuum (Benz, Ridenour and Newman, 2008) and a set of postpositivist-constructivist continuums (Tashakkori and Teddlie, 2008). Tashakkori's and Teddlie's (2008) continuums, reproduced in Figure 3.1, analyse the positioning of research studies in relation to eight different criteria. Ramlo and Newman place Q methodology in the middle of the continuum for every criterion except the objective/subjective (they placed Q as wholly subjective), thus claiming Q as a 'mixed research methodology' (2011, p. 183). I find my own research study much closer to the Quantitative Extreme (Post-positivist) [sic] terminus in relation to these criteria.

Quantitative Extreme (Post-positivist)		***Qualitative Extreme (Constructivist)***
Objective purpose	⟷	Subjective purpose
Explanatory	⟷	Exploratory
Numeric data	⟷	Narrative data
Structured/close-ended	⟷	Open-ended
Statistical analysis	⟷	Thematic analysis
Probability sample	⟷	Purposive sample
Deductive inference	⟷	Inductive inference
Value neutral	⟷	Value rich

Note: Simplified version of the multidimensional continuum of research projects, adapted from Tashakkori and Teddlie (2009) in order to focus on specific aspects of Q methodology.

Figure 3.1: Multidimensional Continuum of Research Projects, adapted from Tashakkori and Teddlie (2009), taken from Ramlo and Newman, (2011, p. 181)

3.3.2.1 Modified objectivity and a value neutral approach

This study maximises objectivity within the constraints of studying opinion. I claim a value neutral approach: I identify and acknowledge my own values and biases to report impartial findings and implications. I acknowledge that my professional roles (within teacher education and professional development evaluation) require me to have asymmetric knowledge of mastery, such as the NCETM's programme, and pedagogic practices prescribed by English education policy, but I make no claim to favour one approach from another. I minimise bias in my participants' responses through anonymous recruitment, conducting the Q sort online, and using a pre-sort questionnaire which identifies characteristics based on choices made by participants, not the researcher. I did not seek to confirm or challenge a particular view of mastery as preferable or problematic.

My choice to adopt a value neutral approach to this contestable and complex subject of mastery in mathematics increases the credibility of this research. I, as the researcher, am responsible for reporting on my educational findings without 'pushing' my own standpoint. However, it would be naïve to assume that 15 years of experience as a mathematics teacher, educator and researcher have given me no philosophical, political, or pedagogical opinions about mastery. My own set of opinions must subconsciously shape my interpretation of the findings.

I did consider alternative approaches, including undertaking the Q study myself. Q researchers who undertake their own study do for pragmatic reasons including identifying 'potential blind spots in observation, or as an acknowledgment that they hold a specific and knowable position (Brown 2020c). However, I do not believe, consciously at least, that I do hold a fixed position in relation to mastery: as I explained in Chapter 1, my reasons for choosing the focus and the methodology are borne from curiosity and a motivation to discover and explain. Undertaking the Q study myself would lead to a different frame of reference for observing and explaining the results. Whilst I do not know how the results or interpretation would be altered, choosing to remove 'myself' from the data interpretation was made with the intention of increasing objectivity in the data analysis and interpretation. I want to know how much my participants relatively 'agree' or 'disagree' with their statements, and it is of no practical significance to know my relative agreement or how much my participants agree or disagree with my own opinion.

Postpositivism claims a modified objective epistemology; although researchers attempt to stand outside their research as a 'regulatory ideal' (Guba and Lincoln, 1989, p. 21) we acknowledge that in practice, this is impossible. Our research design, observations and analysis are indelibly shaped by the world that the researcher, and their participants, live in. We exist within a specific paradigm (Kuhn, 2012) and domain (Toulmin, 1958) thus can only understand truth within the context of the social reality at a point in time.

I reject the claim that Q methodology is inherently constructivist. Watts and Stenner paraphrase constructivism as a dynamic process of observing a phenomenon in relation to ever-changing, self-selecting criteria relating to specific contexts (2012, p. 41). I do not claim to seek a dynamic understanding of a person's view of mastery, only a static representation of their true opinion in the time and circumstances of their sort completion. In the study Q methodology was used to quantify opinions, and what a person thinks about something (in this case, the notion of mastery in mathematics) is of course entirely subjective. This makes subjectivity and Q methodology 'indelibly connected' (Watts and Stenner, 2012, p. 25). However, Q methodology is a study of 'operant subjectivity' and thus is an objective snapshot of subjective decisions made at a particular point in time (Brown, 1980).

I can also, as the researcher, maintain objectivity in how I create the concourse and Q sample, analyse the participant sort data, and report the findings (Durning, 1999). The participants' views are subjective, but I endeavour to report an objective analysis of their views and identify real areas of agreement and difference. The detailed description of the research design and data collection and analysis in Chapters 4 and 5 document how I did not look at a single participant's sort until the collection phase had closed, and that I did not do any analysis of the participant characteristics data until I had finished analysing the findings. I also created an anonymous environment for completing the sort which, as much as possible, provided the same environment for each participant taking part in the study.

I do acknowledge researcher bias and subjectivity in how I describe the groups of opinions, or Factors, in Chapter 6, and in discussing the implications of the findings, in Chapters 8 and 9. To fully eliminate bias would dehumanise both myself as a researcher, and the human research participants that have given up their time to participate in the research. I would also be ignoring the context of my profession as a university lecturer and mathematics educator, and the professional background of the participants as reflective practitioners educated in both mathematics and pedagogy. Indeed, scientists do science in the knowledge that their judgements affect funding decisions and government policy, exposing them to potentially dangerous unconscious bias. If biases are unacknowledged then decisions which affect multiple stakeholders are not made transparently: 'there is no hiding from the swift sword of science' (Guba and Lincoln, 1989, p. 126). Hence acknowledging inherent biases in research, whilst they should be minimised, increases, not limits, objectivity and the strength of claims and conclusions. In this way postpositivist studies maintain an 'epistemic rationality' balance of maximising true beliefs whilst minimising false beliefs (Pritchard, 2013, p. 43).

3.3.2.2 Seeking to explain, using deduction and abduction

The study is explanatory: it was designed to categorise and explain rather than increase notions of mastery within groups of teachers.

Ridenour's and Newman's (2014) continuum distinguishes quantitative from qualitative approaches according to whether data is used to test an existing theory or theorise from existing data. These different approaches represent alternative processes of reasoning and ultimately different notions of truth (Bridges, 1999). Truth as correspondence requires a process of *deductive* reasoning (truth of a proposition established by a chain of facts that lead to the correspondence of the facts with the proposition). Truth as coherence follows from *inductive* reasoning (truth is established by presenting evidence which is then used to make generalisations for a wider population). Ridenour and Newman (2014) associate deductive reasoning, where data is used to verify, with quantitative research (and aligns with the postpositive end of Tashakkori's and Teddlie's (2008) continuum) whilst inductive reasoning, where data is used to theorise, is associated with qualitative research and constructivism. Q studies do not sit clearly within Ridenour's and Newman's continuum as much of the reasoning is *abductive:* data is used not to verify, or to theorise, but to explain existing categories of opinion (Watts and Stenner, 2012).

The explanatory approach to the research study is seen through considering the chosen research design in relation to Brown's (1980, p.69) diagram of the stages of a Q methodology study. (Reproduced in Figure 3.2).

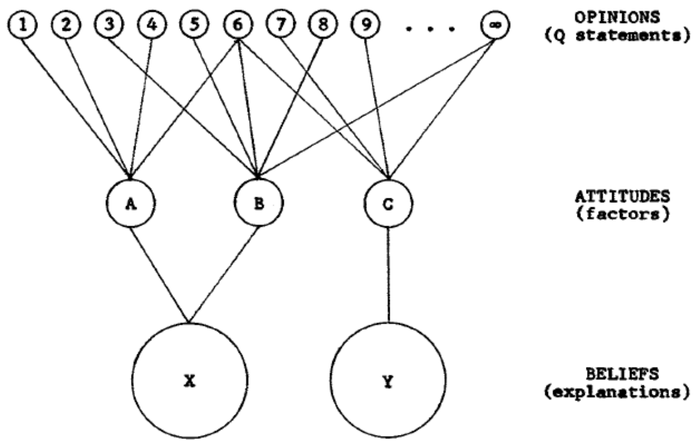


Figure 3.2: Data organisation and reduction in a Q study (Brown, 1980, p. 69)

An infinite potential variation in opinions about mastery (1-infinity in the diagram) was organised by the researcher through systematic collection of the concourse of opinion and selection of the Q sample statements. Research participants had complete freedom to categorise combinations of statements in their Q sort. This was a process of data organisation, presentation, and collection. Data reduction took place in the factor analysis of the Q sorts, to determine how much of the total variability in the data is explained by different groups of people, known as 'Factors' (attitudes A, B and C in the diagram). This is deductive reasoning: I identify characteristics common to individuals whose sorts placed them in a specific factor. Finally, my analysis of the similarities and differences between the factors, and the characteristics of people who load on them, offer explanation of why these divergent views occur, and implications for educational practice and or policy (X and Y in Figure 3.2**Error! Reference source not found.**). An infinite number of opinions is reduced to a smaller number of attitudes, then yet fewer beliefs. Truth is revealed by abduction through pursuit of explanation, rather than description or verification (Watts and Stenner, 2012).

3.3.2.3 Use of numerical data, statistical analysis, and probability sampling

I am clear about the statistical methods I use. Although participants do rank narrative statements, a rigorous and transparent process was used to select them, and pilot studies assessed clarity of meaning of the statements to reduce ambiguity. Participants revealed their views using a probability sample set of structured and close-ended statements. The number of statements and participants was aligned with accepted good practice for Q studies.

The use of Q methodology to answer the research question uncovers different sets of opinions about mastery. The study also looks at the characteristics of the participants who fit into these groups. The findings have applications beyond the study. I do not claim that the study can predict (for instance) the percentage of mathematics teachers might hold a particular opinion, but Q methodology studies can legitimately generalise concepts, categories, theoretical propositions and models of practice for people that share characteristics of the research participants (Watts and Stenner, 2012). Though the number of participants in the research was modest, the 'sample' (in a statistical sense) consisted of the combination of statement order possibilities and is thus large enough for the categorised opinions of identified groups to be significant. Conclusions made for a particular group are generalisable to other persons with the same characteristics of that group (Brown, 1980). Individual opinions of mastery are variable and unpredictable, but reliability of the conclusions as 'systematic subjectivity' (Brown, 2008) is claimed through revealing of distinct categories of opinion and thus within the groups of teachers sharing characteristics identified within the study, a truth as consensus (Bridges, 1999).

3.4 The potential for Q methodology in educational research

Q methodology, as an accepted method for quantitively studying subjectivity, has great potential for addressing some of the polarisation seen amongst educational researchers, who either favour a 'what works' approach (EEF, 2020) or who are clear in their view that 'what works won't work' (Biesta, 2007). This polarisation is damaging educational research, and the profession, by creating 'critical mismatches' in the 'educational ecosystem' of policymakers, teachers and researchers (British Academy/Royal Society, 2018). Q methodology studies have successfully informed healthcare practices and policies (e.g Baker, *et al,* 2010b, Mason, Baker and Donaldson, 2010): my research study can be used as a case study to inform policy and practices in mathematics education.

My application of Q methodology with a postpositivist position harnesses the advantages of mixed method research identified by Guba (1990): the study's findings and discussion encapsulate rigour and relevance, precision and richness, elegancy and applicability and discovery and verification. Individual teachers' opinions of mastery remain variable and unpredictable, but reliability of the conclusions as 'systematic subjectivity' (Brown, 2008) is claimed through the statistical significance of the identified Factor groups, revealing distinct categories of opinion. The findings are valid beyond the study and are of importance to the researcher, the participants, the wider teaching profession, and policymakers.

3.5 Conclusion

This chapter explained the positioning and generalisability of my Q methodology study. I justified the methodological decisions made in this study with an awareness of researcher and participant bias and experimental limitations. I documented my methods and data for others to challenge and disclose subjectivities made in my discussion and conclusions.

The conclusion of this chapter is the end of the 'scene-setting' section of the thesis: I have explained why mastery in mathematics is a valid topic to research, I have analysed the domain and range of views within the topic of mastery, and I have outlined and justified an appropriate methodology to address the research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?'.

In keeping my postpositivist promise of transparency in methodology and methods, the next chapter documents my approach to study design and data collection and reporting. Chapter 4 explains how I systematically created a concourse of statements that encapsulate a complete diversity of views on mastery, and how this was reduced to the final 48 statements in the Q sample. It also explains the strategies for recruiting a diverse group of research participants.

4 Concourse and Q sample

4.1 Introduction

This chapter explains how I designed and tested my Q methodology research study, and how I recruited the participants. I weave together the rationale for the thesis (Chapter 1), the literature review (Chapter 2) and the methodology (Chapter 3) to design an appropriate method for addressing the research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?'

The beginning of the chapter explains how I systematically created a concourse of statements that encapsulate a complete diversity of views on mastery, and how this was reduced to create the final Q sample statements. I detail the two pilot studies used to refine the Q sample statements and the process of online data collection.

In the latter part of the chapter, I explain the planned strategy to recruit study participants, and why these are consistent with my aim of recruiting a diverse group.

This chapter concludes with a summary of the final Q sample statements and administration of the online study. This is the end of the planning stage of the thesis: later chapters are about data collection, results, analysis, and discussion.

4.2 Stages of the creation of the Q methodology research study

The stages of data collection and analysis in this research study are consistent with Q methodology convention (e.g., Watts and Stenner, 2012; Brown, 1980)

1. I created a *concourse,* or set of statements, which represent a 'universe' of possible meanings of mastery. The concourse was generated using the literature review in Chapter 2.

2. I reduced the concourse to a pilot *Q sample* which is a subset of statements from the concourse which represent a balanced sample of statements from the concourse. The concourse is reduced by removing duplicates and refining the language to maximise the range in opinions within each theme, analogous to a balanced-block design (Stephenson, 1993). I identified nine core themes in relation to mastery, which further separated into 'sub-themes' of teaching and learning.
3. I piloted the Q sample with a group of non-specialist participants, to test the clarity of the statements and Q sorting process. This pilot led to refinement of the statements and better Q sorting instructions.
4. I conducted a second pilot with a group of experts, who reviewed the Q sample of statements in relation to concourse coverage. They also reviewed the *conditions of instruction* (Watts and Stenner, 2012) and tested the online Q sort programme.

In the absence having previous Q studies about mastery to refer to, each stage in the process added an element of confidence that my final Q sample was representative of the mastery concourse, which was itself representative of the universe of opinions about mastery.

4.3 Concourse creation

I was responsible for creating my own concourse of knowledge of mastery in mathematics in a robust and replicable manner to ensure the complete range of opinions were included (Watts and Stenner, 2012). Firstly, I checked whether there were pre-existing concourses or Q sets in published literature. Neither my literature review (Chapter 2) nor a search for 'mastery AND Q methodology' on the Canterbury Christ Church University (CCCU) LibrarySearch database, the Q methodology journal 'Operant Subjectivity' archives or Google Scholar found previous Q studies researching mastery (in mathematics or otherwise). There was also no pre-existing framework classification of mastery learning. I therefore decided to generate a concourse from scratch.

Generating my own framework took additional time and placed additional responsibility on me as a researcher (according to Zabala (2014) the concourse, in theory, is infinite). However, the framework I created for the study can be utilised in future Q methodology research studies on mastery. Repeated use of this framework will test the reliability of my research and will strengthen the body of evidence for researchers, practitioners, and policymakers in this area.

Q methodologists agree that concourse development is 'more of an art than a science' (Brown, 1980, p. 186) and there is no one accepted method for generating a concourse. Kenward's (2019) review of concourse development strategies reported that approximately half of the reviewed studies had no obvious theme or structure to their concourses and the remaining studies used researcher-generated themes, a researcher-generated framework or pre-existing frameworks. Concourse development can begin from theory or experimentation (Brown, 1980; Stephenson, 1953), providing that the completed set of statements represent the population of all possible answers to the research question.

4.3.1 Concourse categories

The concourse creation process for this study began with the rationale, described in Chapter 1. I identified that ideas about mastery varied because of a teacher's philosophy of mathematics education, the existence of two original mastery programmes and the use of the word mastery to represent mathematics taught in Singapore and Shanghai. This was the starting point the review of literature. As explained in Chapter 2, the literature review and the methodology sections of this study were written concurrently, and a concourse 'emerged' from this process. The statements that formed the concourse, and ultimately the Q statements originated from one of the pieces of literature reviewed in Chapter 2. Whilst concourse statements can originate from anywhere, including interviews and social media, I felt uncomfortable in considering the relative weighting of different sources, so narrowing the concourse sources to reviewed literature provided structure. Whilst, in theory, this would exclude a mastery viewpoint which exists outside of published literature (so perhaps a very 'new' viewpoint), I dismissed this as unlikely. It would be reasonable to assume that any new mastery viewpoint (as well as other viewpoints that could be collected from interviews or social media) be derived from a combination of existing opinions, and so hence would be captured within the 'net' of the placement of the carefully chosen Q statements.

I organised the reading for the literature review according to Ernest's (1991) framework of educational ideologies, with an intention that these should be the categories for the concourse themes. However, there was significant overlap, and according to the principles for a balanced block design (Stephenson, 1993; Brown, 1980) I should minimise overlap within the categories. Thus, I created my own categories using a combination of Ernest's framework, Boylan, *et al.*'s, 'Mastery innovation timeline and influences' (2019, p. 48, Figure 1), the rationale for exploring mastery outlined in Chapter 1 and my professional judgement as a critical consumer of the mastery literature. I identified three origin categories for mastery, named *philosophy of mathematics education, two origin mastery programmes* and *Singapore and Shanghai.* For each of these I identified distinct categories that exemplified mastery in relation to teaching and learning. I named these categories *types of knowledge, attainment and assessment, curriculum, success for all, whole-class teaching and direct instruction, small steps and variation, multiple representations, flexible fluency, and teacher professional development.* A diagrammatic representation of these categories is seen in Figure 4.1.

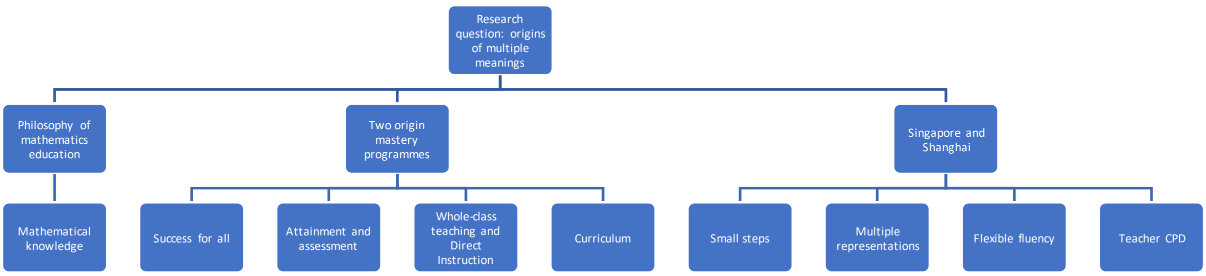


Figure 4.1: Categories of mastery

4.3.2 Concourse statements

After finalising the categories I revisited the literature review to identify and compile a list of statements that define aspects of mastery within each category, minimising overlap and leaving none outside. Although I considered including graphs, diagrams, equations, or other pictorial representations (since they are of course major components of mathematics), as concourse items, I decided to limit the concourse items to text statements. Whilst there was a practical element to this decision (many online Q sort programmes do not facilitate non-text statements), I concluded a text-only selection more likely to result in a set of items that elicit an equal emotional or self-referential response in the participant (Stephenson, 1953). Inclusion of an equation, for example, might lead a participant familiar with or favourable to equations to respond more (or less) strongly to someone who is averse to equations. I limited the statement generation to reviewed literature for the same reasons stated in Chapter 4.3.1: I was nervous about placing undue weight on unverified sources and I felt that pragmatically there was sufficient variation within the reviewed literature to cover the extremities of the universe of opinions. When piloting the Q statements with selected experts they provided reassurance that a category of statements had not been omitted.

After this initial trawl, the concourse had 105 statements, much higher than the Q sample 'house standard of 40-80 items' (Watts and Stenner, 2012, p. 61). A greater number of statements increases the sample size, thus theoretically strengthening the verification aspects of the research, but too many statements makes the sorting process onerous for the participants, thus reducing the accuracy of the judgements. It also makes it more difficult for the researcher to select statements of equal importance within the concourse. I studied each theme in turn, removed duplicates, and tried to select six statements in each of the categories that best represented the limits of opinion within that category (two of the categories did at this point contain seven statements) (see Table 4.1 for an example of this process). These fifty-six statements were the Q sample for a first pilot study.

<table>
<tr><th>Original statement</th><th>Reason for rejection</th><th>Final statement</th></tr>
<tr><td>In mastery lessons there are multiple episodes of teacher talk, teacher-pupil questions and pupil work in one lesson</td><td rowspan="4">Too specific - essence can be covered with wider alternatives</td><td rowspan="2">In mastery lessons, complex problems should be reduced by the teacher into a series of steps</td></tr>
<tr><td>In mastery lessons there will be an episode of teacher talk, teacher-pupil questioning then pupils being set a piece of work dependent on their understanding</td></tr>
<tr><td>Mathematical exercises to develop mastery must be smoothly connected to the previous exercise</td><td rowspan="2">In mastery lessons, complex problems should be reduced by the students into a series of small steps</td></tr>
<tr><td>In mastery lessons, all students should complete all parts of an exercise in order so that the learning journey is smooth</td></tr>
</table>

Table 4.1: Example of rejected and replaced statements

4.4 First pilot study

I always intended my final research study to be administered online, with teachers recruited through electronic advertisements and social media. However, since the dynamics of the Q sorting process as part of the data interpretation are highly valued by Q methodologists (Brown, 1989), and it was my first Q study, I was very keen to do at least one face-to-face pilot Q study to gain experience and critique the importance of Professor Brown's claim.

4.4.1 Aims and method

The aim of this first pilot was to validate the Q methodology process and to elicit feedback about how Q participant novices understand the principles of what they have been asked to do, so contributing to the *conditions of instruction* (the exact wording of the instructions given to participants before they undertake their sort) and administration of the final study. It also gave me a chance to observe the 'dynamics of subjectivity' (Brown, 1990, p. 196) and obtain a degree of insight into the statements that participants found more or less difficult to place and a sense of the overall time it would take to complete the sort. Twelve final year undergraduate students, studying a module called 'Maths in Society' as part of their Education Studies degree (which is not a teacher training course but is sometimes a stepping-stone to becoming a teacher), were invited to complete a Q sort during a seminar session. None of the students had any previous experience of Q methodology and as non-mathematics specialists they would be expected to have little experience of mastery in mathematics. The students knew that they were taking part in a pilot study and gave verbal consent, however for them it was a learning experience to help them think about mathematics education and research methods. It was not an intention that the results of their sorts would be analysed. Hence, the condition of instruction was very open. For the second pilot and final study, I related the condition of instruction much more directly to the research question.

At the start of the session, I gave the students an introduction to Q methodology and presented an empty grid on the classroom whiteboard. I gave each student a set of cards, a large blank sheet of paper and some blue tack.

The conditions of instruction were:

1) I asked the students: *'Please read each card and put it in one of three piles depending on whether you AGREE with, DISAGREE with, or have NO OPINION on each statement'*.
2) After the students completed this, I asked: *'Please sort the cards into a grid as shown on the whiteboard. There are no right or wrong answers.'* [The whiteboard displayed a PowerPoint slide with an empty array as shown in Figure 4.2, with 'most disagree' above the -5 and 'most agree' above the 5.]

The students were asked to work individually but could, and did, discuss the meaning of the cards in the group. As the participants completed their Q sorts I circulated the room, listened to their discussion, and answered queries. I made a note of any pertinent points or questions that I would use to refine the study.

4.4.2 Results and discussion

All twelve students completed the task, although four of the students worked in two pairs. Figure 4.2 shows one of the completed sorts.

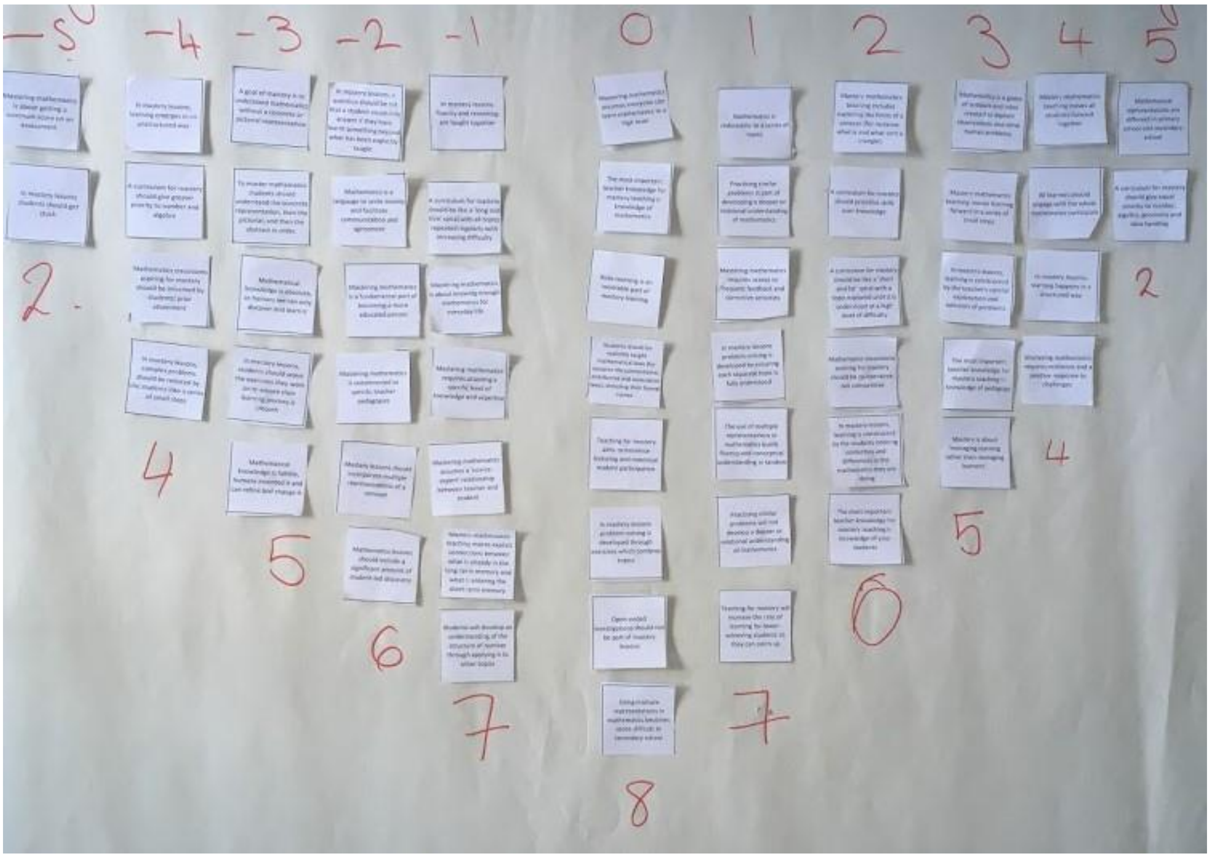


Figure 4.2 - A completed Q set from one of the first pilot participants

The only card that a student asked for a clarification of meaning was: ‘Mathematics is reducible to a series of topics.’. The participants did however comment that fifty-six statements were too many and I observed that they struggled to remain focused by the end. When questioned about this, the students suggested halving the number of statements.

4.4.3 Reducing the number of statements in the Q sample

The significant feedback from the pilot was a recommendation to reduce the number of statements in the Q sample. Noting Watts and Stenner's recommendations that a Q sample be a broad, seamless, and balanced representation of the opinion domain, and remembering that I had struggled (and failed) to reduce the statements to six in each, this proved to be challenging. I considered a variety of methods for reducing the number of statements, as well as leaving the number unchanged. Ultimately, I decided to remove the mathematical knowledge category from the Q sample.

This decision had a large pragmatic element to it: other solutions (such as removing one or two statements from each category) appeared to reduce the essence of each category as there were no obvious duplicate statements. The decision meant the final concourse framework was consistent with a balanced block design and Fisher's design principles (Stephenson, 1993): half of the final themes originated in historical 1960s mastery programmes, and half originated from pedagogical approaches in East Asia. The diagrammatic representation of the categories of mastery, Figure 4.1, shows that whilst the *mastery origin* and *Singapore and Shanghai* categories contain four themes, the *philosophy* category has just one.

I did consider whether, and how, the removal of an entire category from the Q statements affected the validity of the concourse. I felt that whilst there was of course a danger of missing a fundamental element, the 'standalone' nature of the philosophy category minimised this. I felt it was important to continue to consider the potential impact of a teachers' mathematical philosophy on their view of mastery, and so the final question that participants answered following their Q sort was: 'Which of the following statements <choice of three> aligns most closely with your own opinion of the nature of mathematics?' The answers to this question were considered as part of the results and discussion. Hence, whilst the concourse was reduced in size (asking this question post-sort removed the opportunity for the relative importance of mathematical philosophy to be studied), this category was still considered in the study.

I also decided it was important to have exactly six statements in each theme, to reduce the number of statements by another two and to create additional balance. When I looked at the statements, they often 'paired up' as different views upon an aspect of a theme. Where pilot participants had noted ambiguity or confusion in the wording of statements, they were amended. The research question invited teachers to consider mastery in terms of both teaching and learning (of students, although teacher learning is also explored), so I categorised each statement as being primarily about teaching or learning. There were equal numbers of statements in each category. The set of statements for the second pilot, which remained unchanged in the final Q sample appears in Appendix A.

4.5 Online Q sorting

The final research study was undertaken online, using QsorTouch (Pruneddu, 2016). The decision to complete the study online was made quite early in the thesis, primarily because both the researcher and the target participants have limited time, resources, or opportunity to physically meet. Online Q studies are increasing in popularity for practical reasons associated with lower cost and with fewer geographical challenges. Teachers are under a great deal of time pressure, and an option of completing the Q sort in their own time and place and using a computer rather than paper widened the pool of potential participants and potentially increased the response rate.

My postpositivist positioning required that I endeavour to maintain an objective distance between researcher and participants, hence participants remained anonymous. This also reduced the ethical implications. I acknowledge the danger of online participants feeling less committed to, or engaged with, the research project and submitting low quality sorts (Dairon, Clare and Parkins, 2017), including those where the participant completes the study at random, or with less thought and consideration than they would give if completing the study face-to-face. Whilst this danger cannot be eliminated, it was minimised through reducing the number of statements after the first pilot. I also monitored the time taken by each participant to complete their sort and could have removed sorts that were completed in an abnormally short time.

I acknowledge that an online method meant that potential information to be gained by the researcher through verbal or non-verbal ('the silence of the Q sort' (Jeffares, 2019)) methods was lost. Teachers were given no time limit to complete their search, so they had time to reflect deeply on their choices, but of course they may not in fact choose to do so. There were also no post-sort interviews, which are a common facet within Q methodology studies. I had no opportunity to seek further information on common, unexpected, or challenging choices made. I did however include some non-compulsory post sorting questions in the pilot study and the final study. All pilot participants and over 90% of study participants completely or partially responded to these questions. Anonymous elicitation of opinions does, of course, remove the temptation for the postpositivist researcher to interpret participant responses or silences in inconsistent or non-transparent ways.

4.6 Second pilot study

4.6.1 Aims and method

The second pilot was designed to collect feedback from mathematics educators and online learning experts. I wanted feedback about whether the Q sample captured the full range of views on mastery, whether the language used in the statements was appropriate for my target participants (mathematics teachers and educators), and whether the whole procedure 'worked' online.

I sent emails to six professional contacts personally chosen because I knew them as either a practicing and experienced mathematics teacher, mathematics education lecturer or online learning experts and invited them to complete my pilot doctoral study. The email included an online link to the study hosted on QsorTouch software (Pruneddu, 2016). Pilot responses were ostensibly anonymous, although all were invited to correspond by email if they wished, and pilot participants knew that the pilot study would be small enough for me to potentially 'guess' who each sort belonged to.

When participants clicked on the link, they were taken to a summary paragraph explaining the research study, Q methodology and the aims of the pilot. The Q sort comprised a two-step process of dragging the Q statements into an agree/ neutral/ disagree column, before re-sorting the statements into a shape resembling a normal distribution with the continuum comprising 'most disagree' to 'most agree', with 'neutral' in the middle.

The condition of instructions for the pilot were:

'A Q-sort is a bit like a 'card sort' or a 'diamond nine'; you will be shown forty-eight statement cards. You need to decide how much each statement aligns with, or doesn't align with, your own definition of 'mastery' in relation to teaching and learning of mathematics.

For the first part of the study, please read each statement and put it in one of three piles; 'agree', 'disagree' or 'neutral'. It doesn't matter how many cards you put in each pile.

In part two of the study you will place the cards into the grid below. If you scroll down to the bottom of the screen there is a '+' and '-' button that you can use to change the grid size. This may make it easier for you.'

After sorting, the pilot participants were asked three free-text questions:

1) In your opinion, do the statements represent a comprehensive range of views in relation to mastery? If not, what is missing?

2) In your opinion, does the number of statements provide enough variety without being too tedious for the participant? If not, should the number be increased or

reduced?

3) Is the online platform sufficiently easy to use and compatible with your device? If not, what are the major pitfalls?

4.6.2 Results and discussion

I assume that all six participants completed the pilot (there were six completed sorts: it is possible that participants completed the sort multiple times). All six sorts gave at least partial responses to the free text questions.

All six participants agreed the wording of the statements was a good representation of the range of perspectives on mastery, providing primary evidence that the Q sample is fit for purpose. Four participants told me the number of statements (reduced to 48) was comprehensive without being onerous, one found the number of items 'overwhelming' and another reported the 'two-step' process took an 'overly long time'.

The main area of negative feedback participants gave related to the shape and labelling of the distribution. Two of the participants questioned the forced normal distribution shape, one reporting they found it 'constraining'. Three participants questioned the combination of agree/ neutral/ disagree for the first stage of the sort with the 'most disagree-most agree' continuum of the second stage. One participant wrote: 'I felt that by the forced placement of [some of the cards in stage two] my initial opinion of disagree was being disregarded'.

The pilot feedback helped me evaluate my approach and methods. I investigated the efficacy and impact of using, or not using, a forced normal distribution in a Q sort. Statistically, Brown (1971) calculated that it makes little difference to the results. Watts and Stenner stress that it is *not* irrelevant to the participants and that having a forced distribution may be of help, as it allows participants to rank statements that provoke less strong reactions as equally agreeable or disagreeable: asking participants to rank all statements means 'they (the participants) are probably wasting their time and we (the researchers) are probably helping them' (2012, p. 78).

I therefore kept the forced distribution. I removed the term 'fixed normal distribution' from the final instructions in favour of 'place your statements in this grid'. I also removed the numbers from the distribution categories and replaced with a continuum labelled 'most agree with' and 'least agree with'. I made the final conditions of instruction more comprehensive and gave the participants a sorting process 'algorithm', to try and make the sorting less overwhelming. Removing the numbers and the 'neutral' centre section addressed concerns (made by maths teachers) that participants were placing cards in a disagree pile that they did not disagree with in an absolute sense. However, it meant there was no benchmark to compare absolute agreement or disagreement *between* sorts. I concluded that making participants feel more comfortable with their placements was a worthy trade-off for this and was reassured from Brown (1971) and Watts and Stenner (2012) that this would make little statistical difference to the results. I pondered this question with delegates at the ISSSS 2019 conference whilst presenting the project and pilot results and was advised that this change was necessary given the pilot feedback and would not adversely affect the study's findings.

The reviews of QsorTouch were positive, though the two participants who completed the sort using a mobile device reported that not being able to see the whole sort at once was difficult, and one participant reported compatibility issues with their computer operating system.

An unexpected bonus of conducting this second pilot was that all participants offered to promote the invitation to participate in the final study with their own teacher contacts. Three participants commented that this was 'a great topic to investigate', they 'hope I publish the results' and that doing 'a Q' was interesting and enjoyable.

4.7 Participant recruitment

4.7.1 Approaches to participant recruitment in Q methodology

Traditional 'R' factor analysis explores the proportion of the variability in a population that is explained by different factors. In 'R' factor analysis the 'sample' is defined as a selection from the population of research participants. The inverse nature of Q methodology factor analysis means that the study sample is the correlation matrix of Q sorts, and the research participants (who will be grouped into factors) constitute the variables (Brown, 1980; Watts and Stenner, 2012). This means only a modest number of participants is required to make the categorised factors statistically significant and generalisable (only) to other persons with the same characteristics of the participant group (Brown, 1980). Hence, it was important to recruit a diverse set of research participants to 'ensure a variable set of variables' (Watts and Stenner, 2012, p. 71). Brown suggests that there should be 'enough subjects to establish the existence of a factor for purposes of comparing one factor with another' (1980, p. 192). A significant proportion of Q methodology literature refers to the set of participant completers as the 'P-set' (and the set of Q statements as the 'Q-set') but I chose to omit this term to aid reading for non Q specialists.

4.7.2 Methodology of selecting the participants

I needed to balance the simultaneous requirements of anonymously recruiting busy teachers with the need for sufficient diversity to provide appropriate variables. Watts and Stenner (2012) advocate strategic sampling of participants likely to express 'interesting or pivotal points of view' (p. 71). My research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?' required me to recruit specialist mathematics teachers likely to have an interest in the teaching and learning practices surrounding mastery. I wanted to attract current and former teachers with varied professional experiences. Beyond this, I had no preconceived ideas about characteristics of teachers who are likely to provide diverse viewpoints (hence a need for this study).

Q methodology studies have successfully used a sampling framework to recruit numbers of participants with specific characteristics (e.g. van Exel, *et al*, 2015). I rejected this approach because unlike van Exel, *et al*'s study, the theoretical structure underlying the set of Q statements did not offer any immediate clues about likely defining participant characteristics. I did gather data related to participants' geographical location, gender, teaching experience and knowledge of mastery, (see Chapter 4.8.2) but intended to use it in the interpretation phase rather than to stratify the sampling. In addition, significant time and resource constraints contributed to my decision to employ pragmatic recruitment techniques including snowballing, and Watts and Stenner (2012) concede this is common to Q study participant recruitment. I did not have the means or contacts to target the recruitment, nor did I want to exclude sorts from the sample without very good reason because there was no hypothesis linking participant characteristics with their viewpoints on mastery, and it would be unethical to hastily exclude data from a participant who had taken considerable time to provide them.

I set a recruitment target of 40 participants. This number is consistent with the recommendation by Watts and Stenner that 'the number of participants should be less than the number of items in the Q set (sample)' (2012, p. 73), and Stainton Rogers' recommendation of 40-60 participants (1995). I was reasonably confident that I could recruit 40 participants because the rarity of Q methodology within education research means few teachers will have completed a Q study before, so they may be encouraged and motivated to take part for this reason (Nazariadli *et al.*, 2019).

Prior to recruiting participants I received ethical clearance from Canterbury Christ Church University (see Appendix B).

I recruited participants in four ways:

1. Word of mouth: by approaching pre-existing contacts and a link on my email signature and teaching PowerPoint slides,
2. Through social media (Twitter and Facebook),

3. Through professional distribution lists (mathspromLISTSERVE, Q-METHOD LISTSERVE and the NCETM online community),
4. Snowballing (Goodman, 1961): asking recruited research participants to share the link with their networks.

Further information about participant recruitment is given in Chapter 5.2.

Needing the participants to be broadly representative of mathematics teachers, I collected relevant demographic and professional data to be able to describe and analyse the types of participants who completed the sort. Whilst the chosen recruitment methods had the potential of recruiting a wide selection of teachers, the choice of an online sort undoubtedly positively biased those who can use computers and who engage with electronic communication and social media.

4.8 Final Q sample, administration, and conditions of instruction

4.8.1 Final Q statements

The final statements are reproduced in Appendix A and are listed in Table 4.2 below:

No	Statement	Theme	Subcategory	Type	About
1	In mastery lessons all students should be assessed every lesson	Attainment and Assessment	Frequency	Continual	Teaching
2	In mastery lessons assessments should only be used at the beginning and end of topics	Attainment and Assessment	Frequency	Periodical	Teaching
3	All students are capable of achieving a mastery level of attainment	Attainment and Assessment	Distribution	Uniform	Learning
4	In general, 1/3 of students will achieve a mastery standard, 1/3 of students will achieve an average standard, and 1/3 of students will achieve a low standard of attainment	Attainment and Assessment	Distribution	Graduated	Learning

5	Mastery will be easier to attain if children are taught in groups of similar prior attainment	Attainment and Assessment	Grouping	Set	Teaching
6	Mastery will be easier to attain if children are taught in groups of mixed prior attainment	Attainment and Assessment	Grouping	Mixed	Teaching
7	Learners should move through a mastery curriculum at their own pace, moving on once they reach the expected level of attainment	Mindset and Differentiation	Progression	Personalis ed	Learning
8	Learners should move through a mastery curriculum as a group, only moving on once all students have reached the expected level of attainment	Mindset and Differentiation	Progression	Whole group	Learning
9	Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up	Mindset and Differentiation	Gap	Catch up	Learning
10	Teaching for mastery involves students keeping up, not catching up	Mindset and Differentiation	Gap	Keep up	Learning
11	To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	Mindset and Differentiation	Exercises	Same	Learning
12	To achieve mastery, in mathematics lessons students should all be working on different problems	Mindset and Differentiation	Exercises	Different	Learning
13	In a mastery curriculum students will understand the structure of number before applying it to other topics	Curriculum	Topics	Compartm ented	Learning
14	In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	Curriculum	Topics	Connected	Learning

15	A curriculum for mastery should give equal priority to number, algebra, geometry and data handling	Curriculum	Weighting	Unweighte d	Teaching
16	A curriculum for mastery should give greater priority to number and algebra	Curriculum	Weighting	Weighted	Teaching
17	Planning mastery lessons is quicker because there are no differentiated resources to create	Curriculum	Planning	Faster	Teaching
18	Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises	Curriculum	Planning	Slower	Teaching
19	To achieve mastery, students should be explicitly taught mathematical laws (for instance the commutative, distributive and associative laws), including their formal names	Methods	Laws	Teaching	Teaching
20	To achieve mastery, students should understand mathematical laws (for instance the commutative, distributive and associative laws) but do not need them to be explicitly taught	Methods	Laws	Understan ding	Teaching
21	A student is more likely to achieve mastery if a teacher uses a specific pedagogy	Methods	Pedagogy	Specific	Teaching
22	Mastering mathematics is unconnected with specific teacher pedagogies	Methods	Pedagogy	Non-specific	Teaching
23	Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student	Methods	Relationship	Expert	Teaching
24	Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student	Methods	Relationship	Mentor	Teaching

25	In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	Small steps and Variation	Questions	Stretch	Teaching
26	In mastery lessons, all questions set should reflect only what has been explicitly taught	Small steps and Variation	Questions	Content	Teaching
27	In mastery lessons, complex problems should be reduced by the teacher into a series of steps	Small steps and Variation	Reduction	Teacher	Learning
28	In mastery lessons, complex problems should be reduced by the students into a series of small steps	Small steps and Variation	Reduction	Student	Learning
29	Teaching for mastery should minimise lecturing and maximise student participation	Small steps and Variation	Participation	Student	Teaching
30	Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	Small steps and Variation	Participation	Teacher	Teaching
31	Mastery lessons should incorporate multiple representations of a concept	Multiple Representations	Type	Variable	Learning
32	To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	Multiple Representations	Type	Fixed	Learning
33	Multiple representations are not always needed in secondary school teaching for mastery	Multiple Representations	Understandi ng	Hierarchic al	Learning
34	A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	Multiple Representations	Understandi ng	Non-hierarchic al	Learning
35	In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	Multiple Representations	Construction	Teacher	Learning

36	In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing	Multiple Representations	Construction	Student	Learning
37	Rote-learning is incompatible with mastery learning	Flexible fluency	Rote	Unnecessa ry	Learning
38	Rote-learning is an inevitable part of mastery learning	Flexible fluency	Rote	Necessary	Learning
39	Practising similar problems is part of developing a mastery understanding of mathematics	Flexible fluency	Practise	Similar	Learning
40	Practising a variety of problems is part of developing a mastery understanding of mathematics	Flexible fluency	Practise	Variety	Learning
41	In mastery lessons problem-solving is developed through exercises which combine topics	Flexible fluency	Problems	Connected	Learning
42	In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	Flexible fluency	Problems	Compartm ented	Learning
43	Reading, and taking part in, educational research is an important aspect of teaching for mastery	Continued Professional Development	Knowledge	Subject	Teaching
44	Mastery professional development activities should include a high degree of teacher subject knowledge development	Continued Professional Development	Knowledge	Pedagogy	Teaching
45	Mastery professional development activities should include a high degree of specific pedagogy development	Continued Professional Development	Location	Outside	Teaching
46	Teaching for mastery pedagogy is mainly learnt through external professional development	Continued Professional Development	Location	Within	Teaching

47	Teaching for mastery pedagogy is mainly learnt through collaborative in-school professional development with colleagues	Continued Professional Development	Focus	Research	Teaching
48	Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps	Continued Professional Development	Focus	Improvem ent	Teaching

Table 4.2: Final Q statements

4.8.2 Final conditions of instruction

The final conditions of instruction and administration, including the participant consent statement are detailed in Appendix C and below:

1. The participants' invitation, arriving by email or social media, contained a brief introduction to study and a link to the QsorTouch online study, http://tinyurl.com/masteryresearchstudy. The link was active from 8th October 2019 until 21st December 2019.
2. Upon clicking the link, participants arrived at a landing page with more information about Q methodology, the research study, and a consent statement. A sentence on the landing page explaining the importance of the study, and the contribution that each teacher is making to educational research, aimed to increase the likelihood that teachers would remain focused during the sort.
3. Participants answered a pre-sort questionnaire (consisting of some yes/no and some 5-point scale Likert questions) to determine information about their professional background and pedagogical preferences (including questions about their personal philosophy of mathematics).
4. Participants completed their sort. The conditions of instruction for this part were: *'Now for the study. A Q-sort is a bit like a 'card sort' or a 'diamond nine'; you will be shown forty-eight statement cards and will need to make some decisions about how much each statement aligns with, or doesn't align with, your own definition of 'mastery'*

in relation to teaching and learning of mathematics. The study only looks at how you have ranked the statements relative to each other.

For the first part of the study, please read each statement and put it in one of three piles; 'agree', 'disagree' or 'neutral'. It doesn't matter how many cards you put in each pile.

In part two of the study you will place the cards into the grid below. You must put the exact number of cards in each column.

A suggested way to do this is as follows.

1) Sort the 'agree' pile into the right hand side of the grid. Put the card you agree with most in the furthest right-hand column (column I), then work backwards towards the middle. Cards placed in the same column will be judged as being of the same relative importance to you.

2) Sort the 'disagree' pile into the left hand side of the grid. Put the card you disagree with most in the furthest left-hand column (column A), then work backwards towards the middle. Cards placed in the same column will be judged as being of the same relative importance to you.

3) Finally, sort your 'neutral' pile into the remaining spaces in the grid.

You may find some cards easy to place, and some more difficult. Remember that it is the relative importance that matters; you may agree or disagree with all the cards - that is fine. Take your time and remember there are no right or wrong answers.

If you scroll down to the bottom of the screen there is a '+' and '-' button that you can use to change the grid size. This may make it easier for you.'

5. Participants answered three post-sort questions about the cards they agreed and disagreed most with, and their opinion of the nature of mathematics.

4.9 Conclusion

This chapter explained how I systematically created a concourse of statements that encapsulated a complete diversity of views on mastery, and how this was reduced to the 48 statements used in the Q sample. The combination of a literature review and pilot studies resulted in a set of Q sample statements that represented the domain and range of teacher views of mastery.

The two pilot studies provided me with invaluable information about the 'fitness for purpose' of the design of the research study and gave me confidence that the final framework and design would give accurate results and be user-friendly.

In Chapter 5 I detail the experience of participant recruitment. I also explain how I use the raw participant data generated by the sorts to generate the groups, or Factors, that represent the views of mastery as revealed by the participants.

5 Data collection, factor extraction and factor rotation

5.1 Introduction

This chapter bridges the methodology and the results. I explain how I utilised Q methodology analysis to address the research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?'

The start of the chapter outlines the process of recruiting 45 participants to complete the online Q study between October and December 2019. The chapter then explains the analysis I took to turn raw Q sort data into the viewpoints, or Factors, that constitute categories of meanings of mastery revealed by the study. After the data were downloaded, centroid factor analysis indicated four distinct views of mastery. Varimax factor rotation and flagging of significant sorts on each factor provide the instructions needed to calculate the theoretical Q sorts for each factor. The opinions of mastery held by each factor, and the characteristics of the participants who hold these views, is revealed in the next chapter.

5.2 Data gathering time, methods, and pattern

The study was open for completion from 8th October 2019 until 21st December 2019. 45 participants completed the study, five more than the target of 40, and fewer than the number of Q sample statements. The short link to the study, valid for the duration of the data collection, was http://tinyurl.com/masteryresearchstudy. I publicised the link by word of mouth, (such as during teaching sessions, conference presentations and during face-to-face professional meetings), but I anticipated that most of the participants would be recruited electronically. Recruitment by electronic means complements online data collection because participants who choose to take part can move from recruitment to participation quickly, and at their convenience (Hewson, 2017). Asking participants, or viewers of the link to pass on the study's details to their own contacts is known as ‘snowballing’. Snowballing exponentially increases the spread of information, and can increase representation from groups that might otherwise be underserved, since receiving a link from a known contact gives the study ‘cultural competence’, and increases the trust in the validity and importance of the research from the potential participant (Sadler *et al.,* 2010, p.370). This might have created clusters of similar groups.

5.2.1.1 Email

Publicising the study’s link by email to people who know me professionally increased the likelihood that unforeseen errors would be identified and reported to me by early participants (I had tested the link and programme extensively but did not discount the possibility that an error remained undetected). Participants recruited by email were invited to contact me by return to report queries or problems. These participants were likely to be the ‘seeds’ for future dissemination so it was very important that they feel engaged and happy to recruit others in their ‘affinity social group’ (Sadler *et al.*, 2010, p. 372).

On 8th October I sent personal emails to two contacts, on 10th October I sent one personal email to one contact and on 15th October I sent personal emails to a total of 33 named contacts. I also sent a generic email to a mathematics education LISTSERVE (with an unknown number of subscribers) and a Q methodology LISTSERVE (with just over 1000 subscribers). The wording of a typical email was:

Hello xxx,

I hope you don’t mind me posting this.

I am currently collecting data for my Ed. D. thesis all about ‘meanings of mastery held by secondary mathematics teachers’. I am using Q-methodology, which resembles an online ‘card-sort’ exercise.
I am looking for research participants who are secondary mathematics teachers or who work in secondary mathematics education to complete my online study.
If this applies to you, and you would like to take part, I would be very grateful.
The link is www.tinyurl.com/masteryresearchstudy
It is easier to complete on a computer (rather than a tablet or phone, but all should work!) and it will take about 30 minutes.
All data collected will be anonymous.

Thank-you so much. Please could you also pass this invitation on to your mathematics teacher colleagues and connections.

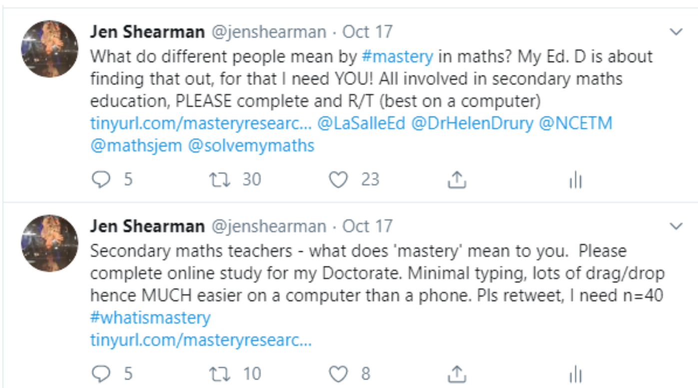
Do contact me personally if you have any queries.

Jen Shearman

5.2.1.2 Social media

My chosen social media platform for recruitment was a social media site called Twitter, in which individuals publish short messages (up to 280 characters) called ‘tweets’, readable to anyone who ‘follows’ their account. Individuals also send and receive personal messages and use keywords (known as ‘hashtags’) to attract like-minded individuals to their tweet. Individual Twitter users have an ‘account page’ which includes a photo and short description. At the beginning of the study my Twitter account (@jenshearman) had a ‘digital footprint’ of about 500 followers. I created my own Twitter account in 2014 for professional reasons and so most of my followers related to mathematics, education, or both. Large numbers of teachers and educational academics use Twitter: the phrase ‘EduTwitter’ to describe tweets and conversations about aspects of education describes a ‘virtual staffroom’ platform for teacher voice, scholarship and activism (McGill, 2019).

I sent the first two recruitment tweets on 17th October 2019. The first one was short, simple and generic so it could be understood quickly and retweeted without edit (I also ‘pinned’ this tweet to my profile, so that anyone looking at my profile would see the recruitment tweet). This first tweet was viewed by 4511 individuals (data correct on March 31st 2020). I sent another similar tweet and ‘tagged’ (added the name of) prominent tweeters of mathematics education: this second tweet was viewed by 12,924 individuals (data correct on March 31st 2020).



I could not link tweets, emails and completed sorts due to participant anonymity (and to a certain extent the snowball recruitment) but I frequently checked the submissions to notice whether direct recruitment activity correlated with submissions. Tweets were used as a 'narrowcasting' (as opposed to broadcasting) device to give an effective call-to-action to a specific audience, so I was mindful to ensure I motivated rather than irritated my intended audience and maintained appropriate professional and ethical conduct with my research participants (Sadler *et al.,* 2010). Once a stream of study completers was established, I sent fewer unsolicited tweets, instead concentrating on replying to others and 'liking' those who retweeted. My tweets (which were not all about the study) were seen over 20,000 times in the first week of recruitment (Figure 5.1), and 73,000 times during the 66 days that the study was active (Figure 5.2). In contrast, tweets sent during the same period the previous year were viewed 27,000 times.

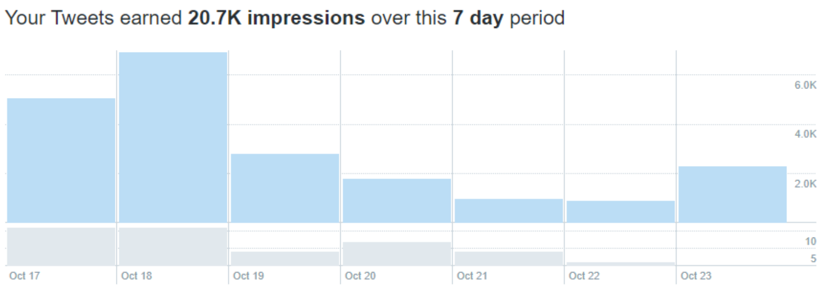


Figure 5.1: Twitter impressions 17/10 to 23/10 2019

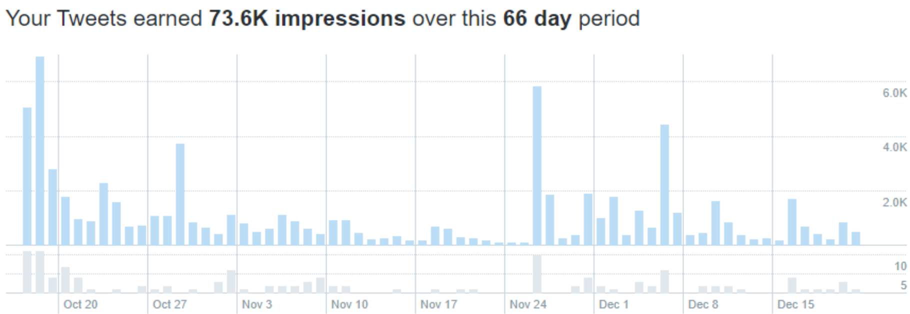
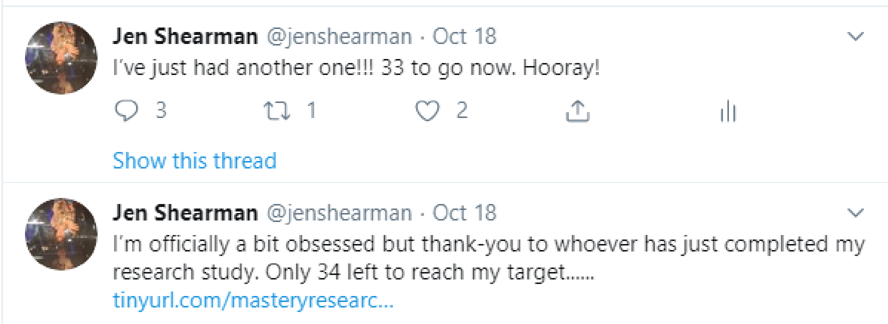


Figure 5.2: Twitter impressions 17/10 to 21/12 2019

Individuals who shared (either by Twitter, email or other means) the study with others, known as 'seeds', are central to the success of the snowball technique and need to be encouraged, supported, respected and acknowledged (Sadler *et al.*, 2010). I therefore tweeted messages of thanks, and milestone updates. I also thanked followers, and shared recruitment statistics once the study closed. I will use Twitter to communicate research outcomes in due course.



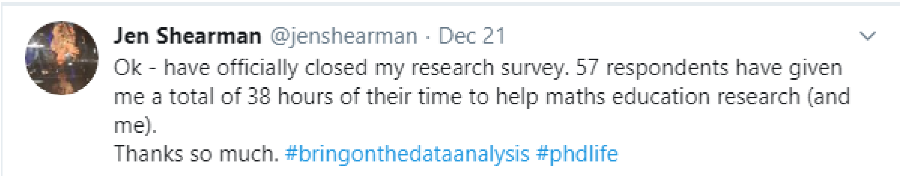


Figure 5.3 shows the number of submissions each day during the data gathering phase. The three engagements in mid-December were most likely responses to 'countdown' or 'thanks' tweets or emails sent after I had reached target and planned to close the study.

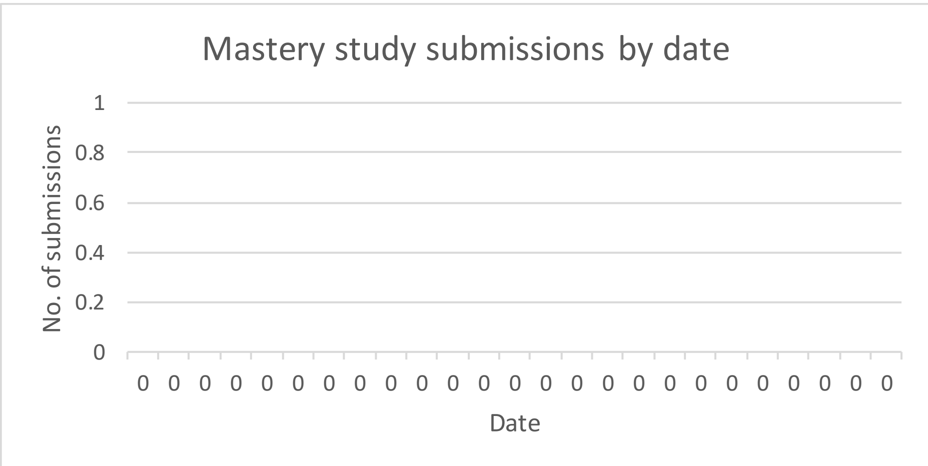


Figure 5.3: Submissions per day, 17/10 to 21/12 2019

5.3 Raw data

Once I had excluded pilot and test sorts, 45 completed sorts were included in the study. The mean average time taken to complete was 45 minutes and 57 seconds, with the fastest sort taking 8 minutes and the longest sort taking 3 hours 20 minutes. A box and whisker distribution of the time taken is displayed in Figure 6.2. I considered removing the 8-minute sort but in absence of any concrete reason for exclusion (it is physically possible to complete the sort in 8 minutes) I decided to keep it in there. This sort did not align with any of the final extracted factors.

I downloaded the completed dataset from QsorTouch on 22nd December 2019. Three separate programmes were used in collecting (QsorTouch; Pruneddu, 2016), analysing (PQmethod; Schmolck, 2014) and presenting (Microsoft Excel), and whilst all three programmes supported the .csv format, considerable time was taken to prepare the data for each individual programme, and to check for errors.

5.3.1 Characteristics of study participants

The characteristics of the research study's participants are displayed in Table 5.1. The nature of my approach to participant recruitment and commitment to anonymity meant that I had no control over who completed the study but I was wary about over-recruitment of particular groups (see Chapter 4.7.2). I compared the characteristics of the study participants with the general population of eligible participants to consider the limited applicability of the study's findings to other groups of people.

	Selected statistics	**Remarks**
N	45	More than target, less than number of Q statements
Gender	60% female	Broadly representative of UK teachers

Ages	11% in 20s 20% in 30s 36% in 40s 27% in 50s 4% 60+	Over-representative of teachers in 40s and 50s
Location	45% London and South 18% Midlands and East of England 31% North 6% National/other	Over-representative of South East and North West
Occupation	64% practicing teachers 20% teacher educators 7% trainee teachers 4% former teacher	Over one quarter of participants not currently teaching
Teaching experience	32% under ten years 36% 10-20 years 33% over 20 years	More experienced than the average mathematics teacher
Type of school mostly taught in	78% non-selective 9% grammar	Over-representative of teachers in grammar schools
Proportion of maths in degree	13% less than one quarter 33% more than three quarters	More mathematical experience than the average mathematics teacher
Training route	67% University PGCE 22% Employment PGCE	High proportion of participants have a PGCE
Completed SKE?	24% Yes	
Mastery CPD undertaken	42% NCETM 12% Complete maths 16% Other 29% No	High proportion of participants have experienced mastery CPD
Opinion of mathematics	A Game' 33% 'A language' 27% 'Absolute' 29% 'Fallible' 11%	Significantly low proportion of participants who think maths is fallible

Table 5.1: Participant characteristics

5.3.1.1 Gender

Twenty-seven female and 17 male participants took part (one participant preferred not to reveal their gender). Whilst this is obviously not representative of the general population, and not all participants were teachers, approximately 60% of teachers in the UK in 2017 were female (OECD, 2018) and thus as a ‘ready reckoner’ this is broadly representative.

5.3.1.2 Age

The participants are over-representative of teachers in their 40s and 50s. The percentage of study completers who identified as in their 20s, 30s, 40s, 50s and 60s is 11%, 20%, 36%, 27% and 4% respectively compared to UK percentages of 31%, 32%, 23%, 13% and 2% (OECD, 2018). This older demographic than perhaps would be expected provides evidence that online methods do not necessarily discourage older respondents.

5.3.1.3 Location

Participants were recruited from all parts of England, with larger representation from the South East (22%), the North West (20%) and London (16%). This broadly corresponds to areas with greater populations although there is over-representation from the South East and North West: in England, 16% of people live in the South East and 13% live in the North West (Statista, 2018). No-one from the West Midlands completed the study. Given that I live and spend much time teaching in the South East, the national nature of the study participants shows the potential for online Q studies to capture rich and rigorous data with fewer geographical constraints than face to face methods.

5.3.1.4 Occupation and school setting

Apart from one Higher Level Teaching Assistant (HLTA) all participants were trained, or training to be, teachers (assumed to be majority mathematics, although I omitted to ask this question). Twenty-nine participants identified as a teacher, three identified as trainees, two as former teachers and nine were teacher educators. Over one-quarter of the participants were not currently teaching in a school at the time of completing the study. This means that the study gave more evidence about those who have left classroom teaching than was planned, and less evidence about what practicing teachers think mastery is. I was able to identify differences in teacher educators' views on mastery compared to teachers, an unexpected finding.

Participants were asked which type of school they spent much of their time teaching in (of course, teachers work in a variety of schools during their career). Thirty participants had spent most of their time in a non-selective comprehensive school, two in a multi-academy trust, one in alternative provision and two selected ‘other’. Four participants had spent most of their time teaching in grammar schools, and five in selective comprehensive schools. Currently, 5% of state-funded secondary pupils attend grammar schools (House of Commons Library, 2020), and so arguably teachers with majority of grammar school experience are over-represented. Two reasons for this are: the over-representation of older teachers (grammar schools were more numerous until the late 1970s) and the over-representation of teachers from London and the South East (in the South East, 13% of secondary school pupils attend grammar schools) (ibid). Since Q methodology favours recruiting a diverse set of participants, greater representation from grammar schools increases the ‘voices heard’ in the study.

5.3.1.5 Qualifications and experience

There is evidence that the study’s participants have more experience of both learning and teaching mathematics than the average secondary mathematics teacher in England. There was an even distribution of years of teaching experience, with either seven or eight participants in each five-year category, and seven teachers with more than 25 years of experience. The five-year retention rate for secondary mathematics teachers in England is now 50% (Sibieta, 2018).

One-third of participants classified at least 75% of their degree as mathematical, with 13% classifying this as less than 25%. Although the classifications are different, estimates suggest that half of secondary mathematics teachers have a ‘relevant degree’ (Sibieta, 2018). Only 6% of participants did not have a Post Graduate Certificate in Education or PGCE (obtained through either a university-based or employment-based route).

5.3.1.6 Mastery-specific CPD

Although no direct comparative data is available (hence the need for this study), there is evidence that a greater percentage of study participants have undertaken mastery training or development than is representative of mathematics teachers in England. As an example, 42% of study participants reported undertaking mastery CPD with the NCETM, yet up to and including the 2019-20 academic year, approximately 1000 secondary mathematics teachers had directly taken part in an NCETM secondary mastery development activity. In 2018 there were slightly over 35,000 teachers of mathematics in England (DFE, 2018). This makes the study's findings, and discussion, particularly relevant and interesting to stakeholders with an interest in mastery professional development, such as schools considering a mastery programme and providers and funders of mastery.

5.3.1.7 Opinions on the nature of mathematics

Again, there is no comparable national number for this question since teachers are not routinely asked about their philosophical position on mathematics. Similar numbers of teachers consider mathematics to be 'a language', 'absolute' or 'a game'. Only five participants consider mathematics as 'fallible'. Q methodology does not advocate statistical tests, but I was interested to know whether the lower number of teachers with 'fallible' views of mathematics was significantly different from a random selection. A chi-square test for this yields a p-value of 0.08, significant at the 10% level. Therefore, in this study, the voice of the teacher who does consider mathematics to be fallible is limited. If a teacher's view on the nature of mathematics is an influence on their opinion of mastery (as I suggest in 1.5 and 2.3), then this finding means that the study will give an incomplete picture of the diversity of mastery opinions.

5.3.1.8 Summary of all participant characteristics

The data presented in 5.3.1 indicates that the participants who completed the study were of both genders, all ages and living across England. Seventy-five per cent of the participants were practicing or trainee teachers and 20% were now teacher educators. The participants had more experience of learning and teaching mathematics than the average English mathematics teacher and had taught in selective and non-selective schools. Just under 70% of teachers had experience of mastery CPD in some form. Participants were less likely to consider mathematics as ‘fallible’ than ‘a language’, ‘absolute’ or ‘a game’.

5.4 Factor extraction and analysis

The 45 completed Q sorts produced arrays of data, showing participants’ scores (between -4 and +4) for each of the 48 statements. Q methodology allows the researcher choice in their method of factor analysis and rotation. It was my responsibility to analyse this data appropriately and transparently, justifying my methodological decisions and ensuring other researchers can replicate, and question, the analysis.

5.4.1 Choice of factor analysis method

I undertook a centroid factor analysis which indicated that there were four factors, or distinct viewpoints present in the data.

Factor analysis is a statistical technique, first used in the 19th century and attributed to Pearson and Spearman, that mathematically attributes the differences or variance in values of a dataset to one or more external variables. Stephenson’s original Q methodology adapted this technique to attribute participants to groups that exhibit shared opinions.

There are two competing methods for extracting the factors. Stephenson’s favoured analysis, reproduced by Brown (1980), is known as ‘centroid’ and uses iteration to identify the first, second, third (etc.) sets of scores that account for as much of the initial (and subsequent, following the extraction of the previous factors) variation in the data as possible. Principal Component Analysis (PCA) is an inversion of ‘R’ analysis, computes the best mathematical solution, and its complement to R and compatibility with statistical packages make it popular with statisticians (Ramlo, 2016).

Debates about which technique to use dominate discourse amongst Q methodologists (Ramlo, 2017, Q-METHOD LISTSERVE, 2014, 2016, 2018, 2019). Academic arguments are contested on the theoretical ground of deductive versus inductive reasoning: PCA is an automated process that extracts factors following statistical decisions, whereas centroid allows the researcher to study the data and make choices about how many factors to extract (Ramlo, 2016). However, considerable weight is also given to arguments about computational ease or availability of software.

My choice of centroid analysis considered all these arguments. Firstly, using centroid analysis meant that I understand the algorithm, what the numbers mean and what they are telling me. Secondly, I used PQmethod that offers centroid analysis. Thirdly, I agreed with Brown's, Danielson's and van Exel's (2015) argument that Q factor analysis is fundamentally different from an inversion of R. One of the assumptions of PCA analysis is that each item in the correlation matrix is correlated at 1 with itself: as Brown (1980) explains, this is not true for a Q sort. Each number in the matrix represents an opinion at a point in time, and people do not always agree with themselves. I felt strongly that I wanted to do a Q methodology analysis, not an 'inverted R'.

In any case, Q methodologists do agree that the choice of factor analysis makes little difference to the results (Ramlo, 2016; McKeown and Thomas, 2013; Watts and Stenner, 2012; Harman, 1976).

5.4.2 Unrotated factors

The centroid factor analysis extracted seven initial factors (this is the maximum possible that PQmethod will extract). Table 5.2 gives the Eigenvalue and percentage of explained variance for each factor. The centroid method normally gives results showing each extracted factor less than the previous one but not always: in this study Factor 5 has a larger Eigenvalue than Factor 4, and Factor 7 has a larger Eigenvalue than Factors 3, 4, 5, and 6.

Unrotated Factors							
Factor	1	2	3	4	5	6	7
Eigenvalue	13.2233	3.128	2.2619	0.192	1.3878	0.0716	1.6031
% explained variance	29	7	5	0	3	0	4
Cumulative % explained variance	29	36	41	41	44	44	48

Table 5.2: Unrotated factor Eigenvalues and explained variance

Twenty-nine per cent of the explained total variance in the sorts was explained by Factor 1 alone. Factors 2, 3, 5 and 7 explain 7%, 5%, 3% and 4% of the data variance respectively.

There is no ‘perfect’ decision as to how many of these factors should be retained as ‘valid’ distinct views of mastery. Watts and Stenner (2012) explain this with a cake cutting analogy: if the top of a cake has two strawberries, three chocolate buttons and five sugar flowers then there is equal justification for cutting the cake into two, three or five pieces. A one-factor solution was considered, since Table 5.2 indicates this single factor currently accounts for nearly 30% of the variance. However, the whole point of Q methodology is to identify shared differences within the premise of a level of agreement and I wanted to explore how two sorts with similar correlations on Factor 1 differed in respect to the other factors. A seven or six-factor solution was also discounted since Factors 4 and 6 contribute little to the explained variance. I therefore needed to decide on a two, three, four or five factor solution.

In Q methodology there is no one method of deciding how many factors is the 'right' number. Stephenson's Q methodology welcomes the researcher's contribution to the decision. Watts and Stenner (2012) suggest the retained factors should account for as much variability as possible, and that all factors retained should provide an important characteristic to the overall solution. There are statistical tests available to guide the researcher's decision, and I chose to use the three tests compatible with centroid factor extraction.

Firstly, the Kaiser-Guttman criterion suggests factors with an Eigenvalue greater than 1 can be retained. The Eigenvalue is a measure of the explained variance of each factor relative to the number of Q sorts in the study. For a study with 45 Q sorts the variance of a single Q sort would be $100 \div 45 = 2.22\%$. A factor variance of 2.22% has an Eigenvalue of 1, hence factors with a lower Eigenvalue than this explain less of the total variance than the average single Q sort. According to this criterion Factors 1, 2, 3, 5 and 7 could be retained.

Secondly, Brown (1980) suggests retaining factors with two or more significant factor loadings. In a 48-item Q sample, a factor loading of $2.58 \times \left(1 \div \sqrt{48}\right) = \mp 0.38$ is significant at the 0.01 level (2.58 is the number of standard deviations above or below the standard error of a set of 48 statements, so a non-significant sort would only be incorrectly flagged 1% of the time). According to this criterion Factors 1, 2, 3 and 7 could be retained.

Thirdly, Humphrey's rule states a factor is significant if the cross-product of its two highest loadings exceeds twice the standard error (Brown, 1980). The standard error of a 48-item Q sample is $1 \div \sqrt{48} = 0.14$ so the product of the two highest loadings should exceed 0.28. According to this criterion Factors 1 and 2 could be retained.

Since all three tests gave different results, I decided to retain Factors 1, 2, 3 and 7. This was the 'middle ground' decision and allowed the voices of more of the sorts to be heard in the analysis and discussion, with the awareness that most sorts reflect at least some of the viewpoint of Factor 1, and that Factors 3 and 7 were less well represented. Before reaching this decision, I did explore factor rotations of a 2-factor, 3-factor and 5-factor solution, however these either produced results with very few significant sorts or produced very high cross-factor correlations. So, I am happy with 'cutting the cake' into four slices.

5.5 Factor rotation

Factor rotation clarifies the distinctiveness of the factors. When the factors are rotated, the relative loadings of *each sort in relation to each other* remain unchanged. However, the loadings of *each sort on each factor* are statistically manipulated to reduce the amount of 'noise' caused by sorts that load on more than one factor and to reduce the number of sorts that no not correlate with any factor. In the case of this study, factor rotation would also reduce the impact of the higher degree of communality displayed in Factor 1.

I used automatic Varimax rotation to rotate Factors 1, 2, 3 and 7. Factor 7 was renamed Factor 4.

PQmethod offers two methods of factor rotation: hand rotation and Varimax. Q methodologists disagree about which rotation to use on the same theoretical and practical grounds as discussed in the centroid and PCA section. Varimax rotation is an automatic calculation that maximises the number of sorts that load on only one factor (Akhtar-Danesh, 2016). Hand rotation is used by Q methodologists who want to theorise based on preconceived ideas and so could rotate a factor to increase the 'voice' of a specific participant's sort. I did not want to privilege any participant above any other: indeed I preferred to know nothing about the characteristics of any sort before all factor analysis was completed. The very nature of my study was to discover and explain rather than theorise (see 3.3.2.2).

Unrotated factors include sorts that have no significant loadings on any factor, and sorts that have similar loadings on all factors (these are known as confounded sorts). Rotating the factor axes has the effect of 'moving' all the sorts (though in fact it is the axes that move) closer towards an axis of one factor or another to increase the significance of the sorts that load on one factor only (particularly factors other than 1). In this way distinct differences in viewpoints are clarified.

Figure 5.4 and Figure 5.5 are graphical representations of the study's unrotated and rotated factors (due to the two-dimensional nature of the representation, only two factors can be displayed at a time, but in reality, all four factors were simultaneously rotated).

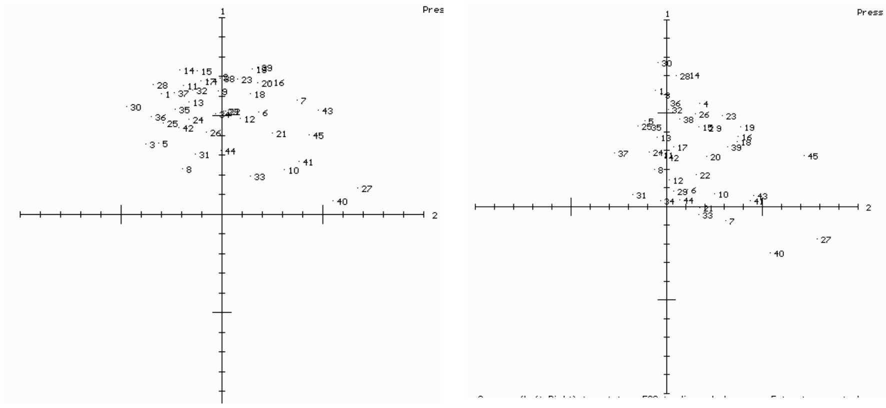


Figure 5.4: Unrotated (left) and rotated (right) Factors 1 and 2

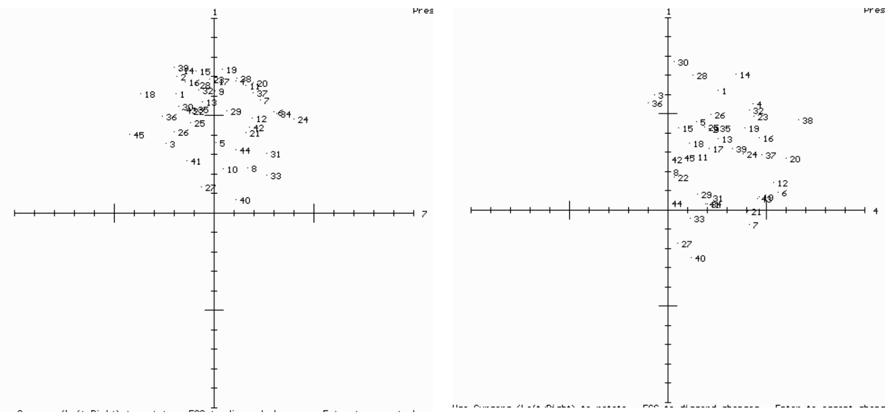


Figure 5.5: Unrotated (left) and rotated (right) Factors 1 and 7 (renamed 4)

The post-rotation Eigenvalues and explained variance are shown in Table 5.2. The factors now explain 44% of the study's variance (now reduced from 48% due to the removal of Factor 5). The dominance of Factor 1 is reduced. Factor 2 accounts for less of the explained variance than the other three factors and represents the views of fewer participants.

Rotated factors				
Factor	1	2	3	4 ('old 7')
Eigenvalues	6.3	3.15	5.85	4.5
% explained variance	14	7	13	10
Cumulative % explained variance	14	21	34	44

Table 5.3: post-rotation Eigenvalues and explained variance for retained factors

5.5.1 Significant sorts and cross-correlations

Following rotation, I 'flagged' significant sorts using the criteria of a loading of at least 0.38 (as explained in 5.4.2) on only one factor (Watts and Stenner, 2012). Each sort number is one participant's Q sort (sort 1 was completed by participant 1, sort 2 by participant 2 and so on). The factor loadings and flagging for each sort are displayed in Table 5.4.

Rotated flagged factor loadings				
Sort No	Factor 1 load	Factor 2 load	Factor 3 load	Factor 4 load
1	0.6210X	-0.0607	0.2963	0.253
2	0.4223	0.2021	0.5276	0.2106
3	0.5990X	-0.0265	0.085	-0.0684
4	0.5515	0.1745	0.0683	0.4304
5	0.4608X	-0.1126	0.0526	0.1455
6	0.0953	0.107	0.2169	0.5589X
7	-0.0749	0.3082	0.5479	0.4138
8	0.1979	-0.0633	0.1147	0.0098
9	0.4238X	0.2388	0.2651	0.2133
10	0.0702	0.252	-0.1429	0.4655X
11	0.2792	-0.0385	0.6697X	0.1317
12	0.1445	0.0151	0.2297	0.5349X
13	0.374	-0.0504	0.4421X	0.2551
14	0.7081X	0.0987	0.2177	0.3451
15	0.4259	0.1668	0.652	0.0564
16	0.3746	0.3722	0.253	0.4655X
17	0.3223	0.038	0.6367X	0.2111
18	0.3486	0.3668	0.5173X	0.1076
19	0.4277	0.3846	0.2384	0.3899

20	0.27	0.208	0.2197	0.5987X
21	-0.005	0.1718	0.2823	0.4024X
22	0.172	0.1529	0.6676X	0.0297
23	0.4887	0.2923	0.1396	0.4373
24	0.291	-0.0901	0.1034	0.3818X
25	0.4342X	-0.1517	0.3355	0.1872
26	0.4966X	0.1509	-0.011	0.2204
27	-0.1703	0.7853X	-0.0065	0.0501
28	0.7017X	0.0504	0.2299	0.1258
29	0.0844	0.0379	0.6888X	0.1522
30	0.7706X	-0.046	0.1355	0.033
31	0.0629	-0.1777	0.3154	0.2109
32	0.5219	0.0114	0.2309	0.4143
33	-0.0438	0.168	0.0727	0.1132
34	0.0347	-0.0307	0.5965X	0.2016
35	0.4292X	-0.1011	0.3734	0.2475
36	0.558	-0.0003	0.3859	-0.1022
37	0.2897	-0.2713	0.5243	0.4787
38	0.4676	0.0662	0.1285	0.6614
39	0.3225	0.3181	0.5945X	0.327
40	-0.2443	0.5422X	-0.0232	0.1169
41	0.0327	0.4379X	0.1081	0.1904
42	0.2661	-0.0092	0.3643	0.0002
43	0.059	0.4532	0.3852	0.4556
44	0.0375	0.0691	0.4268X	0.0008
45	0.2724	0.7204X	0.0913	0.0629
% explained variance	14	7	13	10

Table 5.4: Flagged factor loadings

3/45 sorts have no significant loading on any factor. 12/45 sorts are confounded (they had significant loadings on more than one factor). The opinions of the non-significant sorts will be missing from the analysis and the opinions of the confounded sorts is diluted within the analysis. Therefore, the analysis that follows in subsequent chapters directly considers 30/45, or two-thirds of the completed sorts.

Correlations between factor scores				
Factor	1	2	3	4
1	1	0.0268	0.5548	0.4753

2	0.0268	1	0.1962	0.3472
3	0.5548	0.1962	1	0.5248
4	0.4753	0.3472	0.5248	1

Table 5.5 displays the correlations between the rotated factors.

Correlations between factor scores				
Factor	1	2	3	4
1	1	0.0268	0.5548	0.4753
2	0.0268	1	0.1962	0.3472
3	0.5548	0.1962	1	0.5248
4	0.4753	0.3472	0.5248	1

Table 5.5: Factor cross-correlations

As mentioned previously, unrotated Factor 1 accounted for 29% of the study variance. Rotating the factors reduced the sorts that significantly loaded on Factor 1 and increased the number of sorts loading on Factors 2, 3 and 4. Since rotating the factors preserves the relative loadings of each sort, there was an inevitable increase in the correlations between the factors. Although sorts were loaded away from Factor 1, the importance of this factor in relation to the other factors was not decreased, hence the increase in cross-correlation. The high correlation between Factor 1 with both Factors 3 and 4, and the high correlation between Factors 3 and 4 does mean that some of the participants display aspects of both viewpoints and explains the large number of confounded sorts. The cause of these correlations is the shared opinion displayed in unrotated Factor 1, so high cross-correlations would also have featured in a 2, 3 or 5-factor solution.

5.5.2 Factor arrays

Each factor should be thought of as a 'model sort' of a fictitious participant. PQMethod displays the score given by this fictitious participant for each statement, and a Z-score, which is a score that is weighted according to the number of participants loading on each factor, thus allowing cross-factor comparisons to be made. Appendix D shows the statement Z-scores and Q sort scores for Factors 1-4. I have situated the factor arrays in Chapter 6: Figure 7.1, Figure 7.3, Figure 7.5 and Figure 7.7.

It is conventional within Q methodology studies to (re)create the single Q sort that represents the viewpoint of each factor, known as a *factor array*. After creating each array, I chose to colour code them (and the score sheet shown in Appendix D) as follows:

- Consensus statements (calculated by PQMethod to score not statistically significantly differently on any factor) are green,
- Positive distinguishing statements (calculated by PQMethod to score significantly higher on that factor than any other) are yellow,
- Negative distinguishing statements (calculated by PQMethod to score significantly lower on that factor than any other) are red.

Distinguishing statements calculated by PQMethod highlight statements that are ranked significantly differently by one factor compared with another. This means that not all distinguishing statements will be significantly higher or lower, they may be significantly 'in the middle'. These are marked in grey in Appendix D. I chose not to highlight these on the Factor arrays for ease of analysis. To fit each array on a single page I reduced the words in some statements.

5.5.3 Consensus statements

Stephenson created Q methodology specifically to expose and study the different categories of opinion that would be masked by a single majority viewpoint (Watts and Stenner, 2012). This happens in both the factor analysis and rotation stages: factor analysis maximises the variation that is explained by the extracted factors, and rotation sharpens the distinctiveness of the factors and increases the number of sorts that load on exactly one factor. Other than consensus statements, presenting and analysing the factors is about explaining what is different, not what is the same, about teachers' opinions of mastery.

I chose to use Q methodology for my research study precisely because the study's focus is exploring *differences* in teachers' notions of mastery, rather than similarities. However, centroid factor analysis of the Q sorts calculated that 29% of the total variance in the study was explained by a single unrotated factor. This means that a 'model' sort created to mathematically best represent the opinion of the whole set of participants would account for 29% of the total variability observed in all the participants' sorts. This is of course an average: some sorts would closely resemble this viewpoint and others would be very different. It is important to acknowledge that there are areas of agreement about mastery, and to consider what these are. The factor rotation process reduced the dominance of a single factor, allowing more nuanced viewpoints to emerge. However, this meant there was inevitable cross-correlation between the factors.

I first considered whether participants considered mastery as about *learning* or about *teaching* and found that participants gave higher rankings to statements about learning. This was calculated by comparing the sum of total participant scores for the statements about 'learning' (+32) and the statements about 'teaching' (-32) (see Appendix A).

Secondly, I identified and studied areas of shared opinion through scrutinising the *consensus statements*, calculated by PQMethod. Consensus statements are those that participants in all four factors give scores that are not statistically significantly different. There were five consensus statements, displayed in Table 7.1. Given the existence of high factor correlations, particularly between Factor 4 and the others (0.48, 0.35 and 0.52), the existence of only five consensus statements increased the evidence that the four factors were distinct in nature, rather than different aspects of the same viewpoint. Three consensus statements (starred in Table 7.1) are significant at the 95% level; other statements are significant at the 90% level.

Interpreting these consensus statements (see Chapter 7.6) was complex because four-fifths of the consensus statements have negative scores, indicating areas of ‘shared disagreement’, rather than shared agreement.

5.6 Conclusion

Chapter 5 described in detail the processes for collecting, organising, and analysing the 45 Q sorts completed by my research participants. In relation to Brown’s (1980) data organisation and reduction diagram (Figure 3.2), Chapter 5 represents how the study reduced an infinite possibility of opinions (the Q statements and their relative position in a Q sort) into four attitudes (the Factors).

Throughout the chapter I emphasised the steps taken to maximise transparency and replicability and described and analysed bias. Where a researcher decision was made, such as how many factors to retain, I gave full explanations, utilising statistical techniques. In Chapter 6 detail the method for interpreting these collected and reduced data to address the research question.

6 Data interpretation

6.1 Introduction

This chapter reports the methods I used to analyse and interpret the Q study data, to address the research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?'.

I begin by discussing why there is no single method for interpreting Q study data, and how I devised a method by adapting approaches used by established Q methodologists. In addition to the standard 'factor array' I used colour coding of distinguishing statements and created bespoke infographics. I then detail how I used these visual representations to construct factor narratives.

In the later part of the chapter, I discuss how I integrated the characteristics of the study's participants into the analysis: how and why I separated factor 'attitudes' from those who hold them, whilst developing a complete picture of each viewpoint and how they interact. I present data tables segmenting characteristics of the participants who were flagged on each factor, and the average time taken to complete the sort.

6.2 Factor interpretation strategy and method

The data analysis in Chapter 5 was straightforward and unambiguous: centroid factor extraction and Varimax rotation are mathematical algorithms yielding identical results regardless of how many times they are run. The choice to extract four factors was mine, but the decision was made with the assistance of statistical tests. I 'flagged' significant sorts according to fixed criteria. After factor extraction and rotation were completed PQmethod yielded an enormous amount of statistical information about each factor. It was now the role of the researcher to 'get under the skin' of this data to understand exactly what did constitute the meanings of mastery for each factor.

There is no single method that Q methodologists advise in relation to factor interpretation: Watts and Stenner (2012, p. 147) concede a lack of published literature on this. Brown (1980) emphasises the researcher must focus on the relative scores of the statements as an expression of an attitude and creates an in-depth understanding of each factor separately. Watts and Stenner provide a detailed description of their approach to factor interpretation, as does Ramlo (2014), through her series of YouTube videos, and more briefly in her Q study publications (e.g. 2017a). Contemporary Q methodologists are clear that each factor should be simultaneously analysed standalone and together, to capture the holistic as well as the atomistic nature of each factor (Watts and Stenner, 2012), so my representations needed to facilitate this analysis. My factor interpretation strategy and method used both sources as inspiration but included increased visual representation. A key method for me in interpreting the factors was transforming the statistically generated data tables for each extracted and rotated factor into useful representations which I could explore and explain.

6.2.1 Colour-coded factor arrays

Factor interpretation started with the creation of the factor arrays (Chapter 5.5.2). Watts and Stenner, (2012, p. 140) present example factor arrays with only the statement or item number in each box, and this is where I began. However, I found that constantly 'looking up' each statement number in a table compromised my ability to understand each factor, and I also could not easily see the score on each statement for a factor compared with others. Hence, I created arrays displaying the full statements and with the colour coding described in Chapter 5.5.2. The arrays displayed in Figure 7.1, Figure 7.3, Figure 7.5 and Figure 7.7 are condensed to A4, but were printed a much larger size, and placed side-by-side on a noticeboard, during the interpretation process.

After the factor arrays were created, I created a strategy to rigorously compare the scores on each statement for each factor with all other factors. Watts and Stenner use a 'crib sheet' technique (2012, p. 150). For each factor they identify statements ranked highest and lowest, and statements ranked higher and lower compared to other factors. My colour coding on the factor arrays mirrors this process. Identification of these statements draws the researcher's attention to statements that are ranked both absolutely and relatively high and low.

6.2.2 Factor infographics

I found the factor arrays a necessary but insufficient representation of each factor's viewpoint. I wanted to observe the importance each factor viewpoint placed on each theme and subcategory. I created an infographic for each factor (see for example Figure 6.1, which is a partial reproduction of Factor 1's infographic), as a visual alternative to Watts' and Stenner's (2012) crib sheet. My infographic partly resembles a mathematical 'bar model' which is a representation used as part of mastery, and whilst I did not consciously set out to do this, I have since reflected on whether I deliberately created this model due to the nature of the topic. I found the process of creating the infographic a useful way of deepening my own understanding of the data, and the infographics were primarily created for my use in interpretation. However, mathematics teachers, a likely interested audience, may appreciate this representation. I used a piece of commercial software, Venngage (2011), to create the infographics. To make the data compatible with the capabilities of Venngage, I had to add 4 to each statement score to get rid of the negative numbers. This does change the ratio of the scores to each other in some sense, but I could find no better solution within the time and resource constraints of the research. I also had to abbreviate the themes, categories, and statements for ease of display and due to software constraints.

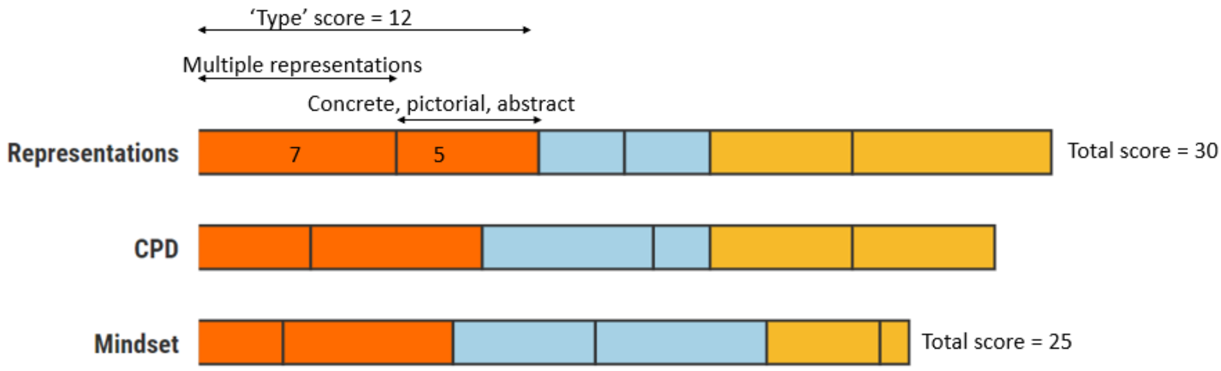


Figure 6.1: Sample infographic representation

The infographic in Figure 6.1 was created as described below.

The **length of each bar** represents the total score for each of the eight themes, ordered highest to lowest. The categories are the same as Table 2.1, with some names shortened for ease of reading. The longest bar unit length corresponds to the score of the highest-ranking theme (in Figure 6.1 the highest scoring theme is 'Multiple Representations', displayed as 'Representations') with other bars sized relative to this (see limitation in above paragraph). The total score for the statements in the 'Representations' theme in Figure 6.1 was +6 (scored in the software as 6 + (4 x 6) = 30) and the total score for the statements in the 'Mindset' theme was +1 (scored in the software as 1 + (4 x 6 = 25).

The **colours on each bar** differentiate the three subcategories on each theme. The 'Representations' theme in Figure 6.1 has three subcategories; 'type,', 'understanding' and 'construction', and the relative length of each coloured section represents the total score for each statement in that subcategory (with 4 added to each statement). The total score for the 'type' subcategory in the 'Representations' bar in Figure 6.1 was +4 (scored in the software as 4 + (4 x 2) = 12).

The **lines on each bar** show the relative score for each of the two statements in each subcategory. In Figure 6.1 statement 31, '*Mastery lessons should incorporate multiple representations of a concept',* abbreviated to 'multiple representations', scored +3 (scored as 7), and statement 32, *'To master mathematics is to understand mathematics using concrete, pictorial and abstract representations',* scored +1 (scored as 5). The higher scoring abbreviated statement title was displayed on the infographic.

I had not seen Q factors represented as infographics before, and these infographics assisted me greatly in understanding what was distinctive and shared within the data: for example, it highlighted the relative importance of representations within mastery in every factor. Creating and developing a series of infographics was a deliberate attempt to make sense of the different types of information individually and holistically. I ensured that the procedures for colour coding and display were transparent and applied consistently to all factors, in accordance with my postpositivist position. The colour coding and infographics will also help non-Q methodologists interpret the data.

Once the participant characteristics had been analysed and the narratives completed (see Chapter 6.3) I created four 'one-pagers' (see Figure 7.2: Factor 1 infographicFigure 7.2, Figure 7.4, Figure 7.6, Figure 7.8) which combined the bars with conventional statistics and charts to provide a useful summary of the findings, especially for non Q-methodology specialist.

6.2.3 Factor narratives

Once I was confident in my own knowledge from the data tables, factor arrays, and infographics, I used the colour-coded arrays and infographics to write a narrative account of what each Factor believes mastery to mean, in relation to their own practice and the learning of their students. I accounted for bias in the narratives by referencing the statements, and their scores, that influenced each observation or description. I wrote a summary paragraph and gave each factor a name (in italics) as an *aide memoir* for the reader. An initial narrative was drafted before the characteristics of the participants aligning with each factor was known, then updated post analysis (see Chapter 6.3).

Q methodologists debate the efficacy of naming factors and I consulted the Q-METHOD LISTSERVE in this matter. Brown (2020a) recommends referring to the factors as (e.g.) Factor 1, 2, etc which 'liberates the investigator to direct attention to meanings that originate from the phenomena rather than social conventions.' However, Kraak (2020) insists that the purpose of the Q researcher is to interpret the mathematics, and that names 'help people who are unfamiliar with this mixed-method research approach to understand and visualize the contribution of the qualitative nature of this methodology'. My decision was to name the factors to help the reader, whilst routinely referring to (e.g.) 'Factor 1' in the thesis where possible. I named the factors before looking at the characteristics of the participants, so the name reflects my interpretation of the Q sort data only.

6.3 Participants holding each factor viewpoint

I completed the colour coded factor arrays, the infographic bars, and a draft factor narrative before interrogating the characteristics of the participants. Brown (1980) demands researchers consider 'attitudes as attitudes quite independently of whoever may have provided them'. And Watts and Stenner (2012) emphasise interpreting each factor 'on its own and together' to explain the whole viewpoint, crucial to the nature of Q methodology and the role of abductive reasoning (Watts and Stenner, 2012). I also wanted to reduce researcher bias in the interpretation of the nature and opinions of each factor, as per my stated claim to a postpositivist position in Chapter 3.3.

Watts and Stenner emphasise the purpose of understanding the defining characteristics of participants who have the views of a particular factor is about *feeling* (2012, p. 159). Whilst I disagree that the researcher should put herself in the metaphorical shoes of the participants, I do consider reasons why a point of view and a characteristic might be linked. When updating the factor narrative I used relevant literature, reviewed in Chapter 2, and the qualitative comments made in the post sort questions as evidence for my explanations.

6.3.1 Characteristics of participants holding the views of each factor

Table 6.1 displays the characteristics of the participants who identify with the views of Factors 1, 2, 3, and 4 and how they compare with the whole group.

	All	**Factor 1**	**Factor 2**	**Factor 3**	**Factor 4**
N	45	9	4	9	7
Gender	60% female	80% female	100% male	67% female	86% female
Ages	11% in 20s 20% in 30s 36% in 40s 27% in 50s 4% 60+	10% in 20s 0% in 30s 40% in 40s 40% in 50s 10% 60+	0% in 20s 50% in 30s 25% in 40s 25% in 50s 0% 60+	22% in 20s 22% in 30s 11% in 40s 33% in 50s 0% 60+	29% in 20s 14% in 30s 58% in 40s 0% in 50s 0% 60+
Location	45% London and South 18% Midlands and East of England 31% North 6% National/other	40% London and South 20% Midlands and East of England 20% North 20% National/other	25% London and South 50% Midlands and East of England 25% North 0% National/other	44% London and South 0% Midlands and East of England 55% North 0% National/other	29% London and South 14% Midlands and East of England 57% North 0% National/other
Occupation	64% practicing teachers 20% teacher educators 7% trainee teachers 4% former teachers	20% practicing teachers 50% teacher educators 10% trainee teachers 20% other	100% practicing teachers	67% practicing teachers 11% teacher educators 11% trainee teachers 11% former teachers	86% practicing teachers 14% teacher educators
Teaching experience	32% under ten years 36% 10-20 years 33% over 20 years	20% under ten years 40% 10-20 years 40% over 20 years	25% under 10 years 25% 10-20 years 50% over 20 years	44% under 10 years 33% 10-20 years 22% over 20 years	43% under 10 years 43% 10-20 years 14% over 20 years
Type of school mostly taught in	78% non-selective 9% grammar	70% non-selective 10% grammar	100% non-selective	100% non-selective	57% non-selective 14% grammar
Proportion of maths in degree	13% less than one quarter 33% more than three quarters	20% less than one quarter 40% more than three quarters	25% less than one quarter 0% more than three quarters	0% less than one quarter 44% more than three quarters	43% less than one quarter 29% more than three quarters
Training route	67% University PGCE 22% Employment PGCE	60% University PGCE 10% Employment PGCE 30% other	100% University PGCE	67% University PGCE 33% Employment PGCE	57% University PGCE 29% Employment PGCE 14% SKITT
Completed SKE?	24% Yes	10% Yes	0% Yes	22% Yes	43% Yes
Mastery CPD undertaken	42% NCETM 12% Complete maths 16% Other 29% No	50% NCETM 20% Other/DNA 30% No	25% NCETM 25% Complete maths 25% Other 25% No	55% NCETM 18% Complete maths 9% Other 18% No	44% NCETM 11% Complete maths 33% Other 11% No
Opinion of mathematics	‘A Game' 33% 'A language' 27% 'Absolute' 29% 'Fallible' 11%	‘A Game' 10% 'A language' 20% 'Absolute' 30% 'Fallible' 40%	‘A Game' 75% 'A language' 25%	‘A Game' 33% 'A language' 22% 'Absolute' 44%	‘A Game' 29% 'A language' 29% 'Absolute' 43%

Table 6.1: Characteristics of participants loading on each factor

6.3.2 Q sorting time

The distribution of time taken to complete each sort differs by factor (Figure 6.2). The differing numbers of participants associated with each factor (ten, four, nine and seven respectively) limits the significance of differences in time taken to complete the sort.

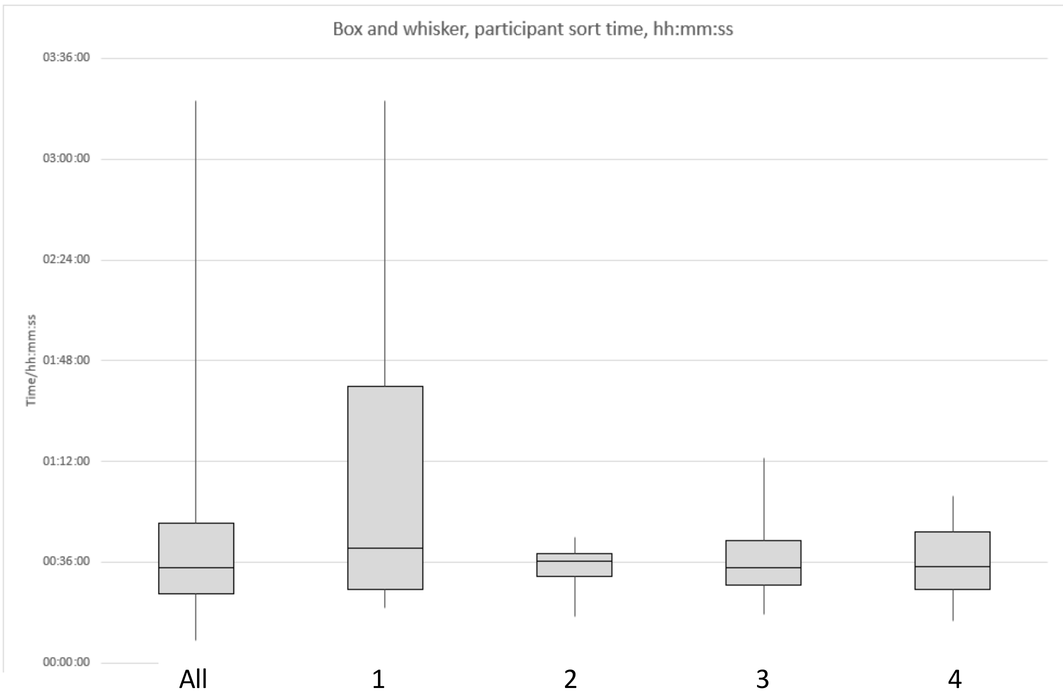


Figure 6.2: Distribution of sort time by factor

6.4 Conclusion

Chapter 6 concludes the data collection, analysis, presentation, and interpretation methods. I described and justified all choices made, including how I used and adapted standard Q methodology presentation methods. Chapter 7 displays the results of this work, and the study. I address the research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?' by presenting viewpoints of mastery held in common, and in conflict, as reported by the study participants.

7 Results: Viewpoints of mastery

7.1 Introduction

Chapter 7 includes, and is limited to, a presentation and interpretation of the viewpoints of mastery extracted from the study data, using the method described in Chapter 6. Each of the four factors is presented in turn and the data displayed in the order of:

1. Factor name and short narrative paragraph,
2. Brief description of the characteristics of the participants whose viewpoints were aligned with the factor,
3. Factor array,
4. Factor infographic 'one-pager',
5. Factor narrative, with reference to statement scores and participant quotes.

Following this, the Chapter presents data on areas of consensus, and interconnection between the four factors.

7.2 Factor 1: 'Travel far, travel together.'

Mastery is a specific pedagogy driven by the belief that all students should attain highly in mathematics. Students should be kept together, with no students allowed to fall behind. Mastery is not necessarily the answer to improving standards and closing gaps in secondary schools.

Teachers should develop a high level of subject and pedagogical knowledge to design and teach lessons that allow students to construct their own notions of mathematical concepts. This is achieved by exploring multiple representations of a concept and by using exercises which expose what is similar and what is different in the mathematics being studied. Teachers do not require specific external professional development to develop a mastery pedagogy.

Ten out of 45 sorts shared a viewpoint consistent with Factor 1, explaining 14% of the variance. There were 16 distinguishing statements for Factor 1. Factor 1 participants most strongly associated mastery with the themes of: Representation, Continued Professional Development and Mindset. They least associated mastery with the themes of: Steps, Methods, and Assessment. Factor 1 participants scored learning-focused statements at +10 and teaching focused-statements at -10. Statements about learning were scored higher by Factor participants than any other factor.

The nine participants representing Factor 1 were older and more experienced (in both mathematics and teaching) than the average respondent. Factor 1 included a high proportion of teacher educators. They were the least likely group to have undertaken mastery CPD. They were the only factor to consider mathematics as fallible. Participants in Factor 1 had the most variable sort times.

-4	-3	-2	-1	0	1	2	3	4
38: Rote-learning is an inevitable part of mastery learning	30: Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	46: Teaching for mastery pedagogy is mainly learnt through external professional development	6: Mastery will be easier to attain if children are taught in groups of mixed prior attainment	24: Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student	32: To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	37: Rote-learning is incompatible with mastery learning	36: In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing	3: All students are capable of achieving a mastery level of attainment
	12: To achieve mastery, in mathematics lessons students should all be working on different problems	15: A curriculum for mastery should give equal priority to number, algebra, geometry and data handling	19: To achieve mastery, students should be explicitly taught mathematical laws, including their formal names	41: In mastery lessons problem-solving is developed through exercises which combine topics	13: In a mastery curriculum students will understand the structure of number before applying it to other topics	44: Mastery professional development activities should include a high degree of teacher subject knowledge development	29: Teaching for mastery should minimise lecturing and maximise student participation	
	4: In general, 1/3 of students will achieve a mastery standard, 1/3 of students will achieve an average standard, and 1/3 of students will achieve a low standard of attainment	17: Planning mastery lessons is quicker because there are no differentiated resources to create	33: Multiple representations are not always needed in secondary school teaching for mastery	20: To achieve mastery, students should understand mathematical laws (for instance the commutative, distributive and associative laws) but do not need them to be explicitly taught	48: Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps	10: Teaching for mastery involves students keeping up, not catching up	31: Mastery lessons should incorporate multiple representations of a concept	
		25: In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	7: Learners should move through a mastery curriculum at their own pace, moving on once they reach the the expected level of attainment	39: Practising similar problems is part of developing a mastery understanding of mathematics	35: In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	40: Practising a variety of problems is part of developing a mastery understanding of mathematics		
		26: In mastery lessons, all questions set should reflect only what has been explicitly taught	22: Mastering mathematics is unconnected with specific teacher pedagogies	11: To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	14: In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	45: Mastery professional development activities should include a high degree of specific pedagogy development		
		2: In mastery lessons assessments should only be used at the beginning and end of topics	21: A student is more likely to achieve mastery if a teacher uses a specific pedagogy	43: Reading, and taking part in, educational research is an important aspect of teaching for mastery	42: In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	8: Learners should move through a mastery curriculum as a group, only moving on once all students have reached the expected level of attainment		
			5: Mastery will be easier to attain if children are taught in groups of similar prior attainment	28: In mastery lessons, complex problems should be reduced by the students into a series of small steps	9: Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up			
			34: A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	1: In mastery lessons all students should be assessed every lesson	18: Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises			
			27: In mastery lessons, complex problems should be reduced by the teacher into a series of steps	23: Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student	47: Teaching for mastery pedagogy is mainly learnt through collaborative in-school CPD			
				16: A curriculum for mastery should give greater priority to number and algebra				

Green=consensus statement, yellow= positive distinguishing statement, red= negative distinguishing statement

Figure 7.1: Factor 1 factor array.

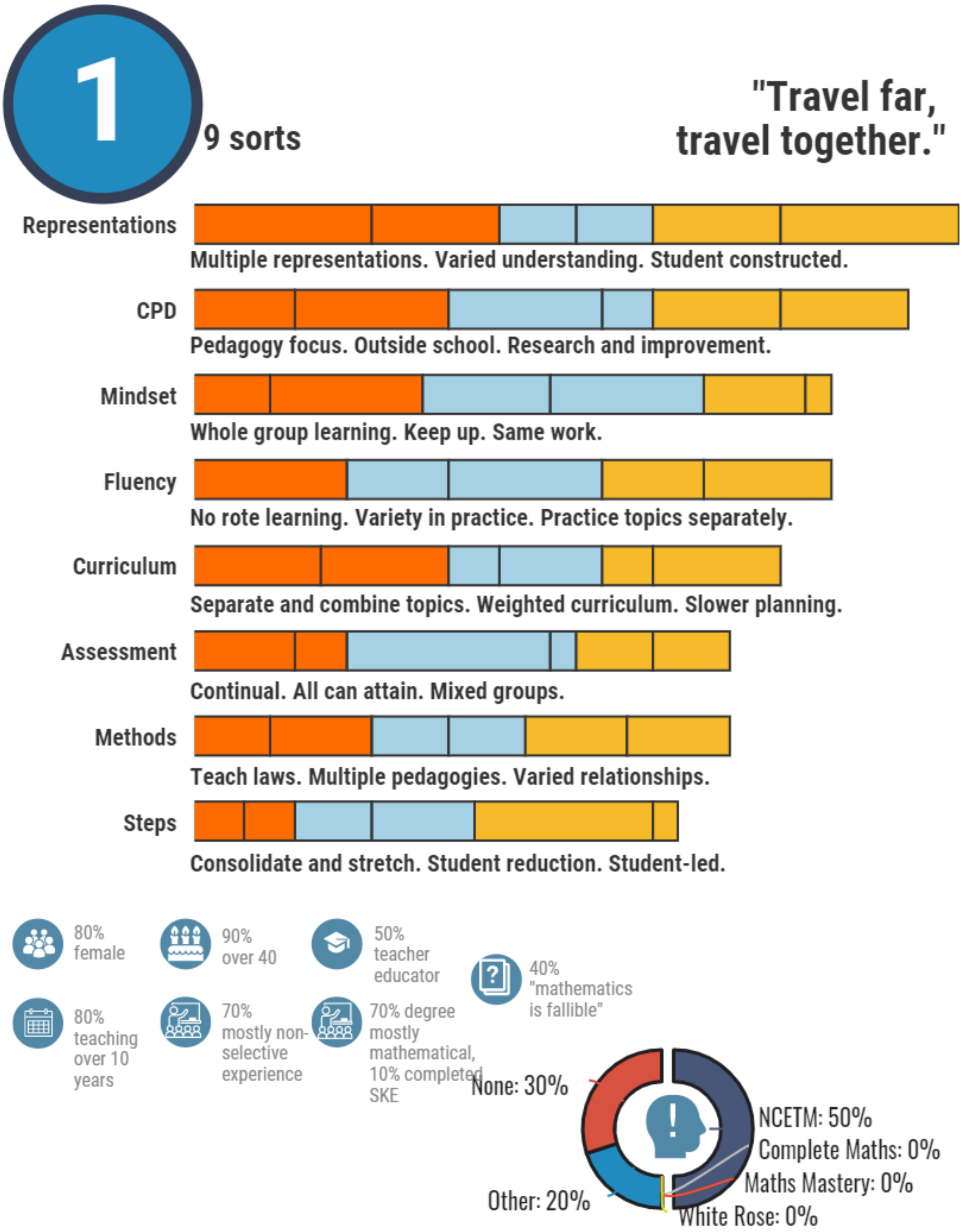


Figure 7.2: Factor 1 infographic

Participants who load on Factor 1 believe more than all other factors that all students can achieve a mastery level of attainment (statement 3, scoring +4). This mindset means that students who find something challenging must be helped to keep up rather than having to catch up with others (statement 10, scoring +2). Teaching for mastery is a way (but not the only way) to close the current attainment gap (statement 48, scoring +1, less than some other factors).

'I believe that everyone can achieve in mathematics, and find the most important take-away from international studies is that high-performing education systems around the world do not have ideas of fixed ability entrenched in society or schooling.'

(Participant 13, Factor 1).

Factor 1 participants feel strongly that teaching for mastery should maximise the opportunity for student participation in the lessons, and that students construct their own learning though noticing similarities and differences in the mathematics that they are doing (statement 29, scoring +3 and statement 36, scoring +3). Participants in this factor feel more strongly than any other factor that students do not learn mathematics by rote (statement 38, scoring -4 and statement 37, scoring +2).

Teachers should develop a high level of mathematical and pedagogical knowledge to enable this learning (statements 44 and 45, both scoring +2). This combination of knowledge enables them to plan lessons that incorporate multiple representations of a concept (statement 31, scoring +3), give students the opportunity to notice (statement 29, scoring +3) and allow students to practice a variety of problems (statement 40, scoring +2).

'I believe that strong, flexible and connected subject knowledge is an essential prerequisite for successful teaching for mastery. Therefore mastery professional development activities must intertwine the development of teachers' subject and pedagogical knowledge.'

(Participant 30, Factor 1)

Teaching for mastery is not about imparting knowledge to students (statement 30, scoring -3).

Factor 1 participants are divided in their views about whether students should be set questions at a level beyond what they have already been taught (statements 25 and 26 both scoring -2). Factor 1 participants do not have strong views about the nature of the teacher-student relationship as either mentor-mentee or novice-expert (statements 24 and 23 both scoring 0).

Factor 1 participants believe a mastery curriculum needs to be carefully designed and weighted, favouring certain topics (statement 15, scoring -2). Curriculum design should allow understanding of number to develop both before, and through an understanding of other topics (statements 13 and 14 both scoring +1). Factor 1 participants believe more than the participants in any other factor that all students in a class should move through the curriculum together, only moving on once everyone has reached the expected level (statement 8, scoring +2). Students should not move through the curriculum at their own pace (statement 7, scoring -1).

'This (moving through the curriculum together) is the basis of mastery maths.'

(Participant 3, Factor 1)

Factor 1 participants do not believe that a teaching for mastery pedagogy is developed though external professional development (statement 46 scoring -2) and do not have strong views about the role of educational research as part of teaching for mastery (statement 43, scoring 0).

'I partly feel that Teaching for Mastery is becoming its own pedagogy-style, but also that it encompasses lots of pedagogic styles and it is more about the opportunity provided for the learners to learn.'

(Participant 9, Factor 1)

7.3 Factor 2: ‘Know your limits, follow the teacher.’

Mastery can be translated as a level of attainment, which not all students will attain. Students are most likely to achieve mastery if they already attain highly and are taught with similar students. Teachers should have good subject knowledge which they transmit to students. Students learn by rote and by practising a variety of problems set by the teacher that are appropriate to their own attainment level.

Mastery is unconnected with specific methods of teaching, so teacher professional development or engagement with research is of limited importance.

Four out of 45 sorts significantly loaded on Factor 2, explaining 7% of the variance. There were 18 distinguishing statements for Factor 2. Factor 2 participants most strongly associated mastery with the themes of: Fluency, Representation and Methods. They least associated mastery with the themes of: Assessment, Steps and Curriculum. Factor 2 scored learning-focused statements at +9 and teaching focused-statements at -9, so favoured mastery as learning over mastery as teaching.

All four participants who loaded on Factor 2 were serving male teachers who had spent most of their time in non-selective schools. They completed the sort much quicker than participants loading on other factors (see 6.3.2). All trained to teach via a traditional university PGCE. Three out of four had taught for over 10 years, and three out of four had a mostly mathematical degree. They had different experiences of mastery CPD. Three out of four believed that mathematics was a ‘game’. The Factor 2 participants had the lowest average sort time.

-4	-3	-2	-1	0	1	2	3	4
21: A student is more likely to achieve mastery if a teacher uses a specific pedagogy	3: All students are capable of achieving a mastery level of attainment	17: Planning mastery lessons is quicker because there are no differentiated resources to create	15: A curriculum for mastery should give equal priority to number, algebra, geometry and data handling	27: In mastery lessons, complex problems should be reduced by the teacher into a series of steps	38: Rote-learning is an inevitable part of mastery learning	22: Mastering mathematics is unconnected with specific teacher pedagogies	40: Practising a variety of problems is part of developing a mastery understanding of mathematics	5: Mastery will be easier to attain if children are taught in groups of similar prior attainment
	37: Rote-learning is incompatible with mastery learning	1: In mastery lessons all students should be assessed every lesson	32: To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	45: Mastery professional development activities should include a high degree of specific pedagogy development	42: In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	35: In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	39: Practising similar problems is part of developing a mastery understanding of mathematics	
	25: In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	29: Teaching for mastery should minimise lecturing and maximise student participation	43: Reading, and taking part in, educational research is an an important aspect of teaching for mastery	46: Teaching for mastery pedagogy is mainly learnt through external professional development	44: Mastery professional development actvities should include a high degree of teacher subject knowledge development	30: Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	33: Multiple representations are not always needed in secondary school teaching for mastery	
		6: Mastery will be easier to attain if children are taught in groups of mixed prior attainment	34: A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	47: Teaching for mastery pedagogy is mainly learnt through collaborative in-school professional development with colleagues	10: Teaching for mastery involves students keeping up, not catching up	41: In mastery lessons problem-solving is developed through exercises which combine topics		
		12: To achieve mastery, in mathematics lessons students should all be working on different problems	14: In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	19: To achieve mastery, students should be explicitly taught mathematical laws including their formal names	31: Mastery lessons should incorporate multiple representations of a concept	23: Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student		
		2: In mastery lessons assessments should only be used at the beginning and end of topics	8: Learners should move through a mastery curriculum as a group	9: Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up	7: Learners should move through a mastery curriculum at their own pace	13: In a mastery curriculum students will understand the structure of number before applying it to other topics		
			11: To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	18: Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises	28: In mastery lessons, complex problems should be reduced by the students into a series of small steps			
			24: Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student	48: Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps	20: To achieve mastery, students should understand mathematical laws but do not need them to be explicitly taught			
			26: In mastery lessons, all questions set should reflect only what has been explicitly taught	16: A curriculum for mastery should give greater priority to number and algebra	4: In general, 1/3 of students will achieve mastery standard, 1/3 of students will achieve average standard, and 1/3 of students will achieve a low standard			
				36: In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing				

Green=consensus statement, yellow= positive distinguishing statement, red= negative distinguishing statement

Figure 7.3: Factor 2 factor array

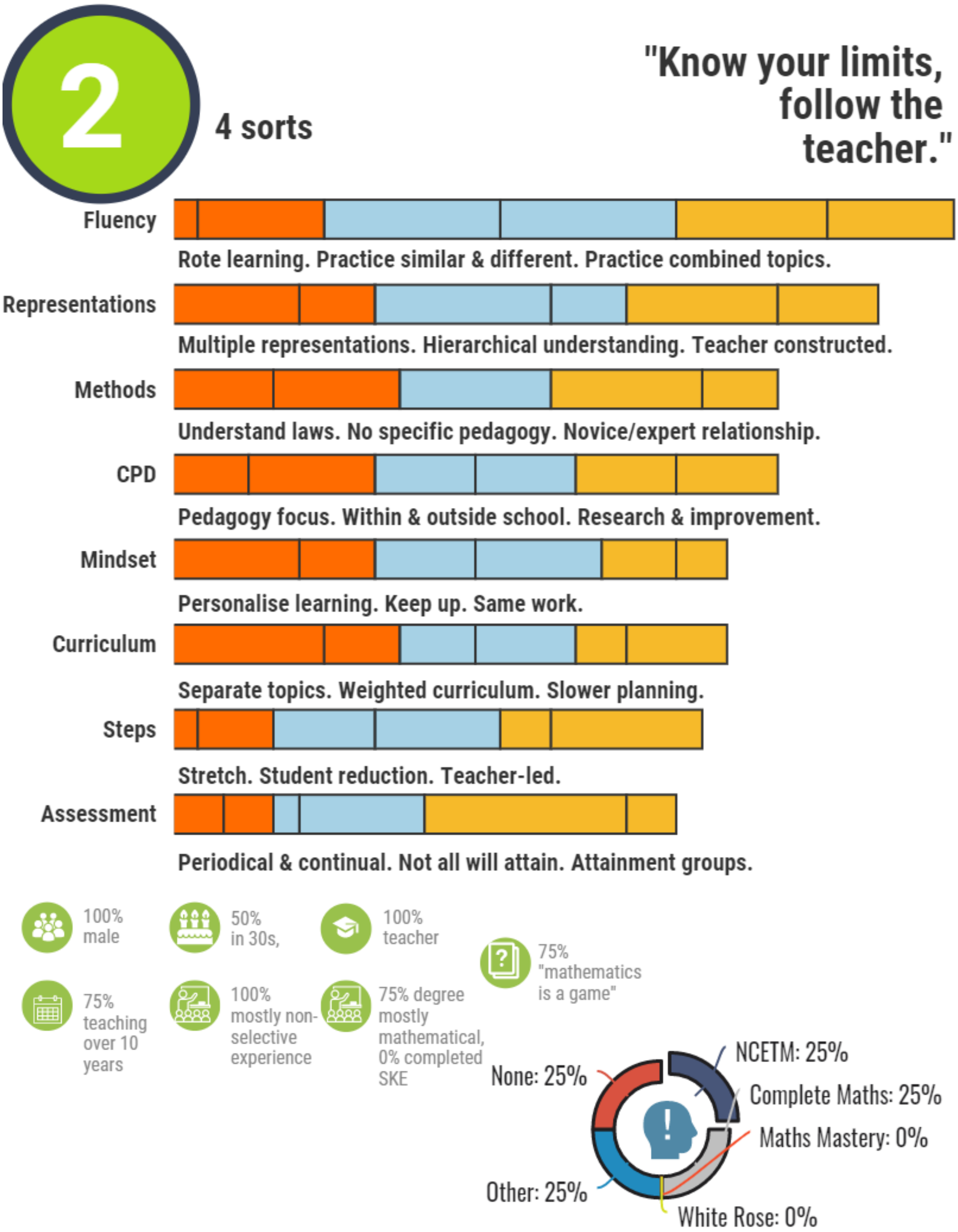


Figure 7.4: Factor 2 infographic

Factor 2 participants strongly believe that mastery is achieved by students taught in groups of similar prior attainment (statement 5, score +4). Mastery has nothing to do with multiple representations (statement 33, scoring +3) or pedagogy (statement 21, scoring -4 and statement 22 scoring +2) and everything to do with students doing lots of practise (statements 39 and 40, both scoring +3).

Factor 2 participants believe more than any other factor that mastery lessons feature a 'novice-expert' relationship between the teacher and the students (statement 23, scoring +2) with teachers imparting knowledge to their students (statement 30, scoring +3 and statement 29 scoring -2). Teachers should carefully explain and select appropriate problems to enable the students to learn (statement 35, score +2). Unlike the other factors, Factor 2 believes that a degree of rote-learning by students is perhaps inevitable (statement 38, score +1) and is certainly not incompatible with mastery learning (statement 37, score -3).

Factor 2 participants, unlike other groups, do not believe that all students are capable of a mastery level of attainment and agree that perhaps one-third of students will achieve mastery, one-third will attain at an average level and one-third will attain at a below average level (statement 3, scoring -3 and statement 4 scoring 1).

'Maybe the achievement gap is inevitable and therefore should be embraced?'

(Participant 41, Factor 2).

Within a group, the students practise problems carefully constructed by the teacher (statement 35, scoring +2) and move through the curriculum at their own pace (statement 7, scoring +1).

Factor 2 participants are the only group who entirely detach mastery from pedagogy (statement 21, scoring -4). Factor 2 participants associate mastery with a high level of teacher subject knowledge (statement 44, score +1) rather than pedagogy (statement 45, score 0) and, perhaps because of this, do not rate either in-school or external Continuing Professional Development highly as a way of developing mastery pedagogy (statements 46 and 47, both scoring 0).

'I give little regard to CPD'.

(Participant 41, Factor 2)

Factor 2 participants do not think that multiple representations are always needed in secondary school to develop mastery (statement 33, scoring +3), nor do they associate mastery with students being able to demonstrate their understanding of mathematics by using different representations (statement 32, scoring -1).

7.4 Factor 3: 'Create a curriculum for interconnected understanding.'

Mastery is needed to both raise overall attainment and close attainment gaps. Mastery learning is an understanding of the structure of number and how number connects with other mathematical topics. Mastery understanding can be developed and demonstrated by exploring multiple representations of a concept, and an abstract understanding should not be a goal.

Mastery is developed in students by teachers who engage with research and professional development to ensure they have good and relevant subject knowledge and pedagogy. Teachers understand the importance of curriculum and lesson design. Teachers design the curriculum that gives priority to developing number and algebra, and design lessons which include careful explanation and exercises which help students notice mathematical structure. These lessons feature small-steps progression and multiple representation.

Teachers do not believe there is a single 'teaching method' or an ideal 'teacher-student relationship' for mastery: if the curriculum and lessons are well designed, all children can achieve mastery.

Nine out of 45 sorts significantly loaded on Factor 3, explaining 13% of the variance. There were 14 distinguishing statements for Factor 3. Factor 3 participants most strongly associated mastery with the themes of: Continued Professional Development, Representation and Fluency. They least associated mastery with the themes of: Assessment, Mindset, and Methods. Factor 3 scored learning-focused statements at +6 and teaching focused-statements at -6. Factor 3's score for learning-focused statements was lower than the other 3 factors.

Factor 3 participants were connected by a lack of commonality in their demographic characteristics. The study participants who held the views of Factor 3 were a mix of different ages and current and former teachers, and just under half had taught for less than 10 years. Two-thirds had a degree that was predominantly mathematical, and they had mixed views about the nature of mathematics. Over half of the participants associated with Factor 3 had undertaken mastery CPD with the NCETM.

-4	-3	-2	-1	0	1	2	3	4
22: Mastering mathematics is unconnected with specific teacher pedagogies	12: To achieve mastery, in mathematics lessons students should all be working on different problems	25: In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	11: To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	23: Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student	41: In mastery lessons problem-solving is developed through exercises which combine topics	44: Mastery professional development activities should include a high degree of teacher subject knowledge development	35: In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	48: Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps
	15: A curriculum for mastery should give equal priority to number, algebra, geometry and data handling	17: Planning mastery lessons is quicker because there are no differentiated resources to create	28: In mastery lessons, complex problems should be reduced by the students into a series of small steps	8: Learners should move through a mastery curriculum as a group.	32: To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	39: Practising similar problems is part of developing a mastery understanding of mathematics	27: In mastery lessons, complex problems should be reduced by the teacher into a series of steps	
	4: In general, 1/3 of students will achieve mastery standard, 1/3 of students will achieve average standard, and 1/3 of students will achieve a low standard	5: Mastery will be easier to attain if children are taught in groups of similar prior attainment	1: In mastery lessons all students should be assessed every lesson	42: In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	19: To achieve mastery, students should be explicitly taught mathematical laws including their formal names	40: Practising a variety of problems is part of developing a mastery understanding of mathematics	31: Mastery lessons should incorporate multiple representations of a concept	
		33: Multiple representations are not always needed in secondary school teaching for mastery	34: A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	30: Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	14: In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	45: Mastery professional development activities should include a high degree of specific pedagogy development		
		7: Learners should move through a mastery curriculum at their own pace, moving on once they reach the the expected level	29: Teaching for mastery should minimise lecturing and maximise student participation	10: Teaching for mastery involves students keeping up, not catching up	3: All students are capable of achieving a mastery level of attainment	36: In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing		
		2: In mastery lessons assessments should only be used at the beginning and end of topics	38: Rote-learning is an inevitable part of mastery learning	26: In mastery lessons, all questions set should reflect only what has been explicitly taught	13: In a mastery curriculum students will understand the structure of number before applying it to other topics	43: Reading, and taking part in, educational research is an an important aspect of teaching for mastery		
			6: Mastery will be easier to attain if children are taught in groups of mixed prior attainment	18: Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises	16: A curriculum for mastery should give greater priority to number and algebra			
			37: Rote-learning is incompatible with mastery learning	21: A student is more likely to achieve mastery if a teacher uses a specific pedagogy	46: Teaching for mastery pedagogy is mainly learnt through external professional development			
			20: To achieve mastery, students should understand mathematical laws (for instance the commutative, distributive and associative laws) but do not need them to be explicitly taught	47: Teaching for mastery pedagogy is mainly learnt through collaborative in-school professional development with colleagues	9: Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up			
				24: Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student				

Green=consensus statement, yellow= positive distinguishing statement, red= negative distinguishing statement

Figure 7.5: Factor 3 factor array

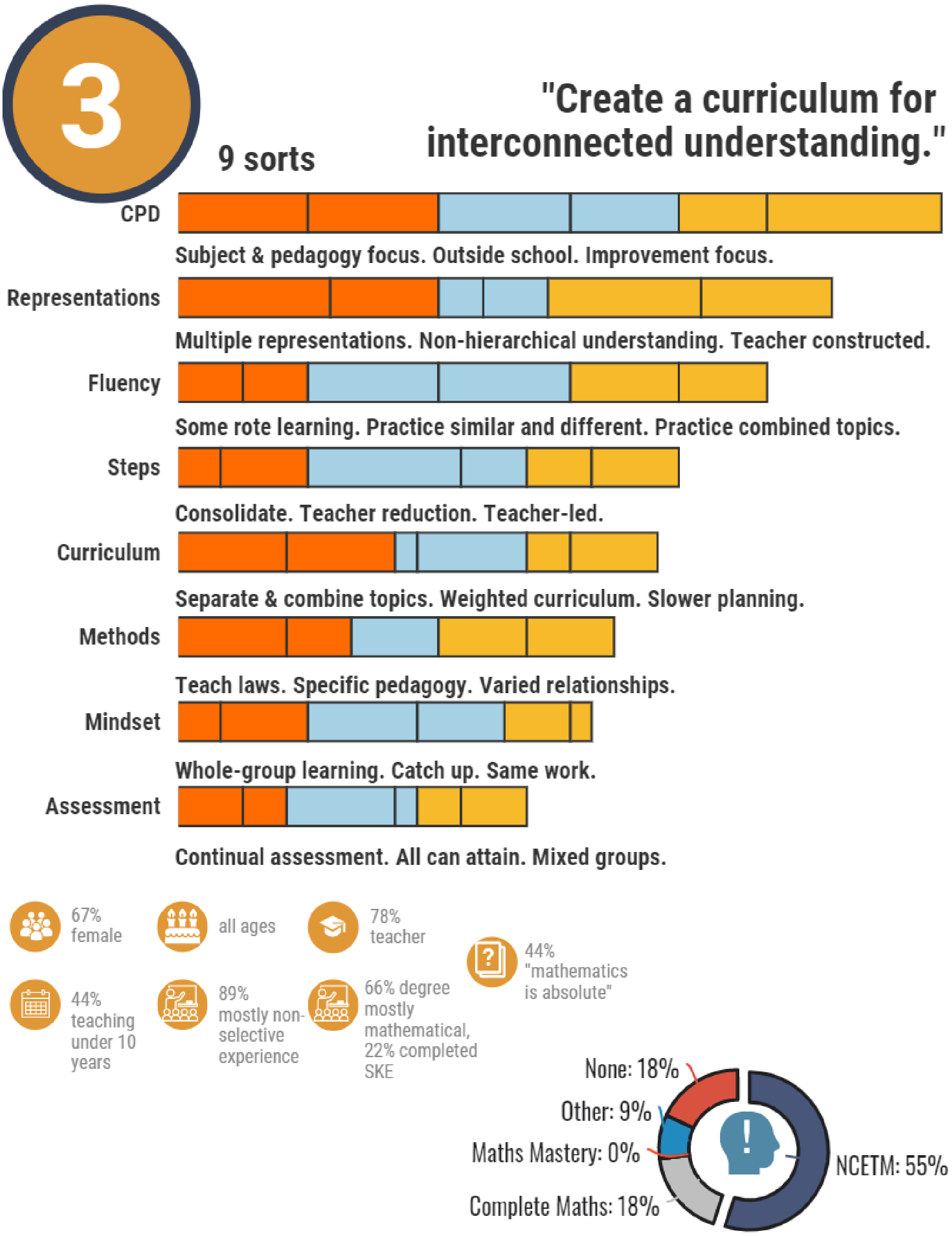


Figure 7.6: Factor 3 infographic

Factor 3 participants are convinced that mastery teaching is needed to improve standards and close attainment gaps (statement 48, scoring +4).

> *'This (statement) is the one I most agree with….. it is not my understanding of Teaching for Mastery but more about how I understand its impact and place in the educational landscape'.*

(Participant 34, Factor 3)

Mastery is about a set of specific teacher pedagogies (statement 22 scoring -4): *teachers* should carefully explain mathematical concepts and select appropriate problems, (statement 35, score +3), reduce complex problems into a series of steps (statement 27 scoring +3) and incorporate multiple representations into lessons (statement 31 scoring +3 and statement 33 scoring -2). They believe this allows the students to notice what is the same and what is different in the mathematics they are doing (statement 36, score +2). Factor 3 participants value the importance of explicitly teaching mathematical laws and vocabulary more highly than any other factor (statement 19, scoring +1).

> *'The point of teaching for mastery is to help students really understand deep structure – and we are the experts to help them do this, so I would not expect them to be able to learn beyond what they have been taught (how is this even possible?)'*

(Participant 39, Factor 3)

Factor 3 participants disagree that the pattern of students' attainment should be equal thirds above average, average, and below average (statement 4 scoring -3) and there is evidence that they believe that all students can achieve a mastery level of attainment (statement 3, scoring +1). They believe that teaching for mastery increases the rate of learning for lower achieving students (statement 9 scoring +2).

To be able to teach for mastery effectively, teachers should engage in professional development that improves both subject knowledge and pedagogy (statements 44 and 39, both scoring +2).

Factor 3 participants value the importance of engaging in educational reading and research more highly than any other factor (statement 43, scoring +2).

Factor 3 participants associate mastery learning as an understanding of mathematics using concrete, pictorial and abstract representations (statement 32 scoring +1), not associating working in the abstract as a mastery goal (statement 34 scoring -1). Curriculum design statements have a higher ranking in this factor, with participants stating that the curriculum should not be balanced equally between number, algebra, geometry and data (statement 15 scoring -3), instead giving greater priority to number and algebra (statement 16 scoring +1). Developing an understanding of number on its own and its connection to other mathematical concepts through curriculum design is important (statements 13 and 14 both scoring +1). Planning for mastery is not quicker (statement 17 scoring -2) but not necessarily slower either (statement 18 scoring 0). Assessment should neither be every lesson (statement 1 scoring -1) nor just at the start and end of topics (statement 2 scoring -2). Students should not be set questions beyond what has explicitly been taught (statement 25, scoring -2).

Factor 3 participants do not necessarily associate mastery with one specific pedagogy (statement 21 scoring 0). Students should be practising both similar and different problems (statements 39 and 40 both scoring +2) and rote learning is either inevitable or incompatible with mastery learning (statements 37 and 38 both scoring -1). They have no relative preference for the nature of the student-teacher relationship (statements 23 and 24 both scoring 0) or the amount of teacher explanation compared to student participation (statement 30 scoring 0 and statement 29 scoring -1 respectively). There is little preference for placing students in same-attainment or mixed-attainment groups (statements 5 and 6 scoring -2 or -1 respectively).

7.5 Factor 4: 'Variety in teaching, learning and achievement.'

Mastery is a deep understanding of the structure of number, achieved through variety in lessons. Students should have the opportunity to tackle lots of mathematical problems, including those which combine mathematical topics.

Teachers should have a good understanding of each student's mathematical needs, achieved through frequent formative assessment. Teachers have excellent pedagogical knowledge and teach lessons that combine careful explanation with setting work which allows students to learn and progress in the way, and at a pace, appropriate to them. This approach will be easier if students are grouped with peers who have similar prior attainment.

Seven out of 45 sorts significantly loaded on Factor 4, explaining 10% of the variance. There were 14 distinguishing statements for Factor 4. Factor 4 participants most strongly associated mastery with the themes of: Curriculum, Representation and Fluency. They least associated mastery with the themes of: Mindset, Methods and Assessment. Factor 4 scored learning-focused statements at +7 and teaching focused-statements at -7, less in favour of learning than factors 1 and 2.

All but one of the seven participants that established the view of Factor 4 were female practicing teachers. These teachers were younger, on average, than participants loading on other factors (one third are in their twenties), and more likely than participants in other factors to mainly teach in a selective school. Forty-three per cent of Factor 4 participants had less than one quarter mathematics in their degree, and 57% had completed a Subject Knowledge Enhancement (SKE) course, more than twice as many than any other factor. Eighty-nine per cent of Factor 4 participants had undertaken some mastery CPD, 44% with the NCETM. They were most likely to consider mathematics as 'absolute', though 29% considered mathematics a 'language', more than any other factor.

-4	-3	-2	-1	0	1	2	3	4
11: To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	12: To achieve mastery, in mathematics lessons students should all be working on different problems	23: Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student	21: A student is more likely to achieve mastery if a teacher uses a specific pedagogy	7: Learners should move through a mastery curriculum at their own pace, moving on once they reach the expected level	47: Teaching for mastery pedagogy is mainly learnt through collaborative in-school professional development	35: In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	40: Practising a variety of problems is part of developing a mastery understanding of mathematics	31: Mastery lessons should incorporate multiple representations of a concept
	46: Teaching for mastery pedagogy is mainly learnt through external professional development	25: In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	22: Mastering mathematics is unconnected with specific teacher pedagogies	20: To achieve mastery, students should understand mathematical laws (for instance the commutative, distributive and associative laws) but do not need them to be explicitly taught	32: To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	45: Mastery professional development activities should include a high degree of specific pedagogy development	14: In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	
	33: Multiple representations are not always needed in secondary school teaching for mastery	6: Mastery will be easier to attain if children are taught in groups of mixed prior attainment	26: In mastery lessons, all questions set should reflect only what has been explicitly taught	39: Practising similar problems is part of developing a mastery understanding of mathematics	42: In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	36: In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing	13: In a mastery curriculum students will understand the structure of number before applying it to other topics	
		4: In general, 1/3 of students will achieve a mastery standard, 1/3 of students will achieve an average standard, and 1/3 of students will achieve a low standard of attainment	34: A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	43: Reading, and taking part in, educational research is an an important aspect of teaching for mastery	3: All students are capable of achieving a mastery level of attainment	41: In mastery lessons problem-solving is developed through exercises which combine topics		
		2: In mastery lessons assessments should only be used at the beginning and end of topics	38: Rote-learning is an inevitable part of mastery learning	29: Teaching for mastery should minimise lecturing and maximise student participation	30: Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	1: In mastery lessons all students should be assessed every lesson		
		8: Learners should move through a mastery curriculum as a group, only moving on once all students have reached the expected level of attainment	19: To achieve mastery, students should be explicitly taught mathematical laws (for instance the commutative, distributive and associative laws), including their formal names	18: Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises	27: In mastery lessons, complex problems should be reduced by the teacher into a series of steps	28: In mastery lessons, complex problems should be reduced by the students into a series of small steps		
			9: Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up	17: Planning mastery lessons is quicker because there are no differentiated resources to create	44: Mastery professional development actvities should include a high degree of teacher subject knowledge development			
			24: Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student	16: A curriculum for mastery should give greater priority to number and algebra	5: Mastery will be easier to attain if children are taught in groups of similar prior attainment			
			10: Teaching for mastery involves students keeping up, not catching up	48: Teaching for mastery is vital in UK secondary schools to improve standards and close gaps	15: A curriculum for mastery should give equal priority to number, algebra, geometry and data handling			
				37: Rote-learning is incompatible with mastery learning				

Figure 7.7: Factor 4 factor array

Green=consensus statement, yellow= positive distinguishing statement, red= negative distinguishing statement

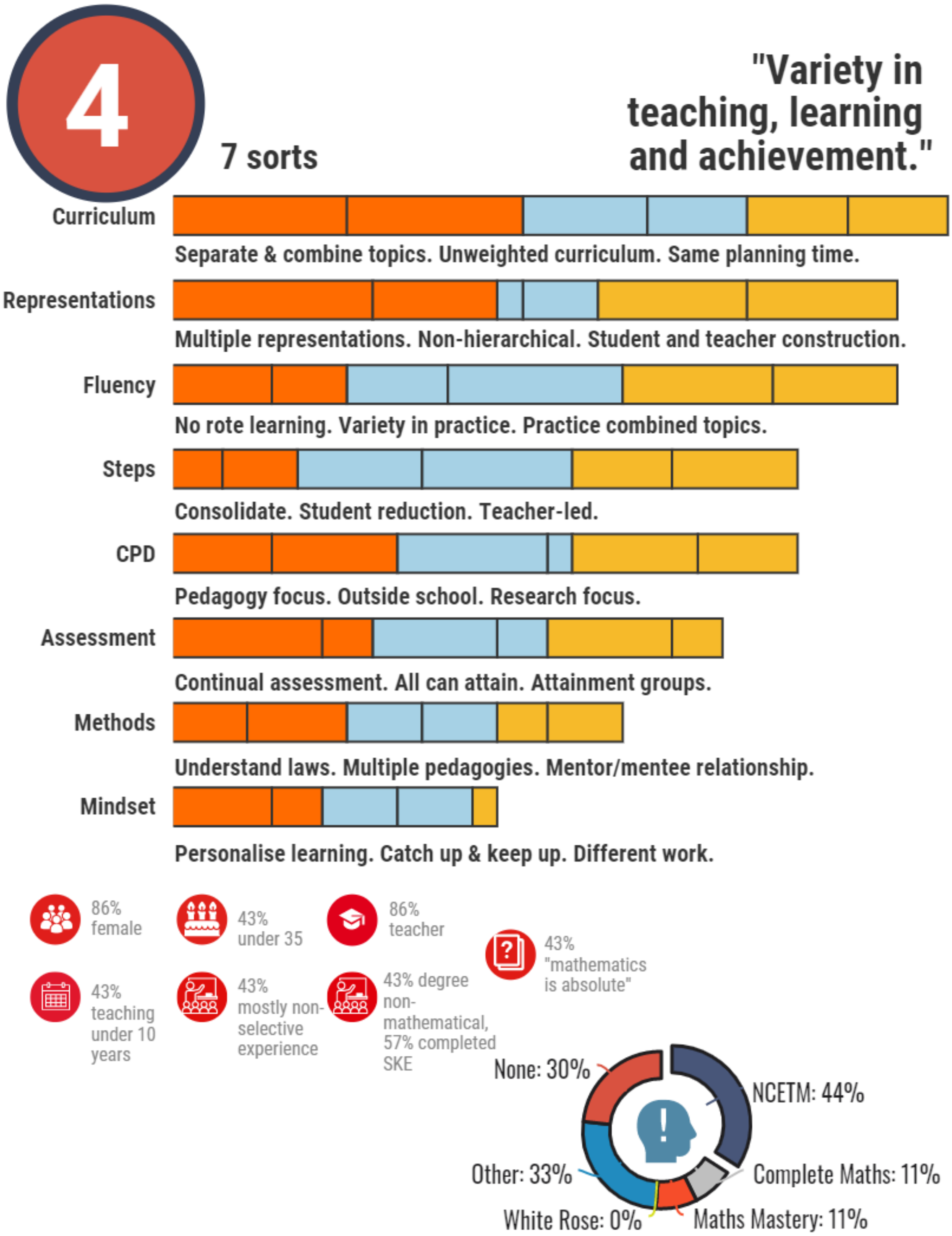


Figure 7.8: Factor 4 infographic

Factor 4 participants associate mastery with variety in mathematics lessons. They believe most strongly that mastery lessons should incorporate multiple representations of a concept (statement 31, scoring +4 and statement 33, scoring -3). Mastery is partly developed through practising a variety of problems (statement 40, scoring +3), and that problem-solving is developed through exercises which combine topics (statement 41 scoring +2). A mastery curriculum should help develop an understanding of the structure of number, both before and through applying number to other mathematical topics (statements 13 and 14, both scoring +3).

Factor 4 participants strongly feel that to achieve mastery students should not be working on the same problems at the same time (statement 11 scoring -4), neither do they feel that students should all be working on different problems (statement 12 scoring -3). Factor 4 participants rate assessing students in every lesson higher than any other factor (statement 1, scoring +2) and prefer grouping students with similar prior attainment together (statement 5 scoring +1 and statement 6 scoring -2). Unlike other factors, they do not believe that teaching for mastery is about low-attainers either catching up or keeping up (statements 9 and 10 both scoring -1).

'It must be organic, you cannot force a learner to move on faster than their own pace and assume that they will have learned it to the same extent.'

(Participant 24, Factor 4)

Factor 4 participants, more than any other factor, do not think that teachers and students should have an 'expert-novice' relationship (statement 23, score -2). They believe that teachers should carefully explain and select problems, and that students should reduce complex problems into a series of small steps (statement 28, scoring +2+). Students construct their learning through noticing similarities and differences in the mathematics they are doing (statement 36 scoring +2). Factor 4 participants mostly agree that all students are capable of mastery attainment (statement 3 scoring +1) rather than the notion of 'one-third low attaining, one-third average attaining and one-third higher attaining' (statement 4 scoring -2). However, they think that practical and resource constraints will impede some students.

'Whilst it is possible for all to attain mastery, the key element here is time. In an ideal world, this would be the case, but in our current educational system we do not have time for 100% to reach mastery in a topic.'

(Participant 19, Factor 4).

Factor 4 participants do not think that teachers learn to teach for mastery through external CPD (statement 46, scoring -3), preferring activities that take place in-school with colleagues (statement 47 scoring +1). They rate professional development activities focused on pedagogy (statement 45 scoring +2) to develop mastery more highly than activities focused on subject knowledge (statement 44 scoring +1). They do not have strong feelings about whether planning for mastery will take more or less time (statements 17 and 18 both scoring 0), whether mastery is associated with teacher engagement with research (statement 43 scoring 0) or whether mastery is vital to increase attainment and reduce attainment gaps (statement 48 scoring 0).

7.6 Consensus and interconnectivity: 'Shared understanding through individual representation.'

Students achieve mastery by acquiring domain-specific knowledge, with problem-solving developing as a result. They use their own representations to support their understanding and working in the abstract is not seen as a goal.

Teachers develop mastery in their students by enabling the whole class to work on the same problems, directly related to what they have been taught, facilitated by their continuous assessment of the class's learning.

Consensus statements					
No.	Statement	Factor 1 Value	Factor 2 Value	Factor 3 Value	Factor 4 Value
2*	In mastery lessons assessments should only be used at the beginning and end of topics	-2	-2	-2	-2
12	To achieve mastery, in mathematics lessons students should all be working on different problems	-3	-2	-3	-3
25*	In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	-2	-3	-2	-2
34*	A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	-1	-1	-1	-1
42	In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	1	1	0	1

Table 7.1: Consensus statements

Consensus statements indicated that participants broadly agree that problem-solving develops from an initial understanding of individual topics (statement 42). This understanding is not necessarily limited to working in the abstract, it could include understanding via a concrete or pictorial representation (statement 34). Teachers should continually assess students' understanding, not just at the beginning and the end of a topic (statement 2) and should set the students work that they should be able to answer using only the mathematics that they have already been taught (statement 25). This work should be set for the whole class, who work on the same questions (statement 12). When the themes that individual factors scored highly on were compared, statements with the theme 'Representations' ranked first or second in each factor.

All Factor viewpoints considered mastery a *level* of shared understanding of mathematics, communicated between the teacher and students in a class. This understanding develops from teacher and student utilisation of appropriate (and perhaps different) mathematical representations. Factors had different viewpoints about *how* this understanding develops and the relative role of the teacher and the student in mastery learning.

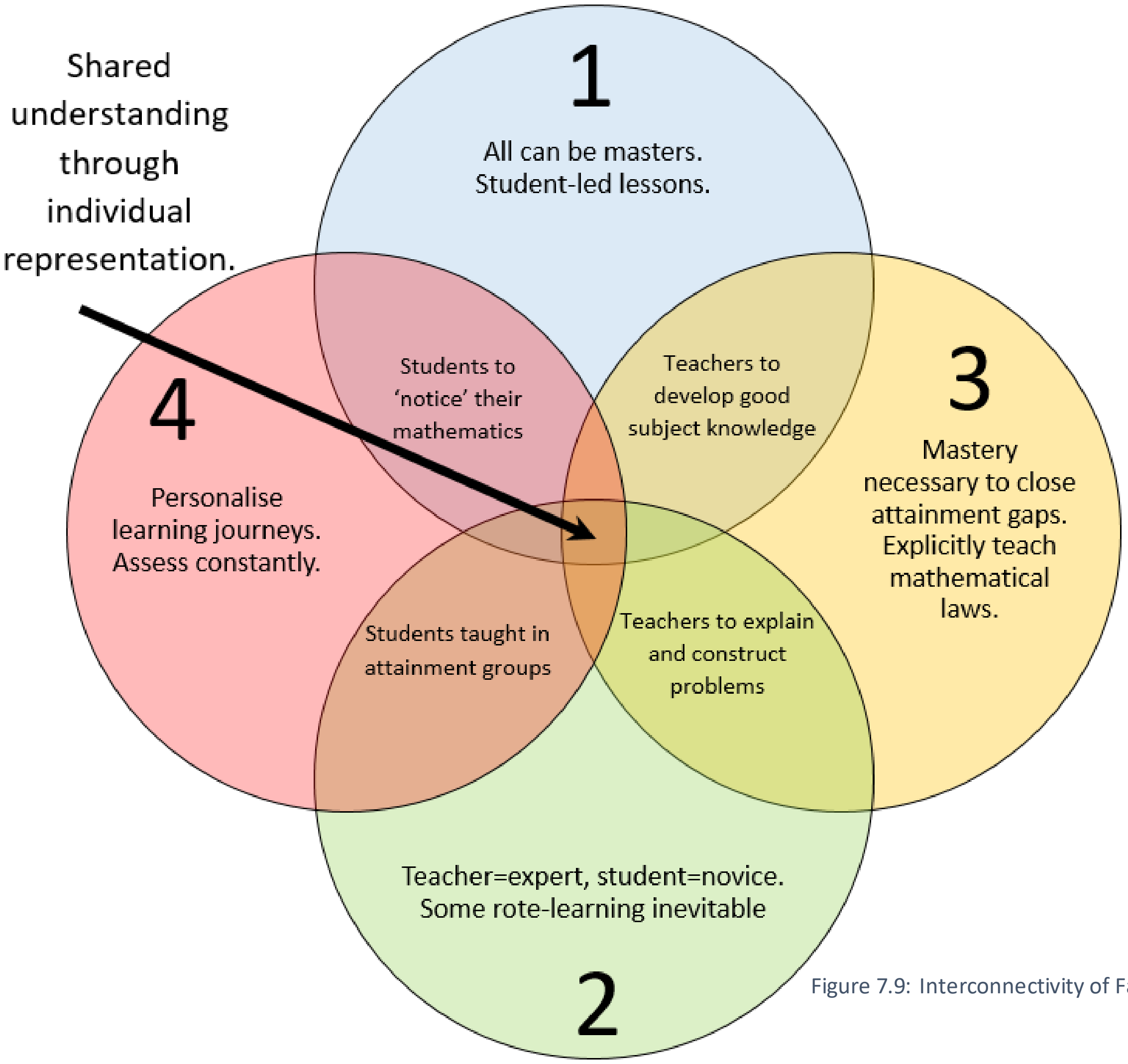


Figure 7.9: Interconnectivity of Factor 1-4

Figure 7.9 is a researcher-constructed simplification of how the four factors interconnect. It has been created with simplicity in mind: the circles are not drawn to scale and the diagram does not show all possible interconnections. Factors 1 and 2 are placed opposite each other as the differences in factor arrays, narrative, and low cross-correlation indicated they had least in common.

Beyond the central area there is some agreement that mastery learning is related to teachers' knowledge of mathematics (Factors 1 and 3), and teachers having the relevant expertise and resources to construct appropriate problems (Factors 2 and 3) which allow students to 'notice' the mathematics that they are doing (Factors 1 and 4). There is also some agreement that this is more likely if students are taught in groups with others working at similar attainment levels (Factors 2 and 4).

Participants in the different factors were least likely to agree on whether all students are capable of a mastery level of attainment and the importance of mastery in the context of closing attainment gaps. Factors had distinct views on the nature of the professional relationship between teacher and student and their interplay in the classroom. Although assessment was generally ranked as less important to mastery as other aspects, Factor 4 emphasised the importance of assessment in relation to teaching and learning.

7.7 Categorising the viewpoints of Factors 1, 2, 3 or 4 as a combination of attitudes about student potential and the role of the teacher

This study identified four different sets of opinions about mastery teaching and learning. These factors are distinguished by attitudes about student potential (whether all students can achieve mastery) and the relative amount of teacher direction in the construction of mathematical representations for understanding. This study found a shared viewpoint of mastery was what it *is* to master. Factor viewpoints differed about how students *become* a master, or *how best to teach* students to master (ibid, p. 23).

This relationship is shown visually in Figure 7.10. Factors 1, 2, 3 and 4 are sections of a 2 x 2 array: the columns distinguish each factor's belief in a growth or fixed mindset, and the rows distinguish belief in a teacher or student construction of mathematical representations. Black text on a white background represents an opinion of mastery and white text on a black background represents a factor characteristic.

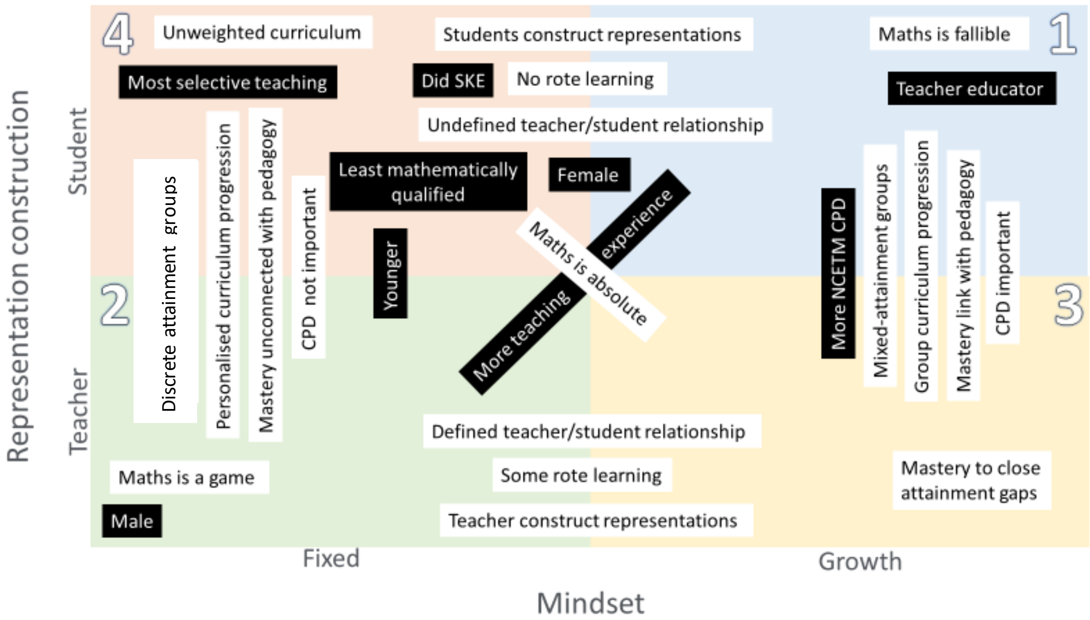


Figure 7.10: Factors 1-4 mapped in a 2x2 matrix

Factors 1 and 3 held a viewpoint that all students have the potential to master mathematics. Factor 1 (the top right-hand quadrant of Figure 7.10) gave a high ranking to statements related to growth mindset: a belief that all students can attain highly in mathematics. This correlates with Block and Burns' historical definition that 'under appropriate instructional conditions, virtually all students can learn well' (1976, p. 4), and the NCETM's rejection that some people 'just can't do maths' (2017). Factor 3 (the bottom right-hand quadrant of Figure 7.10) ranked highly the statements that claim that mastery is achieved if teachers use their knowledge to craft an 'optimum' learning journey for their groups of students.

Thew viewpoints of Factors 2 and 4 were consistent with a more fixed student mindset. Factor 2 (the bottom left-hand quadrant of Figure 7.10) highly ranked the statements that associated mastery with explicit teaching of students grouped by their prior attainment. Factor 4 (the top left-hand quadrant of Figure 7.10) gave high scores to statements associating mastery with teachers that facilitate a personalised approach within discrete attainment groups.

7.8 Conclusion

This results chapter presented analysis of the 45 completed Q sorts, 29 of which were represented in one of four distinct factors. There was good evidence that each factor has a distinct viewpoint of mastery in terms of learning and teaching, and a different relative importance of each mastery theme. There are areas of commonality and interconnectivity, and these were summarised.

In relation to the research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?', there is now good evidence that four distinct viewpoints of mastery exist. The complex relationship between the factors is good evidence of and explanation for the controversy that exists around mastery in English mathematics education (see Chapters 1 and 2). The multiple implications of mastery's 'common ground' and this interconnectivity on the teaching and learning of mathematics is discussed in Chapter 8.

8 Discussion: Implications for mastery mathematics education

8.1 Introduction

In this chapter I identify and examine the implications the study's findings. I have good evidence to address the research question: 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?'. My study was inspired by continued controversy and uncertainty around mastery, and so the chapter highlights clarity about mastery, where it exists and reflects upon the implications of immoveable differences.

Firstly, I consider the finding that all participant groups share a common definition for mastery, and thus the potential for the education community to speak about mastery with a shared language. I then consider that differences between the four factors focus on how mastery is developed the mathematics classroom. The differences in viewpoints about teaching and learning 'toward' mastery influences pedagogical, departmental and developmental choices.

The latter part of the chapter discusses the implications of the findings on those institutions who lead change and development in mathematics teaching and learning: secondary mathematics departments, Initial Teacher Training and providers of mastery continuing professional development. The study's findings should be used by these change-makers to assess their current vision, strategy and provision, and consider how best to develop teachers with different opinions of mastery for the improved learning of students and development of the teaching profession.

8.2 Existence of multiple viewpoints about how mastery is learnt, and by whom

Participants agreed what mastery learning 'is' but disagreed about students' potential to achieve mastery. This suggests that leaders of mathematics education, which includes school departmental heads, Initial Teacher Training (ITT) tutors, and providers and leaders of mathematical professional development could address and discuss these differences to resolve 'mastery's mashups' (Boylan, 2019, p. 14).

> *(The statements hardest to place were) 'Those that refer to specific pedagogy, because I partly feel that Teaching for Mastery is becoming its own pedagogy style, but also that it encompasses lots of pedagogic styles and it is more about the opportunity provided for the learners to learn.'*

(Participant 9, Factor 1).

The ambition of mastery learning, for all students to achieve a given level of attainment, is a challenging one. Different factor viewpoints held different beliefs about whether *all* students can achieve, and how much teachers need to give *explicit instruction* for this to happen. The association of mastery practices as aligned with *either* growth mindset *or* teacher instruction in the literature was explored in Chapter 2. The finding that these beliefs in *combination* are the underpinning influences on mastery views is important. It means that any training or development that promotes a specific mastery curriculum or pedagogical approach will need to focus aligning beliefs of participants with those of the programme.

8.3 Embedding multiple representations into teaching is fundamental to developing and demonstrating mastery

All factor viewpoints agreed that mastery is learning 'one mathematics' through multiple representations.

> *'I understand teaching for mastery to mean teaching for deep and sustainable understanding; knowing **why** as well as knowing **what** and **how**.'*

(Participant 28, Factor 1, emphasis added)

Teachers in all factors agreed that fundamental mathematics knowledge and understanding is universal but that students learn through representing mathematics in different ways as their understanding develops. I illustrated this in the union of Figure 7.9, described as ‘shared understanding through individual representation’.

‘(Multiple representations) are vital for fluency, adaptability and understanding’.

(Participant 35, Factor 1).

Participants agreed that using multiple representations in mathematics is vital for both *developing* and *demonstrating* mastery.

‘To be able to switch between representations of a concept is to fully understand mathematics’.

(Participant 20, Factor 4).

Representations were noted as important regardless of students’ age or prior attainment:

‘There is a tendency for secondary teachers to move too quickly into abstract maths without ensuring that the understanding is well grounded and without supporting students in making connections between different representations.’

(Participant 14, Factor 3).

‘It is (also) a misconception that only ‘low ability’ students require multiple representations’.

(Participant 23, Factor 3).

Hence there is evidence that effective teaching to develop mastery in students *must* include multiple mathematical representations. This area of agreement correlates closely with effective mathematical learning and achievement. Being able to represent mathematics in multiple ways is an indicator of development of both procedural and conceptual knowledge (Rittle-Johnson, Siegler and Alibali, 2001) and teachers who emphasise multiple representations and connections are associated with good student outcomes (Askew, 1997).

All factor viewpoints included a belief that students of all ages and attainment levels should explore multiple representations of mathematical concepts as they develop mastery. There was no evidence of a belief that using different representations indicated a hierarchy of understanding, for instance that students should progress from using concrete manipulatives, to visual representations, and finally to working in the abstract. This opinion is consistent with Bruner's (1965) claim that children use 'enactive, iconic, symbolic' representations to develop their understanding, moving between the domains in a personalised way.

'Students need to be fluent using all representations enabling them to apply concrete/pictorial solutions to an abstract problem to enable them to solve it if necessary'.

(Participant 23, Factor 3).

'It [Concrete, pictorial and abstract representations] means having an understanding of all aspects of a concept.'

(Participant 21, Factor 4)

This finding suggests agreement that concrete and pictorial representations are an important part of mastery mathematics education. An implication of this is that all secondary schools adopting mastery could ensure appropriate concrete manipulatives are available, used and normalised in classrooms. This is not current accepted practice: secondary schools are less likely than primary schools to use concrete manipulatives, despite evidence that they continue to support learning at Key Stage 3 (Hodgen *et al.,* 2018; Carbonneau, Marley and Selig, 2013). This study provides evidence that this situation should change. Since the impact of using manipulatives on learning is cumulative, and dependent on teachers who are themselves confident in using them (Ruzic and O'Connell, 2001; Sowell, 1989), increasing numbers of new and experienced secondary teachers should receive training and support in the effective use of manipulatives. Schools should also embrace pictorial representations such as number lines, bar models and proportion diagrams which complement the concrete and abstract representations used. Where secondary schools have 'feeder' primary schools who embrace elements of mastery teaching such as using manipulatives, a large and increasing number of children will arrive at secondary school already confident in using manipulatives.

8.4 The aims of the National Curriculum are compatible with viewpoints of mastery.

The shared factor viewpoint that mastery is 'shared understanding through individual representation' is entirely compatible with the aims of the secondary mathematics curriculum. Achieving mastery celebrates mathematics as a subject that is 'creative and highly inter-connective', associated with high levels of student fluency, reasoning and problem solving (DfE, 2013, p. 2). Hence, national, or international assessments which accurately assess attainment against National Curriculum outcomes should be appropriate assessments of mastery achievement. Programmes which successfully develop mastery should improve student attainment in national and international tests, thus validating the government policy aim when funding mastery programmes. Since the precise makeup of different mastery programmes currently differs, to maximise the effectiveness of mastery funding more research should be undertaken to compare the effectiveness of different mastery programmes and pedagogies.

8.5 Beliefs about mastery do not necessarily align with a traditional or progressive ideology.

The study found that beliefs about mastery correlate with ideologies, defined as a 'value-rich philosophy or world-view' (Ernest, 1991, p. 111). The opinions of mastery held by Factors 1-4 were compared to Ernest's primary and secondary categories of mathematics education. The results are displayed in Table 8.1, and for ease of analysis I coloured ideologies consistent with traditional education in blue, and progressive in orange. Table 8.1 identifies:

- Viewpoints of Factors 1 and 2 correlated with a progressive (Factor 1, mainly orange) or traditional (Factor 2, mainly blue) view of education,
- Viewpoints of Factors 3 and 4 combined progressive and traditional views (elements of blue and orange.

Educational Ideology element (Ernest, 1991, p. 153, Table 7.1)	**Literature review category name Meanings of mastery: ……'**	**Factor 1**	**Factor 2**	**Factor 3**	**Factor 4**
Primary elements					
Philosophy/ view of mathematics	The philosophy of mathematics	Public educator	Industrial trainer	Old humanist	Mixed
Educational aims/ mathematical aims	The educational aims of mastery programs: The US, Singapore & Shanghai, England	Progressive educator	Old humanist	Public educator	Technological pragmatist
Theory of the child/ theory of ability/ theory of assessment	For all or for some? Norm referenced or criterion referenced?	Public educator	Technological pragmatist	Public educator	Technological pragmatist
Secondary elements					
Theory of learning/ theory of social diversity	A mastery curriculum	Progressive educator	Industrial trainer	Public educator	Technological pragmatist
Theory of teaching mathematics	Mastery and instruction Small-steps and variation Mathematical fluency	Public educator	Industrial trainer	Old humanist	Progressive educator
Theory of resources	Mathematical representation	Progressive educator	Industrial trainer	Old humanist	Progressive educator
Additional element					
	Teachers as professionals	Public educator	Industrial trainer	Public educator	Progressive educator

Table 8.1: Application of factor ideologies in Ernest's adapted framework

Factor 1's opinions of mastery combined a growth mindset with student-led construction of mathematical representations, a high importance given to teachers developing their pedagogy and mixed-attainment groups. This combination sits cleanly with Ernest's Progressive Educator and Public Educator groups. Factor 2's opinions of mastery combined a fixed mindset with a belief in teacher-led constructions of representations, and combines traits of Ernest's Old humanist, Industrial trainer, and Technological pragmatist groups. The 'traditional' or ‘progressive’ mastery positions given by two of the four factors provides evidence for, and a reason why, mastery continues to be politicised.

Factors 3 and 4 however, did not fit wholly within a traditional or progressive ideology. Factor 3’s mastery beliefs were consistent with progressive primary elements and traditional secondary elements: education should ensure all children have the opportunity to be educated, education should encourage creativity and self-efficacy as well as learning factual knowledge, and teachers have a moral and professional obligation to explain, motivate and pass on their knowledge of mathematics. Factor 4’s viewpoint opposed this: beliefs are consistent with a traditionalist opinion that not all children can (or need to) achieve mastery in mathematics, coupled with a progressive stance on teaching that allows students a high degree of freedom in the mathematics they complete in class. Factor 4 is the most passive stance: accepting and respecting the status quo they are confronted with. The finding that the opinions of teachers in Factors 3 and 4 cross the boundaries of established ideologies identifies potential for mastery programmes to fulfil both the traditional educational aim of ‘teaching mathematics’ and the progressive aim of ‘teaching children’.

8.6 Existence of a viewpoint of ‘mastery for all’.

‘The point of teaching for mastery is to help students really understand deep structure – and we are the experts to help them do this, so I would not expect them to be able to learn beyond what they have been taught (how is this even possible?) If problems are selected carefully, and the teacher has really thought about how to deliver these and explain each small step, then hopefully the students will be guided through each small step and have a much better chance if following and understanding the concept in question’.

(Participant 39, Factor 3)

The literature review in Chapter 2 provided evidence that contemporary mastery programmes are built on the notion that all children can achieve in mathematics. Contemporary programmes also suggest specific (and differing) approaches to curriculum and lesson planning, teaching, assessment, and professional development. The opinions of mastery held by Factor 3 did not conform to a progressive or traditional ideology, and Factor 3 teachers make pedagogical decisions for social justice, not political gain. Factor 3's decisions give teachers agency to lead the learning in their classroom, and responsibility to engage in career-long professional development. Factor 3's opinions of mastery, and practices to support the development of mastery in students offer both hope in, and a pathway to, achieving the inspiring and challenging goal of mastery, 'all children to achieve a deep understanding of mathematics within the current English education system. Teacher educators, teachers, schools, and mastery providers sharing the goal of 'all children achieving a deep understanding of mathematics' should study what mastery means to Factor 3 in relation to curriculum, teaching, assessment and professional development. These interested professionals should compare Factor 3 to their own choices and consider similarities and differences. This will inform how they develop, support and challenge colleagues, and will contribute to greater shared understanding of the role of mastery practices as positively contributing to the improved mathematical experiences of all students. Factor 3 holds beliefs closest to that of current mastery providers in England.

Factor 3's viewpoint of mastery suggests approaches that combine progressive educational aims and traditional educational practices for social justice, not political gain.

'It is the importance of the teacher, their knowledge and understanding of how maths works and the craft of the lesson that will ensure the students understand.'

(Participant 29, Factor 3).

Factor 3's opinion of mastery is dominated by their belief that not only all children *can* achieve but *should* achieve. Factor 3 gave their highest (+4) ranking to statement 48: 'Teaching for Mastery is vital in UK secondary schools to improve standards and close achievement gaps'.

'Fundamental to a mastery approach is the idea that all students can learn'.

(Participant 22, Factor 3).

Factor 3's progressive Public Educator beliefs about the primary aims of education, coupled with traditional Old humanist beliefs about the secondary aims of education (Ernest, 1991) mean they believe that all students have the potential to master mathematics, and that they need the mathematical and pedagogical expertise of a teacher to achieve this. Teachers have a moral and professional responsibility to explicitly teach all children to understand the structure of mathematics.

Factor 3 is convinced that specific pedagogical approaches are needed to raise attainment and close gaps: if this were not the case, the gaps would not exist. They exhibit characteristics of a 'connectionist' teacher, feeling responsible for ensuring all children become numerate (or in this case, acquire mastery) and utilising the best strategy to communicate mathematics effectively to them (Askew, 1997). Factor 3 teachers are convinced that pedagogies for mastery are not student-led precisely because they are ineffective and will widen, rather than narrow, achievement gaps.

'(Mastery as a contrast to) ineffective techniques like discovery learning.... (which) are more effective with those who have the most prior knowledge – serving only to widen gaps'.

(Participant 17, Factor 3)

The existence of Factor 3 suggests that some teachers' views of mastery are influenced by more than a progressive or traditional ideology. Mastery teaching offers a new way for some teachers to consider how their subject and pedagogical expertise is directly used to help students learn in a structured, but not pre-determined way. Mastery could be a modern, English answer to the 'paradox of the Chinese Learner' (Chapter 2).

Factor 3 teachers are convinced that the multiple representations that teachers rank as fundamental to mastery learning need to be constructed by teachers who direct students' attention to the mathematical structure which enables them to learn. The teacher lays a 'Pu

Dian' (Huang and Li, 2017) of small steps during the lesson for the whole group of students to follow. This allows students to 'notice' the mathematics they are doing, and teachers to 'notice' areas of deep understanding developing in the students, and areas that need to be further reinforced (Mason, 2002). Students' attention is deliberately directed towards the mathematics that they need to be thinking about using similar, and then non-similar examples. Once a new concept has been introduced and practiced, students are given an unfamiliar problem, or a 'Don Nao Jin' (Huang and Li, 2017) to examine and test the limits of this conceptual and procedural learning (Watson and Mason, 2006).

Factor 3 places responsibility on the teacher to give all students, regardless of starting point, the environment and structure to learn a specific mathematical concept utilising the best available teaching strategies. Factor 3 offers a pathway of achieving the only definition of mastery that encapsulates all subsequent iterations that was introduced in Chapter 1: mastery as 'an explicit philosophy about learning and teaching' in which 'under appropriate instructional conditions, virtually all students can learn well' (Block and Burns, 1976, p. 4).

Factor 3's passionate belief that all children can master mathematics, but only if the teacher leads the learning in classrooms, gives Factor 3 teachers potential to make mastery a reality. These teachers acknowledge that directing students with different prior attainment levels to notice a mathematical representation in a way that is appropriate for all is a challenge. However, these teachers believe overcoming these challenges is hindered by ideological arguments. Participant 17 articulated this point when describing why they found the statements about attainment groupings hardest to place.

'There's no truly applicable research in this area. Only political posturing and overstated claims.'

(Participant 17, Factor 3)

The position of Factor 3 teachers offers hope that despite polarising debates about the practices of traditional or progressive teachers, mastery can be achieved by adopting aspects of both positions. Therefore, schools and teacher educators who share a vision of mathematics achievement for all could look to the mastery practices favoured by Factor 3 teachers.

8.7 Mastery approaches can give teachers permission to be leaders of their own learning, as well as their students'.

> *'Teaching for mastery gives back to the teachers the professional aspects of taking ownership of their craft, of appreciating and developing their subject knowledge (and how they share that) in order to benefit the understanding, learning and self awareness of their students.'*

(Participant 11, Factor 3).

Factor 3 teachers associated mastery with lessons where the teacher is active in leading the learning journey, rather than providing opportunities for students to learn at their own pace. Teachers lead the learning out of a sense of responsibility and professional pride, not because they lack belief in their students' work ethic. Ensuring that teachers make the best pedagogical decisions for students requires a career-long commitment to effective professional development. Schools wanting all children to acquire and demonstrate a deep understanding of mathematics must provide the environment, opportunity, and expectation that their mathematics teachers will access this professional development.

Factor 3 teachers attempt to ensure all students acquire mastery by constructing the best pathway for students to learn mathematics effectively. It is the teacher's role to ensure that students are explicitly taught each part of a concept in the right order, referred to as 'atomisation' (McCrea, 2019, p. 41). Teachers think deeply and make conscious decisions about the structure of lessons, topics, and the curriculum because they are confident in their knowledge of mathematics and pedagogy.

Factor 3 teachers feel a professional responsibility for investing in their own and their colleagues' professional development. Factor 3 considers all types of professional development and engagement with educational research important to mastery teaching. Professional development starts with teacher training then continues throughout a teacher's career. School departments must integrate teacher development through collaborative curriculum and lesson planning, as well as providing external opportunities for professionals to engage in research-informed development, which they then 'bring back' to school and use to develop themselves and others.

8.8 Most viewpoints did not advocate 'mastery for all'.

'I've had to work with resources which are among the worst "professionally" developed which I've ever seen and therefore I'm not entirely sure what is meant by Mastery.'

(Participant 40, Factor 2)

Factor 3 teachers believe that mastery is 'a shared understanding through individual representation' (Chapter 6), and the shared aim of mastery providers that 'all children to acquire and demonstrate a deep understanding of mathematics' (Chapter 2). This finding means that teacher educators and schools wanting to adopt or embed mastery that fulfils a vision of 'mastery for all' could consider how best to develop teachers' beliefs and practices aligned with Factor 3.

However, most study participants did not share all or any of Factor 3's opinions of mastery. Factor 3 explained 13% of the total variance in the completed Q sorts, and one-fifth of the participants in the study, expressed views closer to Factor 3 than any other factor. The rationale for the study (Chapter 1) and the literature review (Chapter 2) offered evidence that there is more about mastery that divides English mathematics teachers than unites them, and the findings reinforce this evidence.

The findings of the study categorised these differences, enabling further study. Figure 7.10 and Table 8.1 represent Factor 3 in relation to the other factors. Teacher educators, schools and mastery providers can use these categorisations to understand different beliefs about mastery that may be held by trainees and colleagues. This is an important first step in understanding, influencing, and changing teachers' beliefs and practices to align with the specific vision for mastery that might be advocated by a department, training route or mastery development programme.

8.9 Opinions about mastery correlate with beliefs about mathematics

The study asked teachers their opinions on the nature of mathematics. Overall, a similar number of respondents described mathematics as 'a game', 'absolute' or 'a language'. Participants are least likely to consider mathematics fallible, so mathematics is assumed to be a body of knowledge which, given effective teaching, students can learn and thus 'master'. However, amongst the factors, Factor 1 considered mathematics as fallible, Factor 2 considered it a game, and Factors 3 and 4 consider it absolute. There is evidence of correlation between a mathematics educator's views on mathematics and their opinions of mastery learning.

In Chapter 2.3 I discussed how different views of mathematics could be associated with different views of mastery. The finding that Factor 1 participants, who include a high proportion of teacher educators, believed mathematics to be fallible is consistent with their belief that mastery is about student-led exploration. Factor 2's choice of mathematics as 'a game' might be influenced by gender (Factor 2 participants were all male) or with their mathematical confidence and assertion that mathematics cannot be mastered by everyone. The game of symbols and rules to solve human problems is inaccessible to some, but enjoyable and fulfilling to those with the requisite competencies. Factor 2 perhaps considers mastery as analogous to the work of a professional mathematician: seeing mathematics as network of connected concepts. Once this network is understood the master can create and use models for understanding and problem-solving. Factors 3 and 4 are interesting: they have very different mastery views but share the belief that mathematics is absolute. Perhaps this represents a ‘naïve position’ for Factor 4: the less confident the mathematicians, the more likely they are to believe that mathematics ultimately yields a ‘right answer’. For Factor 3, a predominant understanding that mathematics ‘is always right’ is consistent with mastery as something that can be acquired by students. This supports the development of a specific curriculum ‘for’ mastery and learning goals that are assessed.

8.10 The findings have implications for secondary school mathematics departments

The findings of this study are important for heads of secondary school mathematics departments in England interested in understanding more about mastery or who want to embed mastery approaches. Schools and departments might be motivated to adopt mastery to increase student attainment or narrowing attainment gaps and wanting to emulate practices in higher achieving countries. Schools and departments might also be motivated by mastery providers, who have financial or contractual reasons to recruit schools to their programmes. This study gives good evidence that ‘mastery’ does not share a common philosophy or vision and can mean very different things for different teachers. Schools need to make decisions about mastery based on whether the curriculum, teaching approaches and development opportunities of the specific mastery programme being considered matches the school or departmental vision for learning and teaching. Mathematics departments also need to understand the individual mastery opinions of teaching staff and consider whether and how to change the beliefs and approaches of teachers who have opinions of mastery different to the department’s vision and policy.

8.10.1 Mastery and departmental vision

Effective mathematics departments have a clear vision for student learning and achievement and associated long and short-term goals. These are influenced by the individual opinions and practices of the head of department and their colleagues, expectations of senior leaders, and government guidelines. Once a department's vision is clear and explicit, policies regarding curriculum and lesson planning, student grouping and pedagogical choices that support this vision can be made and articulated to colleagues, senior leaders, students, and parents.

The study's findings could be used to compare an existing departmental vision and policies with the different types of mastery opinions revealed by the study. This would clarify the department's meaning of mastery and facilitate communication to stakeholders. Alternatively, the Q set compiled for this study, or a presentation of Factors 1-4 could be used in schools to investigate opinions of mastery held by the head of department, senior leaders, colleagues, students, and parents. This would aid the development of a departmental vision and policy for mastery. A mathematics department that has a clear vision and clear goals can then make informed decisions about their engagement in a particular mastery programme or how they want their own mastery practices to develop over time.

8.10.2 Informing mastery development decisions

The makeup of typical secondary school mathematics departments includes qualified teachers with a variety of mathematical expertise and teaching experience, and pre-service teachers undergoing training. The study's finding that mathematical knowledge, teaching experience and previous experiences of mastery correlate with different opinions of mastery means that there will be a variety of professional opinions of mastery present in a department.

The study indicated four distinct opinions of mastery. This means that some department members are likely to be opposed to a specific mastery approach. The NCETM's teaching for mastery programmes are built on the notion that all pupils can achieve, but Factors 2 and 4 disagree. Teachers who share Factor 2 or 4 opinions of mastery will question the efficacy of the NCETM's programmes. An extreme example is the opinion expressed by Factor 2 participant 41: they ranked statement 48, 'Teaching for mastery is vital in UK secondary schools to improve standards and close attainment gaps' as the lowest scoring statement. Mismatch between an individual teacher's mastery beliefs with their department could be identified by teachers undertaking their own Q sort or by familiarising themselves with the four factors and considering which they identify with. Departments can use areas of mismatch to design and facilitate appropriate and transformative professional development for departmental colleagues. Schools considering adopting the NCETM (or any other) mastery approach must consider how best to manage departmental colleagues who will be cynical or resistant to the programme due to differences in beliefs and opinions about mastery.

8.10.3 Mastery and government expectations

Chapter 1.6 outlined that mathematics departments have 'supported autonomy' (DfE, 2016a, p. 4) in deciding their vision, strategy, and policy. The current (2019) Ofsted school inspection framework judges the quality of education in English state schools by the quality of its curriculum, and how it enables the learning of all children. The study's findings indicate that Factors 1 and 3 express mastery consistent with the achievement of all learners, and Factors 2 and 3 express mastery consistent with pedagogies that support the Ofsted definition of learning as an alteration in long-term memory. Schools who claim a mastery approach thus have an incentive to adopt practices consistent with Factor 3.

Ofsted's endorsement of a specific definition of learning that is rooted in neuroscience (see Chapter 2.9.2) privileges educational research that promotes improving memory, such as Cognitive Load Theory (Willingham, 2009; Kirschner, Sweller, and Clark, 2006). This branch of research states that learning is best if teachers employ direct instruction techniques. Rosenshine's (2010) '10 principles of instruction' emphasises the importance of understanding students' prior knowledge, the use of modelling to expose misconceptions, the emphasis on connections and scaffolding, and the importance of questioning. These practices are consistent with the opinions of Factor 3.

8.10.4 Mastery and attainment groupings

The study's findings indicated that Factor 3's belief that mastery can narrow achievement gaps is associated with a belief that students should be taught in mixed-attainment classes. Mixed attainment mathematics classes are uncommon in English schools. Schools could use the study's findings to critically assess whether moving to mixed-attainment classes will be effective in narrowing the achievement gap and enable more students to acquire mastery in mathematics.

Students in England enter school with different mathematical knowledge and experiences, and despite following a national curriculum the range of attainment in mathematics remains wide. Statutory attainment tests taken by Year 6 students (in their final year of primary school) in 2019 show that 27% of students achieved 'higher than the expected standard', 52% of students achieved 'the expected standard' and 21% did not reach the 'expected standard' (DfE, 2019b). 2020 Key Stage 4 assessment of students aged 16 indicated that 42.5% of students attained lower than a 'threshold' grade 5 and 14.6% of students attaining the top grades 8 and 9 (National Statistics, 2020). Secondary schools are not yet closing attainment gaps: across English and mathematics 83.5% of students with high prior attainment scored at least a grade 5 whereas for lower prior attaining students 2.7% reached this threshold. The percentage of secondary schools that teach mixed attainment mathematics classes may be as low as 5% (Tereshchenko *et al.,* 2019).

There is no doubt that a vision of all children achieving a deep understanding of mathematics is a challenging goal for a secondary school that has a wide range of student attainment levels on entry to the school. Closing this gap means that lower-attaining learners need to accelerate their rate of learning compared to higher-attaining learners. There is evidence that mixed attainment classes will facilitate this: setting by attainment has a negative impact for lower and middle-attaining students, coupled with positive impact on higher-attaining pupils (EEF, 2018b; Ireson, 2000). Children from poorer backgrounds are more likely to be in lower attainment groups

(Shaw *et al.,* 2016), and children placed in low sets experience low expectations from teachers, leading to poor motivation and a negative attitude towards mathematics and their own potential to succeed (Nardi and Steward, 2003; Boaler, Wiliam and Brown, 2000). Schools that are under pressure to increase the number of students who reach specific grades may also be more likely to place less experienced or non-specialist teachers with lower attainment groups (Francis *et al.,* 2019). Tereshchenko, *et al*., (2019) found that most students themselves had positive attitudes towards mixed attainment groupings in mathematics: lower-attaining students felt empowered and high attaining students appreciated the opportunity to help others, providing they did not feel bored or unchallenged themselves.

The findings of my own research study, that the set of mastery opinions held by both Factor 1 and Factor 3 favoured mixed attainment groupings, should encourage more secondary schools to address the challenges of mixed-attainment mathematics teaching in a different way beyond 'not doing it'. Factor 3 teachers address this by advocating a mathematics curriculum that all children move through at the same pace. In Chapter 2.8 I described how mastery providers facilitate this: 'dimensions of depth' (Drury, 2018, p. 6), and designing and teaching lessons 'where all students in the class are thinking about, working on and discussing the same mathematical content' (NCETM, 2017). Interestingly, Factor 3's pedagogical choices are compatible with McCourt's 'corrective teaching' and 'topic enrichment and enhancement' (2019, p. 26), despite McCourt's opposition to mixed attainment grouping. These principles will require departments to develop new practices if their current curriculum design, and the pedagogical knowledge and skills of departmental teachers, do not yet align with these ideas. If changing grouping practices completely is rejected by a school, then long-term departmental planning could include phased introduction of mixed attainment teaching (perhaps starting in year 7), redesigning schemes of work where all sets cover the same core content, or a new approach to within-lesson differentiation, same-day intervention, or pre-teaching. If all children really can achieve mastery, then positive action by departments to ensure the learning opportunities and experience for all students is necessary. There are already indications that this is expected at a primary education level: the 2020 DfE Primary mathematics curriculum guidance and ready-to-progress criteria commands that ‘pupils must’ (p. 5).

8.11 The findings have implications for Initial Teacher Training (ITT)

Study participants who were trainee teachers or teacher educators were not likely to have Factor 3’s opinions of mastery. However, the government expectations of Initial Teacher Training curriculums do align with Factor 3. A mismatch between educators, trainees and government means that Initial Teacher Training does not yet develop mastery beliefs and practices aligned with mastery aims of schools, mastery development programmes or national expectations. This misalignment is most pronounced for trainee teachers who are also less confident mathematicians. This finding provides evidence that ITT should introduce a more consistent view of mastery. This should include an increased focus on the importance of teachers’ knowledge of mathematics and specific mathematical pedagogies associated with ensuring all children can learn.

8.11.1 Participants who were teacher educators had different opinions of mastery compared to most practicing teacher participants, and the ITT Framework expectations

Research participants who were teacher educators were most likely to have the viewpoint of Factor 1, associating mastery with pupil-led constructions of representation and a growth mindset mentality. Participants who lacked experience in teaching and less qualified mathematicians were most likely to share the viewpoint of Factor 4, associating mastery with a fixed mindset and pupil-led constructions. Conversely, teachers who lacked teaching experience but were confident mathematicians often had views consistent with Factor 3.

Mastery in teaching and learning is an important theme in the ITT core framework, referencing the evaluation of the Ark Academy’s mastery learning programme (Jerrim and Vignoles, 2016) as an example of the best available education research. The study found the opinions of mastery held by teacher educators in the study were different from expectations of the Initial Teacher Training (ITT) core framework, a document detailing the 'minimum entitlement of all trainee teachers' set by the English Department for Education (DfE) (2019a, p. 3).

Aspects of the ITT core framework likely to be associated with mastery include trainees knowing that 'effective teaching can transform pupils' knowledge, capabilities and beliefs about learning' and demonstrating 'high expectations to all groups, and ensuring all pupils have access to a rich curriculum.' (DfE, 2019a, pp, 17 and 21). Trainee teachers should 'ensure all pupils access the full curriculum, and know adaptive teaching is less likely to be valuable if it causes the teacher to artificially create distinct tasks for different groups of pupils or to set lower expectations for particular pupils.' (Ibid, p.20).

Research participants with the viewpoint of Factor 1 associated mastery with a growth mindset, and more than half of the teacher educators completing the study held opinions consistent with Factor 1. As indicated in Figure 7.10, practices include encouraging measures to ensure all children access the whole curriculum (including the possibility of mixed-attainment groups), encouraging learning about multiple pedagogical strategies, and emphasising the importance of professional development in a teaching career.

Teacher educator participants did not mostly associate mastery with teacher-led approaches to mathematics education. An example of this is the assumed professional relationship between the teacher and their students. Factor 1 professionals do not consider an expert/novice relationship as compatible with mastery. This contrasts the ITT framework which refers to pupils as novices once and experienced teachers and ITT professionals as expert 107 times.

Teacher educators sharing Factor 1's opinions of mastery did not rate the explicit instruction strategies such as teacher-introduced mathematical representations. However, the ITT framework states that trainee teachers should be 'explicitly teaching pupils the knowledge and skills they need to succeed' (DfE, 2019a, p.13). Lessons should include 'using modelling, explanations and scaffolds, acknowledging that novices need more structure early in a domain' (Ibid, p. 17) and 'using partially completed examples to focus pupils on the specific steps' (Ibid, p.11).

A potential mismatch between teacher educators and framework expectations means that trainee teachers who lack confidence in their ability to change students' mathematical trajectories may not have their beliefs and practices challenged. Factor 4 teachers considered mastery as providing an environment for students who are already able to construct appropriate mathematical representations to progress rather than being a direct influence on rates of learning. Factor 4 teachers concurred with Keller's (1968) opinion of mastery as an individual learning journey rather than Bloom's (1968) mastery which keeps groups of students working together. Factor 1 teacher educators are unlikely to challenge their trainee's perception of mastery as aligned with student construction of presentations, minimal instruction, and a less delineated student/teacher professional relationship.

8.11.2 ITT could include a mastery approach to teacher subject knowledge development

The study found that inexperienced, less mathematically confident participants held opinions consistent with Factor 4. Therefore, addressing lower levels of subject knowledge early in ITT could influence the mastery beliefs of teachers as they develop. ITT curriculums could increase their focus on improving pedagogical content knowledge of trainee teachers, particularly those with more limited subject knowledge.

Chapter 2.11 identified that the landscape of mathematics teacher recruitment and training is very different in England than other high-performing countries: England has a shortage of mathematics teachers and potential entrants to the profession have less than degree-level mathematics qualifications. Pre-ITT Subject Knowledge Enhancement (SKE) courses have been available for the last 15 years for eligible participants. SKE courses vary in length and nature, and participants report that they have an impact on their knowledge of general mathematics, specialist mathematics and how to teach it (Edwards *et al.,* 2015). My research study found that less confident mathematicians were likely to hold Factor 4 mastery opinions and are less likely to consider themselves willing or able to lead the development of mathematics learning in their students. This finding means that SKE courses should prioritise developing the participants' deep understanding of mathematics, which is also the aim of mastery learning programmes. SKE courses could therefore consider having a mastery approach themselves. According to Edwards, *et al.*, SKE participants reported that developing their knowledge of specialist mathematics helped them see connections between topics (ibid). Subject knowledge development aspects of ITT courses themselves could also prioritise a mastery learning approach.

8.11.3 ITT could address ideological challenges about mastery by critically debating relevant research and exploring the viewpoint of Factor 3

Identified differences in mastery beliefs and practices indicate potential for aspects of mastery to be missing from teacher training and for inconsistent messages to be given. This exacerbates long-standing tensions between the government, schools and university education departments (e.g. Young, 2014; Ball, 2003) regarding the purpose of, and curriculum for, ITT. Mathematics ITT must address the expectations of the ITT framework and should do this through critical engagement with educational research about mastery. I identified and demonstrated in Chapters 1 and 2 that the literature surrounding mastery is complex and fragmented, and so the adapted Ernest framework in Table 2.1 or Table 8.1 and the collection of literature reviewed in Chapter 2 could be used to explore different positions on aspects of mastery learning and teaching.

Table 8.1, which indicates the opinions of mastery expressed by Factors 1-4 'sit' in relation to Ernest's categories of mathematics education indicates how teachers' professional choices are influenced by, and contribute to, personal and societal ideologies. Addressing the impact of ideologies on teacher practices will expose trainee teachers to the study's finding that more experienced teachers hold views of mastery that are wholly traditional (Factor 2) or wholly progressive (Factor 1). Trainees could consider reasons for this and why the mastery approach described by Factor 3 offers a way of teaching and learning that goes beyond ideological boundaries. Addressing this issue is important because of the finding that, ultimately, teachers are united in understanding the purpose of mastery to be increased student engagement and achievement in mathematics. Hence, mastery practices that transcend ideological positions provide a platform for resolving polarisations of views about specific teaching strategies.

Evidence that mathematics teachers develop traditional or progressive stances on mastery over time means that teachers tend to continue to engage only with research that concurs with their developing ideological beliefs. The study's evidence that the word 'mastery' is currently synonymous with several ideological positions is confusing for teachers, as indicated by the Twitter conversation from May 2020 reproduced in Figure 8.1.

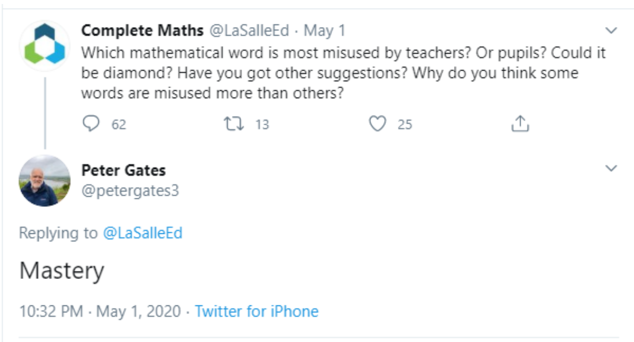


Figure 8.1: Tweet indicating mastery controversy

ITT could explore the differences of opinions in mastery, what causes them, and how differences can be resolved. The study's finding that teachers agree that mastery learning is defined as 'shared understanding through individual representation' (Chapter 7.6) is evidence that that the definition of learning as 'a change in the long-term memory' is necessary, but not sufficient for mastery. Students who are learning for mastery need to be able to make connections between the 'new' knowledge with their existing knowledge. This then supports the pedagogies of a connectionist teacher, which include aspects of explicit instruction with opportunities for student-led exploration (Chapter 0). Trainee teachers are in the perfect position to consider that beginners do not think in the same way as experts, so there needs to be an aspect of modelling explicit teaching, whilst ensuring the learners' attentions are drawn to key aspects of mathematics. Trainees could explore Factor 3's opinion that mastery is about small steps which students need to climb for themselves: explicit teaching therefore is about forming the steps and making connections, not path-smoothing. They could compare Rosenshine's (2010) principles of effective teaching (influenced by cognitive load theory and laboratory-based classroom research) with Swan's and Swain's (2007) 'eight principles for effective teaching' (influenced by Askew's (1997), Swan's (2006) and Boaler's (1998) classroom-based research). The two documents are comparable in their emphasis on teacher questioning, building on prior knowledge and the use of appropriate models and methods, and differ in their approach to individual versus group work and the relative importance of 'correct' answers.

8.12 The findings have implications for mastery CPD

Mastery providers in England are well-placed to influence teacher's knowledge, belief, and practice in mathematics teaching. Whilst some teachers representing all factors had engaged with mastery CPD, teachers with opinions of mastery matching Factor 3 were most likely to have undertaken CPD with NCETM and the Maths Hubs, and valued CPD that developed subject knowledge and pedagogy together. The teachers were professionally driven to be the 'best' teacher they can be, and they value engagement with educational research. Factor 3 teachers' beliefs about mastery are influenced by evidence of good practice in mathematics rather than political leaning. The teachers' non-alignment with traditional or progressive positions leaves them to engage with all types of educational research. They are free from positionality biases and willing to adopt practices whose efficacy is based on a variety of evidence.

8.12.1 Mastery CPD providers do not need to compete

Participants in all four factors claimed some engagement with mastery training, and there is evidence of agreement amongst teachers about what mastery learning is, and how mathematics can be represented in multiple ways. Hence, there is evidence that all mastery providers offer support to develop teacher's pedagogical content knowledge, a classroom environment and curriculum that gives all children the potential to succeed and promote pedagogy that ensure students notice the underlying structure and connections in the mathematics they are learning. The nature of a mastery curriculum and teacher pedagogies are different, but are similar enough to foster collaboration and culture, not competition. All mastery training providers in England recognise the challenges faced by inexperienced mathematicians who enter mathematics teaching. The NCETM prioritises teachers using their knowledge of mathematics to create learning opportunities that expose the underlying structure of mathematics and linking subject knowledge designing exercises that include variation (NCETM, 2017). Mathematics Mastery reassures schools that less mathematically experienced teachers can become experts and support departments to develop these members of staff. They promote a space for departments to routinely discuss teacher misconceptions and offering CPD structured around professional learning communities (Drury, 2018). McCourt (2019) acknowledges that teachers with lower baseline subject knowledge may be anxious about mathematics and pass this onto students, supporting the findings of Carey, *et al*. (2019).

8.12.2 Mastery providers could use the factor viewpoints to clarify their offer.

This study provided important information about what a group mathematics educators in England consider mastery to be. Providers could study the opinions of mastery expressed by teachers in Factors 1, 2, 3 and 4 and against their own principles. A direct comparison between the different mastery CPD providers was not a focus of this thesis but the study's literature review provided evidence of a fit with Factor 3. Specifically, there is agreement that mastery is about creating conditions for all students to understand mathematics at a deep level coupled with an acknowledgement that some students find mathematics more challenging. Teachers and schools therefore have a responsibility to ensure all students maintain the same learning trajectory. The set of principles and pedagogies that support this vision are not universal, and so providers should be very clear on the specific goals of their mastery programme. Such clarity of goals would allow teachers, departments, and schools to make informed judgements about which providers they choose to work with.

8.12.3 Mastery providers should compare their approaches on using representation

The study found agreement amongst participants that individual student representations of mathematics did not represent a hierarchy of understanding. This finding is at odds with mastery providers: Drury claims that 'students need to be able to do the mathematics without the representation' (2018, p. 145) and McCourt wants 'all pupils to be able to work with all mathematical ideas efficiently and effectively in the abstract/symbolic form.' (2019, p. 124). The finding also contradicts the Singapore mathematics approach of 'concrete, pictorial, abstract' (Leong, Ho and Cheng, 2015).

The participants' notion that representations are not hierarchical, and contribute to a shared understanding of mathematics, mean that multiple representations are viewed as different aspects of a single mathematics concept. This is analogous to visualising a 3-dimensional shape by looking at a plan, front elevation and side elevation. Each representation is a different, not better, model of a mathematical concept, so working in the abstract is not a goal. Knowledge of the universal mathematical 'whole' appears as more representations are explored and more connections are made, a mathematical example of the parable of the blind men and the elephant. Sweller (1988) uses schemas to explain this phenomenon, claiming that knowledge deepens as the frameworks of comparison, or schemas, are enlarged and refined. Whilst teachers do not agree whether specific pedagogies support mastery, they are united in the belief that multiple representations must be used to develop and assess deep understanding in students of all ages and all levels of prior attainment.

The different uses of mathematical representations in English mastery programmes could be researched to explore the exact role of multiple representations in the development of mastery learning perhaps using schema research as a starting point.

8.12.4 Implications for the NCETM

The study found that Factor 3 teachers were most likely to have undertaken CPD with the NCETM, a viewpoint compatible with the expectations of the ITT framework. There is evidence that programmes run by the NCETM, through the Maths Hubs network, have the potential to contribute to the interventions that this study claims are needed in Initial Teacher Training and ongoing teacher development. The role of the NCETM's mastery programmes could be expanded to include elements of Initial Teacher Training, perhaps pre-service Subject Knowledge Enhancement courses, or working with non-specialists.

The study's finding that Factor 3 teachers make pedagogical decisions based on research, not ideology, creates an interesting position for continued DfE funding of specific approaches. For the NCETM's Teaching for Mastery programme to develop using the best available evidence, there must be a transparent and healthy relationship between the NCETM programme team, the Maths Hubs, and the DfE. Each stakeholder should be using the best available evidence to make decisions about programme planning, classroom practice, school and educational policies, and use of resources. Teachers and schools should be in a position of influence with DfE policymakers. The NCETM and the Maths Hub network can make this connection between practitioners, researchers, and government but this is not without challenges. Mastery research and practice remains relatively new and the findings indicate that different practices are being used in classrooms. Future programme innovations need to inform and be informed by the ongoing development into English mathematics mastery practices.

To proceed positively, the NCETM, the DfE and the Maths Hubs should evaluate their practices and relationship against the eight recommendations of the British Academy and Royal Society's (2018) 'Harnessing Education Research' report. The three most pertinent recommendations are: improving collaboration between policymakers, researchers, and school leaders; ensuring a bipartite relationship between mastery teaching and mastery research; and considering how best different types of impact evidence are transparently synthesised.

8.13 Conclusion

Chapter 8 explored in detail the implications of investigating what specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students. There is evidence that beliefs about mastery can, but do not have to be, bound up with educational ideologies, and potential to address controversies about mastery. Schools, Initial Teacher Training institutions and mastery professional development providers can critically assess their vision, strategies, and practices to have a sharper focus on ensuring that teaching for mastery facilitates the development of a deep understanding of mathematics for all students. The findings present a model of mastery for all, and this aligns with government expectations, and the practices of government-funded mastery development programmes. I also identified areas where stakeholders in mastery learning could increase their collaboration and engage in further research about mastery learning and associated professional practices.

Chapter 9 is a second discussion chapter highlighting the importance of Q methodology in this research study: how only a Q study could have identified and explained the observed differences in teachers' opinions of mastery. This finding leads to my claim that Q methodology is currently underutilised in education research.

9 Discussion: Implications of the study for Q methodology in education

9.1 Introduction

In this chapter I identify and examine the implications of the study's design, implementation, and findings as a case study for recommending Q methodology studies within educational research. In Chapter 8 I discussed the implications of addressing the research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students?', for mathematics education. Chapter 9 celebrates the use of Q methodology in the study. I claim that only a Q study could have identified and explained the observed differences in teachers' opinions of mastery with rigour, richness, and clarity. I also claim that Q methodology should be a welcome addition to established regular research methodologies widely used in the field of education.

The first part of this chapter is a reflection on the research study and the unique perspective that Q methodology gave in addressing the research question. The group of participants who completed the study were diverse and anonymous, yet their completed sorts yielded good evidence about distinct groups of views on mastery and who is likely to hold them. A rigorous and transparent approach to creating the Q set, recruiting the participants, administering the study, and analysing the results leads to insights that are trustworthy and that can be utilised appropriately by expert and non-expert stakeholders.

The second part of this chapter is my petition for an increased use of Q methodology in educational research in England. There is an acknowledgement amongst Q methodologists that Q is currently underused within education. My extensive experience as a teacher, teacher educator and evaluator of mathematics professional development, coupled with a new knowledge of Q methodology, has led me to believe that Q methodological studies could and should feature more heavily, and hence this study provides good evidence for the 'qualiquantology' Q methodology as a credible addition to quantitative, qualitative, or mixed-methods research.

9.2 Reflections from the research study on the benefits of using Q methodology in an educational research project.

9.2.1 Q methodology identified distinct voices present in a diverse group of educators.

The existence of Factors 1-4 as distinct sets of opinions of mastery held by teachers will be useful to the mathematics education community. Large professional communities of mathematics educators, for example the teachers involved with their local Maths Hubs, will have groups of teachers that identify with the views of the four factors. Teachers in individual mathematics departments may hold one or more of the views. Q methodology studies identify majority and minority views and give rich qualitative detail about their subtle similarities and differences. This allows the researcher, the educational professional and the policymaker apply a Q study's results in diverse ways.

9.2.2 Q methodology enabled the collection of rich and rigorous data without recruiting large numbers of participants.

The 'reverse factor analysis' undertaken in Q studies means that statistically valid data is generated without the need to recruit large numbers of research participants. In a recent conference presentation about my research findings, which limited presenters to one static slide, I demonstrated both the capability of Q methodology and the essence of Factors 1-4 with just four hypothetical statements generating four factor arrays (see Figure 9.1). The diagram illustrated how rich and trustworthy conclusions (in my case, about mastery) can be drawn from a set of carefully chosen statements sorted by a modest number of participants.

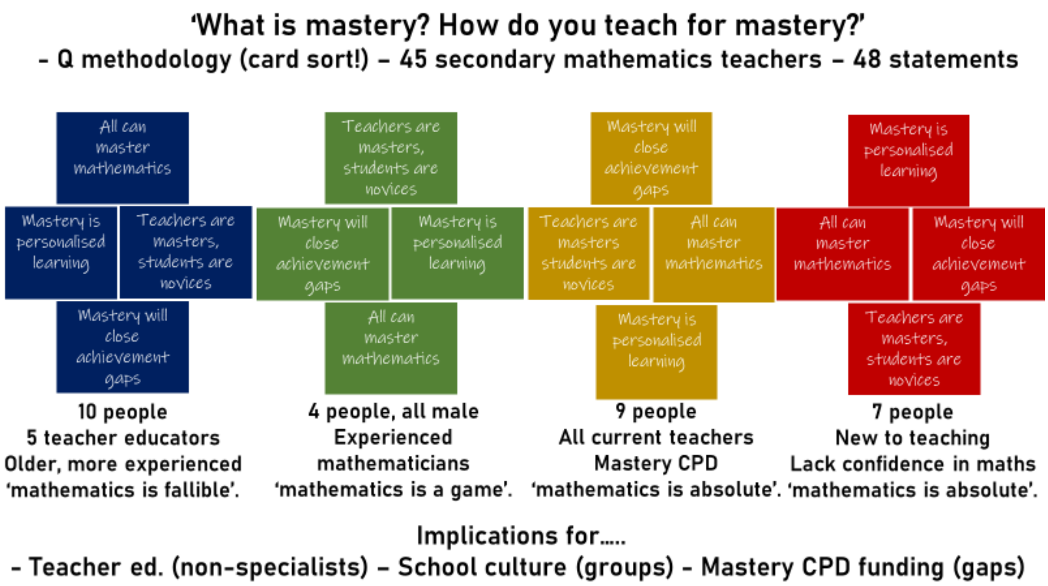


Figure 9.1: Research study factors as four hypothetical arrays

Figure 9.1 shows:

1. That even when there are few issues (in this case four statements) their relative importance leads to large differences in overall opinion. Education is a subject built on broad, contrasting theoretical bases and personal and professional values. Q methodology offers a way of systematically analysing an almost infinite set of complex interrelating issues. If the four arrays in Figure 8.1 were actual Q sorts of four teachers in neighbouring classrooms there would be huge differences in their student grouping, curriculum, and teaching style. The relative placement of statements, viewed holistically, gives the researcher insight into the complex set of opinions that a Factor represents.

2. Q study results are easily presented using different visual representations. Displaying each factor as a hypothetical factor array is standard practice (e.g. Watts and Stenner, 2012). My additional representations (e.g. **Error! Reference source not found.**Figure 7.9, Figure 7.10, Figure 9.1) are an example of how the results can be communicated to a variety of specialist and non-specialist audiences. Teachers in England regularly

utilise a ‘card sort’ or ‘Diamond 9’ activity as a pedagogical strategy, so will be able to interpret the results of a Q study effectively.

3. Q studies give a sense of the relative distribution of viewpoints within a group without overly favouring one factor viewpoint over another: this prevents the ‘tyranny of the majority’ and gives awareness and clarity to minority viewpoints. Figure 8.1 presents a sense of the range of opinions of mastery that exist and the relative numbers and characteristics of teachers which hold them. Different stakeholders can use the statistical information relating to the factors in the way most appropriate to their own needs. I make it clear in my study the views of Factor 2 were held by only four participants.

9.2.3 Q methodology ensured retained factors represented a true set of opinions

I was able to represent the infinite concourse of opinions of mastery, which has no agreed definition, by 48 statements. There is statistical confidence that the four Factors identified in the data are real manifestations of teacher opinion and not the result of chance. Even with only four statements and a simplified array of ‘one top, two middle and one bottom’ ranking there were still 12 possible combinations, hence a sample size of 12. Two participants sorting the cards in the same way are more likely to share opinion rather than it be chance. As the number of statements increases this likelihood increases exponentially (depending on the choice of distribution used). Brown (1980, p. 266) claims that a 33 item Q sample has between 4×10^{13} and 8×10^{36} different combinations, depending on the distribution and the conditions of instruction. High levels of correlation within factor sorts, optimised by the factor extraction and rotation processes, indicate a high probability of true shared opinion. This means that the mathematics education community can feel confident that the four distinct viewpoints of mastery, personified in the four Factors, will be present within mathematics teachers. The choice of Q methodology for the study allowed similarities and differences to be presented and analysed combining statistical confidence with a rich and searching narrative.

9.2.4 Q methodology allowed research participants to reveal their opinions through relative, not absolute judgements

The researcher's responsibility in designing their study is creating a Q set that is a balanced representation of the concourse. In the absence of previous Q studies on mastery in mathematics teaching and learning in secondary schools, I used a framework, categories, and subcategories to ensure the set of statements evenly represented different categories of mastery. The participants however looked at all the statements in a random order, and made comparative judgements in any way they chose, free of external influence. Evidence that participants were comfortable in completing the sort, and perhaps even enjoyed, the opportunity includes the relative ease in which participants were recruited and the 36 minutes average sort time. Participants also understood that their contribution was completely anonymous. The finding that two out of the four Factors revealed opinions that challenged progressive and traditional ideologies is evidence that participants made their choices with an open mind. Brown (2020b) links Q methodology with 'nudge theory': the sorting and placement process reveals participant preferences rather than likes or dislikes, and I believe this frees the participants from ideological shackles and allows their opinions to emerge.

9.2.5 Q methodology allowed quantitative and qualitative data analysis to happen together, not in conflict

My research study categorised the infinite potential opinions of mastery held by 45 teachers through iterations of data reduction and abductive logic. In Chapter 3, I reproduced Brown's (1980, p. 69) diagram to illustrate how the stages of the study revealed and explained shared opinions (see Figure 3.2**Error! Reference source not found.**). Reflecting upon the completed study, I note how the processes of data reduction and abduction are comparable but not identical to Brown's.

I was able to represent possible OPINIONS (top of Figure 3.2) through the processes of:

Appendix A Generating a concourse from the corpus of knowledge of mastery by using the literature review (Chapter 2),

Appendix B Generating a structure for reducing the concourse data in a transparent and representative fashion and articulating how the Q statements would be selected (Chapter 3),

Appendix C Selecting the final set of Q statements (Chapter 4).

I reduced these opinions to participant ATTITUDES (middle of Figure 3.2) by:

Appendix D Extracting, analysing, and rotating the four factors (Chapter 5),

Appendix E Considering the attitudes of the Factors 1, 2, 3 and 4 through the factor arrays, infographics and narratives (Chapter 6),

Appendix F Understanding the characteristics of the participants whose sorts are associated with them (Chapter 6).

It is at this point that data reduction ended and abduction started. The nature of Q methodology permits (and celebrates) the 'emergence of unanticipated behavior' (Stephenson, 1961, quoted in Brown, 1980, p. 36).

I employed abductive reasoning to explain mastery BELIEFS (bottom of Figure 3.2) by:

Appendix G Linking the aspects of mastery found to be relatively important for each factor with the participant characteristics, to understand what might influence beliefs about mastery (Chapter 6),

Appendix H Considering what the different groups of opinions about mastery mean for mathematics education (Chapter 7).

The above reflection is good evidence that my rigorous and transparent application of Q methodology utilised a method that is replicable and insights that are trustworthy.

9.3 Why Q methodology should feature more within educational research

9.3.1 Q methodology remains underutilised within school educational research

My claim that Q methodology is underused in primary and secondary school education is shared by other Q methodologists (e.g. Lundberg, de Leeuw and Aliani, 2020; Rodl, Cruz and Knollman, 2020; Walker, Lin and McCline, 2018). Lundberg *et al.*'s systematic research review identified 74 published primary and secondary education research studies which had utilised Q methodology since 2009, 13 from the UK. They concluded that Q is a flexible, yet rigorous, approach to identifying and evaluating hitherto marginalised groups. Q studies are not totally unknown within education: Coogan and Herrington provide an overview of Q methodology in the book *Research in secondary education* (2011) and list four education journals that had published Q studies. They advocate Q for research that seeks to know what points of view exist on a particular topic.

My research study aligned with one of the four recommendations of Lundberg *et al.*'s review, which stated: '…teacher educator perspectives, either as a single cohort or as part of mixed participant groups, are largely missing. This gap suggests the need for new research to supplement current educational research on improving communication between teacher education providers and their recipients thus highlighting more voices.' (2020, p. 13). Indeed, my own inclusion of the teacher educator voice in my research study was largely unplanned, and this additional voice led to an important discovery of an identified mismatch between teacher educators and serving practitioners.

Durning's (1999) paper advocating the use of Q methodology (with a post positivist positionality) in policy analysis specified five examples. These are: understanding likely reactions to policy decisions, defining understandings of 'fairness' in relation to a policy, understanding perceptions of an issue, identifying preferences, and evaluating policy effectiveness. These examples are applicable and useful in relation to education research and my own study discussion feature aspects of these examples. Baker, *et al's*, (2014) paper highlighted potential for Q to simultaneously investigate the nature and distribution of opinions by investigating observed correlations between participant characteristics and placement within a factor. Their approach would be useful as a follow-up to my own study.

9.3.2 Q methodology studies should feature in practitioner-based research

Engagement with and contribution to educational research by practitioners is highlighted as important for professional development, teacher effectiveness and teacher retention (BERA/RSA, 2014). However, teachers in England do not yet claim to have a good knowledge of research evidence, use research extensively in their decision making, or engage in their own research (Nelson *et al.*, 2017). Lee (2020) claims Q studies are useful for academic research, solving problems and as tool for classroom learning. These are three aspects of a teacher's role and so indicate that Q methodology studies should be plentiful in research about teachers and students, including research undertaken by practitioners themselves.

Lundberg *et al.*'s, 2020 review of Q education research found that Q offered opportunities for a participatory approach to policy formation. They identified potential for Q studies undertaken wholly or completely by teachers to evaluate specific aspects of learning or to gain a better understanding of their students. This participatory research has the potential to link up the 'education research ecosystem' of practitioners, researchers and policymakers, as recommended by the 'Harnessing Educational Research' report (British Academy/Royal Society, 2018). My study was devised using a researcher-generated concourse and Q set and analysed using publicly available software. Classroom teachers have successfully used Q studies to understand student and teacher beliefs (e.g Kotuľáková, 2019; Burke, 2018). The methodology and methods of my research study should be shared as a case study of the feasibility and opportunities of Q studies to enhance teacher participatory research.

As discussed in Chapter 7, teachers' views on education will be partly shaped by their conscious or unconscious educational ideology. This will apply to their choice of research method. A study can apply Q methodology in ways that favour the quantitative or qualitative extremes of Tashakkori and Teddlie (2009, see Chapter 3.3.2), thus valuing the preferences of the research study and the researcher. Q results can also be presented in ways that suit the researcher and the research consumers who may be novices to research and/or Q methodology.

9.3.3 Q methodology represents an evolution, not a hybrid, in educational research

Q studies in the field of education justify their methodological choices through situating education as within the social sciences, which is an inherently subjective field. The ongoing debates about credibility in educational research mean that Q should be celebrated as an addition to current quantitative and qualitative research methods. Ramlo (2015) claims Q is evolving: from firstly being lauded as the 'answer' to objectively measuring subjectivity and beyond the doldrums of being 'not properly statistical'. Q embraces the advantages of mixed-methods studies whilst remaining a single methodology.

The objective and subjective aspects of my research study were clearly delineated. Aspects of the study which could be objective remained so. A review of published and publicly available literature and a transparent framework enabled me to create the concourse and statements (Chapter 4). The participant recruitment strategy was explicit, and participants remained anonymous. The statistical analysis in extracting and presenting the Factors was robust.

Other parts of the study celebrate subjectivity. When participants sorted the statements, they placed cards in relation to each other to reveal their opinion of mastery at a single point in time and space. After extracting the Factors, I made a choice about how many to retain for the study. After rotating the Factors using Varimax I scrutinised the data and was open to the possibility of further hand rotation. The abductive discussion of the implication of the existence and nature of the four Factor groups was made by the researcher. I am confident that the four groups of views, or Factors, of mastery will be present in a group of professionals who share the characteristics of the research participants: I am also confident that different interpretations of the implications of Factors would be made by another researcher. I welcome the data being used in this way.

My own application of Q methodology, which embraced a postpositive position within a subjective subject area, gave good evidence that such studies can categorise preferences amongst a diverse body of people to obtain policy insights, as recommended by Durning (1999). Q methodology traditionalists emphasise the advantages of in-person sorts and post-sort interviews and embrace the subjectivity of interpretation. However, even Professor Brown, perhaps the most eminent advocate of traditional Q methodology, recently acknowledged that Q methodology, just like other statistical research methods, has room for multiple methods and applications (2020). Lundberg, *et al.*, suggest that ‘the full potential of Q’s flexibility in terms of study designs is not yet tapped’ (2020, p. 12). I have shown that a contemporary application of Q methodology, which utilises technology and is systematic in data collection and analysis, can collect data which is open to scrutiny and replication.

9.4 Conclusion

This chapter explored the implications of my choice of Q methodology to investigate what specialist mathematics teachers consider mastery in mathematics to mean, in relation to their own practice and the learning of their students. Using a specific application of Q methodology to identify the four factors was crucial to understand the trustworthiness of the results and thus the strengths of the implications of the factor views on mathematics education.

The importance of Q in my research study, coupled with its relative rarity in educational research studies in England, leads to my claim that Q studies should play a bigger role in educational research. The similarity of Q to a ‘Diamond Nine’, the availability of free-to-use Q analysis software, and the visual possibilities of Q sort data presentation mean that Q studies should be accessible to research novices as well as experts and could be part of practitioner research. An increased acceptance within the Q community to explore variations in the positionality and methods of Q studies mean that the methodology can include a variety of theoretical and practical constructions.

10 Conclusion

10.1 Meeting the aims of the research

Chapter 1 of this thesis explained that my research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their practice and the learning of their students?' was inspired by the increased popularity and funding of 'mastery' in mathematics in England despite there being no common definition or approach. To address this question, I needed to understand the evolution of mastery in mathematics over time and across countries and design a study for ascertaining different views of mastery held by teachers.

I was able to organise the diverse body of literature in relation to mastery with a framework, adapted from Ernest's (1991) classification of mathematics teacher ideologies. I was able to trace the different origins of mastery from seminal contemporary literature including Boylan *et al.*'s, 'Mastery innovation timeline and influences' (2019, p. 48, Figure 1). The framework enabled me to construct the final sample statements using a balanced block design. Piloting the statements with mathematics education specialists provided evidence that key aspects of mastery had not been overlooked.

Choosing Q methodology for the research study enabled me to study the subjectivity of teacher perceptions in a systematic way. I designed the study as a postpositivist application of a mixed-method methodology, and my study methodically observes and explains patterns that emerge from a single, unknown reality. My study appears to be the first mastery Q methodology research study and there were no previous sets of Q statements to use or adapt. I used the framework and findings from the literature review to devise a set of statements, whose placements by the research participants encapsulated a snapshot opinion within mastery's vast set of possible characteristics. A diverse selection of anonymous participants was recruited using social media and snowball techniques. Participants completed the Q sort thoughtfully in their own time and space.

I used centroid factor analysis and Varimax rotation to extract four different viewpoints of mastery, or Factors. My analysis indicated shared beliefs about what constitutes mastery in relation to students' demonstration of a deep understanding of mathematical structures. The four different viewpoints expressed by Factors 1-4 were underpinned by beliefs about student potential and teacher choice of pedagogical strategies. These beliefs correlated with the research participants' characteristics, knowledge, and experience. Interpreting the findings through Ernest's (1991) framework supported a link between mastery beliefs and teacher ideology.

10.2 Closing the circle: summary of the findings through the lens of Chapter 1

Chapter 1.2 outlined my stimulus considering multiple meanings of mastery: the Nrich activity, 'Make 37'. Dr. Morgan noticed two approaches to solving the problem, only one of which, from her perspective, demonstrated mastery. I conclude this thesis by returning to the same scenario and demonstrate how my findings offer new insights.

The narrative below represents how teachers with mastery viewpoints corresponding to Factors 1-4 might approach 'Make 37' as part of a typical mathematics lesson.

10.2.1 The approach of Factor 1

The experienced teachers and former teachers who share the viewpoint of Factor 1 portray mastery as: 'Travel far, travel together'. These teachers relish giving students 'Make 37' and are be happy to let students take the lead in working through it. They believe all students capable of tackling the problem and place them in mixed prior-attainment groups.

The Factor 1 teacher poses the problem and gives out a variety of resources to help the students, perhaps multilink cubes. The groups are left for a time to contemplate the problem and how to represent it using the resources. The students are encouraged try out ideas on paper, using the resources and talking to each other.

The teacher uses their professional judgement when deciding when, and how to intervene. They ask students to describe their progress and draws upon their subject and pedagogical knowledge to help students notice important aspects of their activity, such as what the numbers 1, 3, 5 and 7 have in common and what happens when '10 lots of the numbers' are added together. The teacher values all mathematically relevant student insights and is not pre-occupied with how many of the students solve the problem so long as they all engage in appropriate mathematical thinking which furthers their knowledge. All students who have developed their mathematical thinking have progressed on their journey to mastery.

10.2.2 The approach of Factor 2

The experienced male teachers who share the view of Factor 2 describe mastery as: 'Know your limits and follow the teacher.' These teachers make a judgement about which students can solve the problem and place the students in groups of similar attainment. Lower-attaining students are given something simpler to do.

The Factor 2 teacher, who immediately spots the answer to the problem, recognises that fluency in times tables will make this problem easier to solve, so starts the lesson with some whole-class times table practise. He then displays the problem for the class without any additional resources. He asks the class, 'what is the highest possible total?' and 'what is the lowest possible total?'. The teacher then tells the students to draw a table in their books (with columns labelled numbers used and total) and asks students to try different combinations of numbers, suggesting they work systematically and individually.

After 5 minutes one of the students puts their hand up and suggests that the problem is impossible. The teacher asks the student why they believe this, and the student replies that all the totals they have calculated are even numbers, and 37 is an odd number. After praising the student, the teacher asks how many other students had worked it out. About one-third of the class put up their hands. The teacher writes on the board, 'all pairs of odd numbers sum to an even number' and proves it using algebra. Students copy the statement and the proof into their books. Students who remember the statement have learnt something, but only the students who understand the proof have mastered the mathematics in the lesson.

10.2.3 The approach of Factor 3

The inexperienced but mathematically competent teachers who share the view of Factor 3 describe mastery as: 'Create a curriculum for interconnected understanding.' These teachers believe that by the end of the lesson all students will be able to solve the problem if they plan the lesson properly. The lesson was collaboratively planned by all mathematics teacher in the department. The students are placed in mixed-attainment groups.

The focus of the Factor 3 teacher's lesson is understanding the nature of odd numbers and their multiples. The teacher starts by discussing the nature of '3'. Different representations of 3 are displayed on the board: 3 shoes, the Numicon number 3 shape, 3 eggs in an eggbox. The class discuss each image and agree that 3 is an odd number as there 'is one left over' when items are paired. Multilink cubes are given out and the class hold up their own odd number. The different shapes are compared. The teacher then asks the class what 'two lots of 3' are and uses the same representations to demonstrate that 2 x 3 must be even as 'the leftovers are put together'. The class chant the 3 times table and the teacher draws attention to the odd and even multiples of 3. The class then chants the 5 times table, and the teacher draws attention to the odd and even multiples of 5. This is repeated with the 7 times table. The teacher is confident that the whole class understand the pattern of odd and even numbers in the 3, 5 and 7 multiplication tables.

The teacher then put the images of the bags on the board (but not the question) and asks the students to work in pairs on mini whiteboards. The teacher asks the students to pick any ten numbers from the bags, add them together and write the calculation and the answer on their board. After three minutes, the teacher strategically selects some of the whiteboards and asks the students to stand up and explain their calculation and the sum. The teacher draws attention towards what is the same, and what is different about the sums. The teacher then poses the 'make 37' problem and asks students to work on the problem individually for five minutes. Most of the class demonstrate that the problem is impossible with different levels of sophistication. Students who understand that pairs of odd numbers will always be an even number have mastered the lesson content.

10.2.4 The approach of Factor 4

The inexperienced and mathematically unconfident female teachers who share the view of Factor 4 describe mastery as, 'Variety in teaching, learning and achievement.' These teachers believe that all students should be able to attempt the problem if they want to, but not all will be able to solve it in the limited time available. The students, who attend a selective school, are placed in ability groups.

The Factor 4 teacher sets 'Make 37' as a task that can be attempted by students after they have completed some textbook questions. The teacher introduces the task briefly then spends most of the lesson helping individual students with whichever activities they are working on. She uses her effective pedagogical skills knowledge to keep the students on task and help individual students who are stuck. After half an hour the teacher notices that about six students are stuck on the problem and calls them over to a table to work together. No students have found the answer and one student asks if it is impossible. When the teacher tells them that it is indeed impossible the student asks why. She then directs the question back to the group of students, who notice that all the totals they have worked out are even. One of the students suggests that all possible totals must be even, and between them, the teacher and the student group work out the reason for this. This group of students, and the teacher, have mastered something about the structure of odd numbers through discussing the problem.

10.3 Contributions to professional knowledge and practice

The nature of a Q methodology research study is to expose and explore differences in opinions, and this study was inspired by observations of tension in discussions of mastery in mathematics. The contributions to professional knowledge and practice arise from understanding how the nature of these differences impacts mathematics education.

10.3.1 The study identifies a model for mastery as social justice

The study offers evidence that certain teachers have a view of mastery consistent with Factor 3. This Factor rates, above all, statement 48: 'Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps'. Factor 3 combines progressive views about the aims of mathematics education (all students can and should gain a deep understanding of mathematics) with traditional views of teaching mathematics (students should progress through a curriculum together, and lessons should be explicitly teacher-led). Teachers who present this viewpoint are not necessarily experienced teachers but are confident in their mathematical knowledge and passionate about professional development. The existence of Factor 3 offers a model for improving mathematical understanding and achievement in English schools. The finding that Factor 3 holds views that transcend ideologies explains why attainment gaps persist, and that intervention is needed to close these gaps.

10.3.2 The study identifies a necessity to refine and join up teacher training and subject-specific professional development

Whilst at least two out of the four Factors have a belief that all students can master mathematics, mathematics attainment data indicates this has not yet been achieved. The study indicates that Factor 3 teachers may be inexperienced in the classroom but report a good level of mathematical knowledge. Inexperienced teachers who report a lower level of mathematical knowledge are prevalent in Factor 4. Experienced teachers correlated with the views of Factor 1 or 2. Closing the attainment gap requires more teachers with the views of Factor 3. Therefore, teacher training should have an increased focus on developing subject knowledge and confidence which empowers new teachers to take more responsibility for explicitly leading the learning in their classrooms. Professional development for qualified teachers must maintain a teacher's belief in a growth mindset and their responsibility and competence to provide and teach appropriate representations to ensure all students understand the mathematics content they are being taught. Creating and maintaining appropriate conditions for Factor 3 teachers to flourish will address teacher recruitment and retention.

10.3.3 The study highlights a need for more Q methodology studies in educational research in England

The research question, 'What do specialist mathematics teachers consider mastery in mathematics to mean, in relation to their practice and the learning of their students?' necessitated synthesising a diverse body of knowledge and eliciting opinions from a diverse body of teachers. The structured findings of the research study, and the insights offered, could only have resulted from a Q methodology study. My own research stance, occupying the postpositive end of Tashakkori's and Teddlie's (2009) continuum, takes a specific and unique position in different segments of research. The 'qualiquantology' nature of Q identified distinct sets of opinions which were not unduly influenced by preconceptions or ideology. The information was gathered without placing undue time pressures on busy teachers. My own use of infographics and pictorial representations of the findings, and my clear and transparent analysis process highlight how Q data can be easily interpreted by teachers not yet expert in reading educational research. My research design and findings indicate Q methodology should be used to gather information about teachers, teacher educators and students, including practitioner-based research.

10.4 Limitations of the study

Addressing my research question required me (a new researcher) to make sense of a diverse and contestable topic using a relatively unusual and hitherto unknown methodology. The topic of mastery in mathematics was (and remains) important within mathematics education and my findings are significant in this field. However, I realise with hindsight that I set myself an enormous challenge.

10.4.1 Literature review and creation of statements

The literature review, undertaken first in the research, needed to serve multiple purposes. These included investigating the nature, history, and nature of mastery in mathematics and providing the corpus for the concourse and Q set. Q methodology is about explaining an observation in terms of new insights rather than proving a hypothesis, and no previous Q study existed on mastery, so there was no existing model that I could use to organise my reading. Ernest's framework, and Boylan, *et al.*'s, timeline were effective structures and seeds for the review, but I could have considered other frameworks. I also could have been more strategic in choosing initial key words for database searches and when choosing practitioner resources to review. The nature of mastery in mathematics as a relatively new topic for a research study meant finding an obvious 'gap' in the literature complex to articulate. I also removed one of the categories of mastery – the nature of mathematics – from the Q set. Whilst I articulated sound reasons for doing so, this meant that the Q set could not represent the complete universe of opinions of mastery.

10.4.2 Positionality, data collection and data reduction

Taking a postpositivist position in a Q methodology research study was a risky strategy for a new researcher. I am aware that my background in mathematics and economics biases me towards realist, objective, quantitative approaches to research. I accept, though I have endeavoured to remain transparent and subjective in analysis and interpretation, the very construction of a narrative about mastery by an experienced mathematics educator (who works with the NCETM) suggests a degree of bias and interpretivism that postpositivists would rightly interrogate. I could have explored my own biases further by doing the Q sort myself.

I made choices about the (online) Q sort process and Conditions of Instruction without prior experience of conducting a Q sort. Whilst I did consult with methodological literature and reflect on two pilot studies, this inexperience will inevitably have led to impreciseness in the Conditions of Instruction and thus lower quality Q sorts by the participants.

My status as a novice Q researcher meant that I made decisions partly for pragmatic reasons rather than methodological ones. I chose Centroid factor analysis over PCA (the mathematically 'better' solution) because I had a better understanding of the mathematics underpinning the technique. I used Varimax rotation instead of hand rotation because of the security of a 'mathematically correct' solution and because I lacked confidence in applying hand rotation.

10.4.3 Data analysis and interpretation

Q methodology literature embraces its 'qualiquantology' nature, however this means that neither the rigour of the quantitative aspect nor the richness of the qualitative aspect can be overplayed. The study's participants were not randomly chosen, nor did they have a 'voice' in the statements, which they sorted according to instructions dictated to them by the researcher. At best, they were able to provide a 'best fit' representation of their views of mastery within the constraints I had set.

I have not been immune to the temptation of over or underestimating the significance of the findings. I found one solution that best fits the revealed relative preferences of one group of teachers at a single point in time and I acknowledge that another researcher would use a different factor rotation to identify a different set of viewpoints, with different significant sorts. I have also been tempted to overplay the generalisability of these results: whilst the four factors represent 'real' viewpoints of mastery, other viewpoints do exist, and the factors may not exactly fit the viewpoint of any one person. Factors 1-4 are not the complete picture, and Q methodology was not created to provide one. In the same vein, although a researcher should notice and report interesting features of the participants who identified with a viewpoint, I have at times ventured into making connections between viewpoints of mastery and the characteristics of the participants who shared the viewpoint. These connections are limited to the group of research participants and the results of the study alone are not generalisable to larger populations.

10.4.4 Teacher beliefs not teacher practices

This study identifies teacher viewpoints of mastery. Research participants expressed their thoughts on the curriculum, pedagogy, assessment (and so on) that supports mastery teaching and learning, and the factors are used to tell four different ‘stories’ (see Chapter 10.2). I use these findings to identify changes that could be made in schools, ITT programmes and mastery programmes.

This study did not examine teacher practices. So, whilst conclusions are made about teachers’ expressed preferences, I cannot examine the relative effectiveness of these practices. For example, teachers with the viewpoints of Factor 3 may favour mixed attainment classes to narrow achievement gaps, but I have no way of knowing whether they do exhibit these practices, nor whether they are successful in their aims.

10.4.5 Beliefs about mastery or beliefs about mathematics?

In Chapter 8 I align the viewpoints of the factors with ideologies using Ernest’s (1991) framework. This framework was created to study viewpoints of learning and teaching in mathematics, not any specific aspect. The study did not ask teachers to consider their philosophical position in relation to mathematics and so it is impossible to know the extent a teacher’s viewpoint of mastery is distinct from their philosophy of mathematics.

10.4.6 Limitations of Q methodology

Q methodology offers *a*n explanation, not *the* explanation, for observed differences in participant opinion. The abductive reasoning inherent in Q methodology explains viewpoints which are derived 'from the subject's standpoint as *he* (sic) understands them, and not from the external standpoint of the observer' (Brown, 1980, p. 321). Brown's (1980) diagram, reproduced in Figure 3.2, shows these limitations. All standpoints are true for a point in time only, so the set of opinions 1-infinity are transitory. The data reduction methods, whilst mathematically rigorous are not unchallenged, so the attitudes A, B, C are (rightly) open to questioning. The abductive reasoning to explain the beliefs X and Y are interpretive in nature. Hence, all conclusions from this study, a specific and flawed application of Q methodology, should be treated as limited.

10.5 Recommendations for future research

10.5.1 Research within mastery in mathematics

An important aspect of a postpositivist position is replicability. I welcome other researchers wanting to use the Q sample or raw data for their own research into mastery, both in England and internationally. I would be very interested to explore whether the distribution of teacher characteristics in each viewpoint corresponds with a larger population.

I would be interested in the implications of exploring the unrotated single factor solution, and different factor rotations (perhaps to investigate the significance of a particular thematic viewpoint, such as problem-solving or mixed-attainment classes). Since one-third of the participant's viewpoints were not directly with Factors 1-4, this research would take more mastery viewpoints into account.

This study made claims about distinct teacher beliefs of mastery in mathematics. It made no claims about teacher effectiveness. Future research could investigate the relative effectiveness of teachers who share the beliefs of each Factor. Research could also investigate the beliefs about mastery in mathematics of students, parents, teacher educators (specifically) and other stakeholders.

This study did not ask teachers to specify the age of the students they were thinking about when they considered the statements. It would be very interesting to consider whether, and how, beliefs about mastery teaching and learning change depending on the age of the students. Teachers of particular year groups, Key Stages and attainment levels could be asked to consider the statements in relation to their specific students.

A completed Q sort captures a participant's views about a topic at a point in time. Repeated sorts by attendees of mastery professional development (including trainee teachers) could be undertaken, to capture important information about whether, and how, teacher views and beliefs about mastery teaching and learning change as a teacher's career develops.

10.5.2 Wider research ideas

The notion of mastery could be investigated in relation to other school subjects. The extent to which mastery is subject-specific or represents more holistic knowledge and skills would also be important and interesting to consider, especially in times when the words 'cultural capital' and 'knowledge-rich curriculum' are used, perhaps inconsistently, by educators. Teachers' beliefs about these words, or similar could be investigated.

Teachers and students in English schools may have limited knowledge of Q methodology but are very used to 'Diamond 9' activities. This research study could be replicated with adjusted instructions; asking teachers to place their statements in a Diamond 9 configuration instead of a Normal distribution. There is potential to use a suitably adapted Q methodology to capture student voice.

Q methodology has the potential to provide a unique perspective on professional development evaluation. A research study could examine whether, and how, a teacher's beliefs about an aspect of education change following a sustained professional development activity. Q methodology could also be used to examine departmental or school changes over time. A suitable adapted study could be used to examine changes in student beliefs over time.

Much of this thesis was written in 2020: a year of uncertainty, home-schooling, and online professional development. Q-methodology could be utilised to elucidate student and teacher viewpoints on different aspects of education in the time of COVID-19. This includes attitudes on home-learning, remote teaching, and the increase in mixed-attainment classes due to the constraints of the pandemic.

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Glossary

Centroid	A specific computational method of extracting factors from the completed participant Q sort data, traditionally used in Q methodology.
Concourse	A set of statements that encompass all possible viewpoints of the subject being studied in a Q methodology research project.
Conditions of Instruction	Specific instructions given to participants undertaking a Q sort, e.g.: 'sort these statements so that those least representing your view are at the left-hand side of the diagram, and those most representing your view are on the right-hand side.'
CPD	Continuing Professional Development. A term that encompasses all training received by that serving teachers.
DfE	Department for Education. The government department responsible for children's services and education, including early years, schools, higher and further education in England.
Factor	A distinct viewpoint about the topic being investigated in a Q methodology project, usually represented by a 'theoretical' factor array.
Factor array	A theoretical completed Q sort, showing the placement of each Q sample statement by a hypothetical person who completely represented the opinion expressed by a particular factor.
Factor rotation	A statistical technique which adjusts the makeup of the retained factors to (for instance) maximise the number of participants who identify with only one factor.
ITE or ITT	Initial Teacher Education or Training. The two terms are used interchangeably in education and policy literature. The period of training undertaken by pre-service teachers.
Key Stage (KS)	The age group division of a child, corresponding to levels of the English National Curriculum. KS1=5-7, KS2=7-11, KS3=11-14, KS4=14-16, KS5=16-18.
Mastery	A term which encompasses a range of pedagogical strategies and approaches associated with developing school students' deep understanding of mathematics.

Maths Hubs	A network of 40 hubs, funded by the DfE and locally led by an outstanding school or college, to develop and spread excellent practice in mathematics teaching and learning.
NCETM	National Centre for Excellence in Teaching Mathematics. A government-funded organisation that provides and oversees professional development of mathematics teachers in England.
PCA	Principal Component Analysis. A specific computational method of extracting factors from the completed participant Q sort data.
PGCE	Post Graduate Certificate in Education. A qualification studied by most pre-service teachers in England.
Q methodology	A research methodology which applies quantitative techniques to classify subjective viewpoints.
Q sample	The final set of statements used in a Q methodology research study.
Q sort	A complete set of statements that has been sorted by a research participant in a Q methodology study.
SKE	Subject Knowledge Enhancement (course). A programme to develop mathematical subject knowledge for teaching. Usually undertaken by potential pre-service teachers or by qualified teachers wanting to become mathematics specialist teachers.
Varimax	A specific computational method of factor rotation which maximises the amount of study variance accounted for by the factors.

Appendix A Final Q sample statements

No	Statement	Theme	Subcategory	Type	About
1	In mastery lessons all students should be assessed every lesson	Attainment and Assessment	Frequency	Continual	Teaching
2	In mastery lessons assessments should only be used at the beginning and end of topics	Attainment and Assessment	Frequency	Periodical	Teaching
3	All students are capable of achieving a mastery level of attainment	Attainment and Assessment	Distribution	Uniform	Learning
4	In general, 1/3 of students will achieve a mastery standard, 1/3 of students will achieve an average standard, and 1/3 of students will achieve a low standard of attainment	Attainment and Assessment	Distribution	Graduated	Learning
5	Mastery will be easier to attain if children are taught in groups of similar prior attainment	Attainment and Assessment	Grouping	Set	Teaching
6	Mastery will be easier to attain if children are taught in groups of mixed prior attainment	Attainment and Assessment	Grouping	Mixed	Teaching
7	Learners should move through a mastery curriculum at their own pace, moving on once they reach the expected level of attainment	Mindset and Differentiation	Progression	Personalised	Learning
8	Learners should move through a mastery curriculum as a group, only moving on once all students have reached the expected level of attainment	Mindset and Differentiation	Progression	Whole group	Learning
9	Teaching for mastery increases the rate of learning for lower-achieving students so they can catch up	Mindset and Differentiation	Gap	Catch up	Learning

10	Teaching for mastery involves students keeping up, not catching up	Mindset and Differentiation	Gap	Keep up	Learning
11	To achieve mastery, in mathematics lessons all students should be working on the same problems at the same time	Mindset and Differentiation	Exercises	Same	Learning
12	To achieve mastery, in mathematics lessons students should all be working on different problems	Mindset and Differentiation	Exercises	Different	Learning
13	In a mastery curriculum students will understand the structure of number before applying it to other topics	Curriculum	Topics	Compartmente d	Learning
14	In a mastery curriculum students will develop an understanding of the structure of number through applying it to other topics	Curriculum	Topics	Connected	Learning
15	A curriculum for mastery should give equal priority to number, algebra, geometry and data handling	Curriculum	Weighting	Unweighted	Teaching
16	A curriculum for mastery should give greater priority to number and algebra	Curriculum	Weighting	Weighted	Teaching
17	Planning mastery lessons is quicker because there are no differentiated resources to create	Curriculum	Planning	Faster	Teaching
18	Planning mastery lessons is slower because it takes a long time to craft the small-steps teaching and pupil exercises	Curriculum	Planning	Slower	Teaching
19	To achieve mastery, students should be explicitly taught mathematical laws (for instance the commutative, distributive and associative laws), including their formal names	Methods	Laws	Teaching	Teaching

20	To achieve mastery, students should understand mathematical laws (for instance the commutative, distributive and associative laws) but do not need them to be explicitly taught	Methods	Laws	Understanding	Teaching
21	A student is more likely to achieve mastery if a teacher uses a specific pedagogy	Methods	Pedagogy	Specific	Teaching
22	Mastering mathematics is unconnected with specific teacher pedagogies	Methods	Pedagogy	Non-specific	Teaching
23	Teaching associated with mastery assumes a 'novice-expert' relationship between teacher and student	Methods	Relationship	Expert	Teaching
24	Teaching associated with mastery assumes a 'mentor-mentee' relationship between teacher and student	Methods	Relationship	Mentor	Teaching
25	In mastery lessons, a question should be set that a student could only answer if they have learnt something beyond what has been explicitly taught	Small steps and Variation	Questions	Stretch	Teaching
26	In mastery lessons, all questions set should reflect only what has been explicitly taught	Small steps and Variation	Questions	Content	Teaching
27	In mastery lessons, complex problems should be reduced by the teacher into a series of steps	Small steps and Variation	Reduction	Teacher	Learning
28	In mastery lessons, complex problems should be reduced by the students into a series of small steps	Small steps and Variation	Reduction	Student	Learning
29	Teaching for mastery should minimise lecturing and maximise student participation	Small steps and Variation	Participation	Student	Teaching
30	Teaching for mastery should maximise the opportunity for teachers to impart their knowledge to students	Small steps and Variation	Participation	Teacher	Teaching

31	Mastery lessons should incorporate multiple representations of a concept	Multiple Representations	Type	Variable	Learning
32	To master mathematics is to understand mathematics using concrete, pictorial and abstract representations	Multiple Representations	Type	Fixed	Learning
33	Multiple representations are not always needed in secondary school teaching for mastery	Multiple Representations	Understanding	Hierarchical	Learning
34	A goal of mastery is to understand mathematics without needing a concrete or pictorial representation	Multiple Representations	Understanding	Non-hierarchical	Learning
35	In mastery lessons, learning is constructed by the teacher's careful explanation and selection of problems	Multiple Representations	Construction	Teacher	Learning
36	In mastery lessons, learning is constructed by the students noticing similarities and differences in the mathematics they are doing	Multiple Representations	Construction	Student	Learning
37	Rote-learning is incompatible with mastery learning	Flexible fluency	Rote	Unnecessary	Learning
38	Rote-learning is an inevitable part of mastery learning	Flexible fluency	Rote	Necessary	Learning
39	Practising similar problems is part of developing a mastery understanding of mathematics	Flexible fluency	Practise	Similar	Learning
40	Practising a variety of problems is part of developing a mastery understanding of mathematics	Flexible fluency	Practise	Variety	Learning
41	In mastery lessons problem-solving is developed through exercises which combine topics	Flexible fluency	Problems	Connected	Learning
42	In mastery lessons problem-solving is developed by ensuring each separate topic is fully understood	Flexible fluency	Problems	Compartmented	Learning

43	Reading, and taking part in, educational research is an important aspect of teaching for mastery	Continued Professional Development	Knowledge	Subject	Teaching
44	Mastery professional development activities should include a high degree of teacher subject knowledge development	Continued Professional Development	Knowledge	Pedagogy	Teaching
45	Mastery professional development activities should include a high degree of specific pedagogy development	Continued Professional Development	Location	Outside	Teaching
46	Teaching for mastery pedagogy is mainly learnt through external professional development	Continued Professional Development	Location	Within	Teaching
47	Teaching for mastery pedagogy is mainly learnt through collaborative in-school professional development with colleagues	Continued Professional Development	Focus	Research	Teaching
48	Teaching for mastery is vital in UK secondary schools to improve standards and close achievement gaps	Continued Professional Development	Focus	Improvement	Teaching

Appendix B Ethical Clearance

02 August 2018 Ref: 17/Edu/21C

Jennifer Shearman
c/o School of Teacher Education and Development (SOTED)
Faculty of Education

Dear Jennifer,

Confirmation of ethics compliance for your study - *What is Mathematical Mastery? Secondary Teachers' perceptions of Mathematics Mastery and its relationship to mathematical knowledge, pedagogy and teacher development*.

I have received your Ethics Review Checklist and appropriate supporting documentation for proportionate review of the above project. Your application complies fully with the requirements for proportionate ethical review, as set out in this University's Research Ethics and Governance Procedures.

In confirming compliance for your study, I must remind you that it is your responsibility to follow, as appropriate, the policies and procedures set out in the *Research Governance Framework* (http://www.canterbury.ac.uk/research-and-consultancy/governance-and-ethics/governance-and-ethics.aspx) and any relevant academic or professional guidelines. This includes providing, if appropriate, information sheets and consent forms, and ensuring confidentiality in the storage and use of data.

Any significant change in the question, design or conduct of the study over its course should be notified via email to red.resgov@canterbury.ac.uk and may require a new application for ethics approval.

It is a condition of compliance that you **must** inform me once your research has completed.

Wishing you every success with your research.

Yours sincerely,

Tracy

Tracy Crine
Contracts & Compliance Manager
Email: red.resgov@canterbury.ac.uk

CC Dr Anne Nortcliffe
Ms Lynn Revel

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Professor Rama Thirunamachandran, Vice Chancellor and Principal

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Appendix C Final Q sort administration, questions, and conditions of instruction

Final Q sort administration, questions and conditions of instruction

Introduction text

Thank-you for clicking on the link.

My name is Jen Shearman and I am a Senior Lecturer in mathematics education at Canterbury Christ Church University.

I am undertaking a research study for my Doctorate thesis using Q-methodology to investigate secondary mathematics teachers' meanings of 'mastery' in relation to teaching and learning.

Please complete this study if you are a current or former teacher of mathematics in a secondary school or college (any ages from 11-18), a trainee teacher or a teacher educator.

If you choose to complete the study you will firstly be asked a few questions about your professional background.

You will then be presented with forty-eight statements about mastery which you will place in a grid according to how well each statements aligns with your personal beliefs about mastery. There are no right or wrong answers.

Once I have collected data from all participants in the study I will analyse all the grids to identify meanings of mastery shared by groups of people.

This study will take about thirty minutes to complete and all responses will by anonymous.

I hope you will enjoy completing the research.

Pre-sort questionnaire

Question	Choices
What gender do you identify with?	Male Female Prefer not to say
What is your current age bracket?	18-24 25-29 30-34 35-39 40-44 45-49 50-54 55-59 60-64 65 and over Prefer not to say
Which area of the country do you currently work in?	North West England North East England Yorkshire and the Humber East Midlands West Midlands East of England South East England London South West England Northern Ireland Scotland Wales Other
Please tick the box that best describes your current role	Teacher of mathematics Trainee teacher of mathematics Mathematics teacher educator Former teacher of mathematics Other
How many years in total have you spent teaching pupils aged 11-18?	Number of years

Which characteristics from the list below describe the type of school you have spent most of your career so far teaching in (select all that apply)?	Comprehensive school (in a non-selective area) Grammar School Independent school Alternative provision (eg. a PRU) Special School Comprehensive school(in a selective area) Multi-academy trust Single academy school Local Authority School Single-sex school Other
What is the title of your first degree?	Title
Approximately what percentage of your first degree involved mathematics?	0-24% 25-49% 50-74% 75-100%
Which of the choices below closest matches your training route as a mathematics teacher?	University-based PGCE Assessment only Employment-based QTS only (eg. School Direct, GTP, OTT) Employment-based PGCE (eg GTP, Teach First, School Direct) Other I did not train to be a mathematics teacher
Did you undertake a Subject Knowledge Enhancement (SKE) course either before or during your training?	No Yes
Have you undertaken any CPD specifically related to mastery?	Yes - during my training Yes - from Complete Mathematics/La Salle Education Yes - from the NCETM or Maths Hubs Yes - from White Rose Maths Yes - from Maths Mastery/ARK schools Yes - other No

Pre-sort text

Now for the study. A Q-sort is a bit like a 'card sort' or a 'diamond nine'; you will be shown forty-eight statement cards and will need to make some decisions about how much each statement aligns with, or doesn't align with, your own definition of 'mastery' in relation to teaching and learning of mathematics. The study only looks at how you have ranked the statements relative to each other.

For the first part of the study, please read each statement and put it in one of three piles; 'agree', 'disagree' or 'neutral'. It doesn't matter how many cards you put in each pile.

In part two of the study you will place the cards into the grid below. You must put the exact number of cards in each column.

A suggested way to do this is as follows.

1) Sort the 'agree' pile into the right hand side of the grid. Put the card you agree with most in the furthest right-hand column (column I), then work backwards towards the middle. Cards placed in the same column will be judged as being of the same relative importance to you.

2) Sort the 'disagree' pile into the left hand side of the grid. Put the card you disagree with most in the furthest left-hand column (column A), then work backwards towards the middle. Cards placed in the same column will be judged as being of the same relative importance to you.

3) Finally, sort your 'neutral' pile into the remaining spaces in the grid.

You may find some cards easy to place, and some more difficult. Remember that it is the relative importance that matters; you may agree or disagree with all the cards - that is fine. Take your time and remember there are no right or wrong answers.

If you scroll down to the bottom of the screen there is a '+' and '-' button that you can use to change the grid size. This may make it easier for you.

Post-sort questions

For most agreed and most disagreed cards – why did you place this card here?

Question	Choices

Which statement did you find hardest to place? Please explain why	Free text
This is the last question. Which of these statements closest aligns with your own opinion of the nature of mathematics?	Mathematics is a language to unite society and facilitate communication and agreement Mathematics is a game of symbols and rules created to explain observations and solve human problems Mathematical knowledge is absolute, as humans we can only discover and learn it Mathematical knowledge is fallible, humans invented it and can refine and change it

Post-sort information

Thank-you so much for completing this study.

By clicking 'submit' you are consenting to be included in my research (please see details below).

Your contribution is completely anonymous.

Please do email jennifer.shearman@canterbury.ac.uk if you have any comments or queries. If you wish to be withdrawn from the study please give the date and time that you completed the sort.

I would be delighted if you would share the link to my research with your mathematics colleagues.

The link is https://tinyurl.com/masteryresearchstudy

This research study is being conducted at Canterbury Christ Church University (CCCU) by Jennifer Shearman, as part of her Doctorate in Education thesis. This research will uncover the meaning of ‘mathematical mastery’ for secondary mathematics teachers. Groups of teachers who share similar opinions will be identified, and discussed in relation to the research findings and current government policy advocating the development of a mastery approach within secondary schools. On the legal basis of consent all data and personal information will be stored securely on the QsortTouch server. No unrelated or unnecessary personal data will be collected or stored. The following categories of personal data will be processed; gender, age category, academic qualifications, area of residence, teacher training route, category of employing school. Personal data will be used to categorise distinct opinions in relation to the research question for the purposes of this research only. Data can only be accessed by the researcher, QsorTouch owner and supervision team. After completion of the study, all data will and held for a maximum period of 5 years. The results of this study will be disseminated through the researcher's thesis. The research may also be submitted as a paper to peer-reviewed academic journals. If you have any questions or concerns about the nature, procedures or requirements for participation do not hesitate to the researcher or her supervisor. Should you decide to participate, you will be free to withdraw at any time without having to give a reason. Withdraw through contacting the researcher or her supervisor, giving the date and time that you completed the sort. Please contact the researcher by email at jennifer.shearman@canterbury.ac.uk, or by writing to her at Christ Church University, North Holmes Road, Canterbury, Kent. If you wish to contact Jennifer's supervisor, she may be contacted at anne.nortcliffe@canterbury.ac.uk or through the university address above.

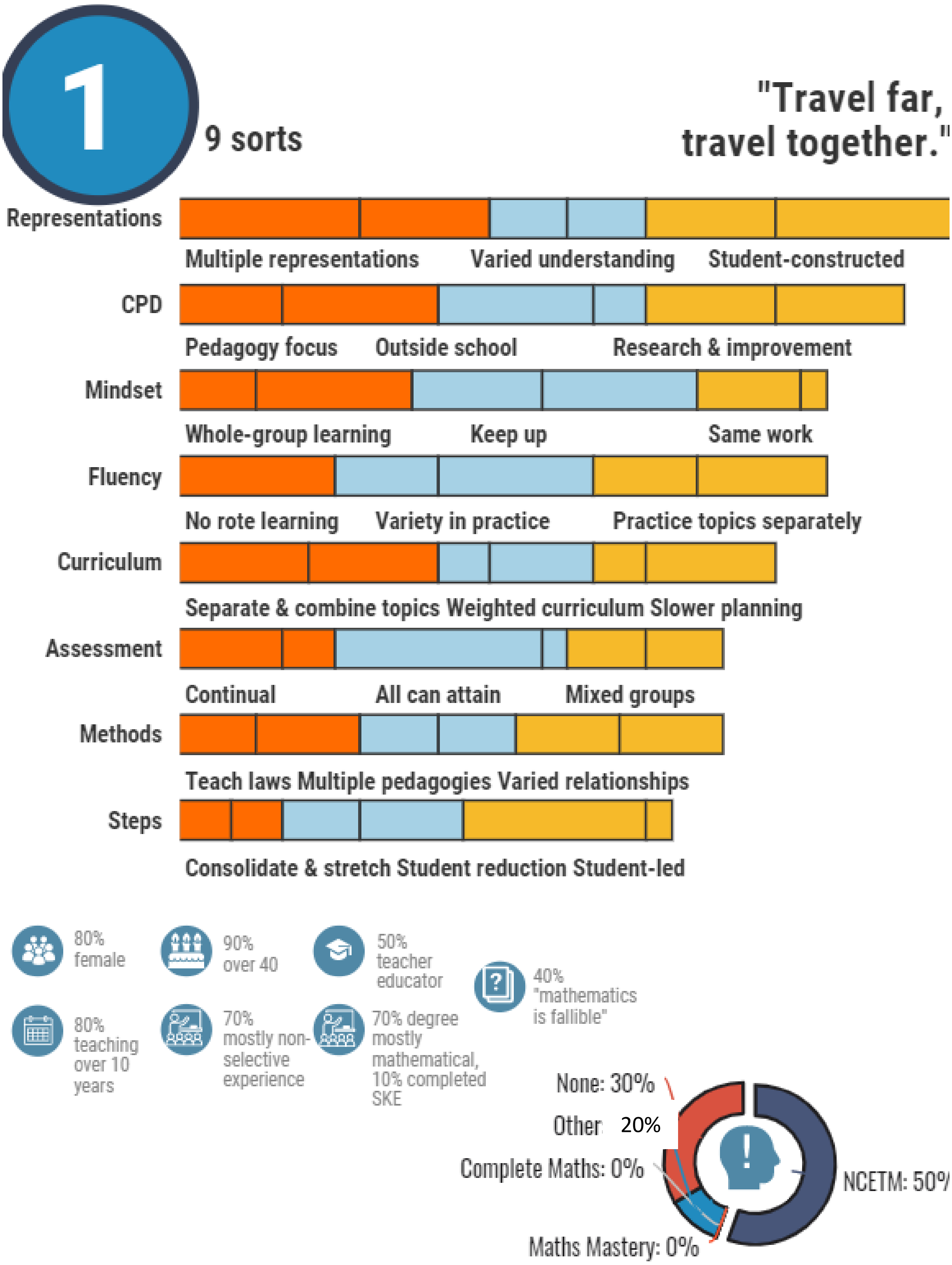
Appendix D Z-scores and Q sort scores for Factors 1-4

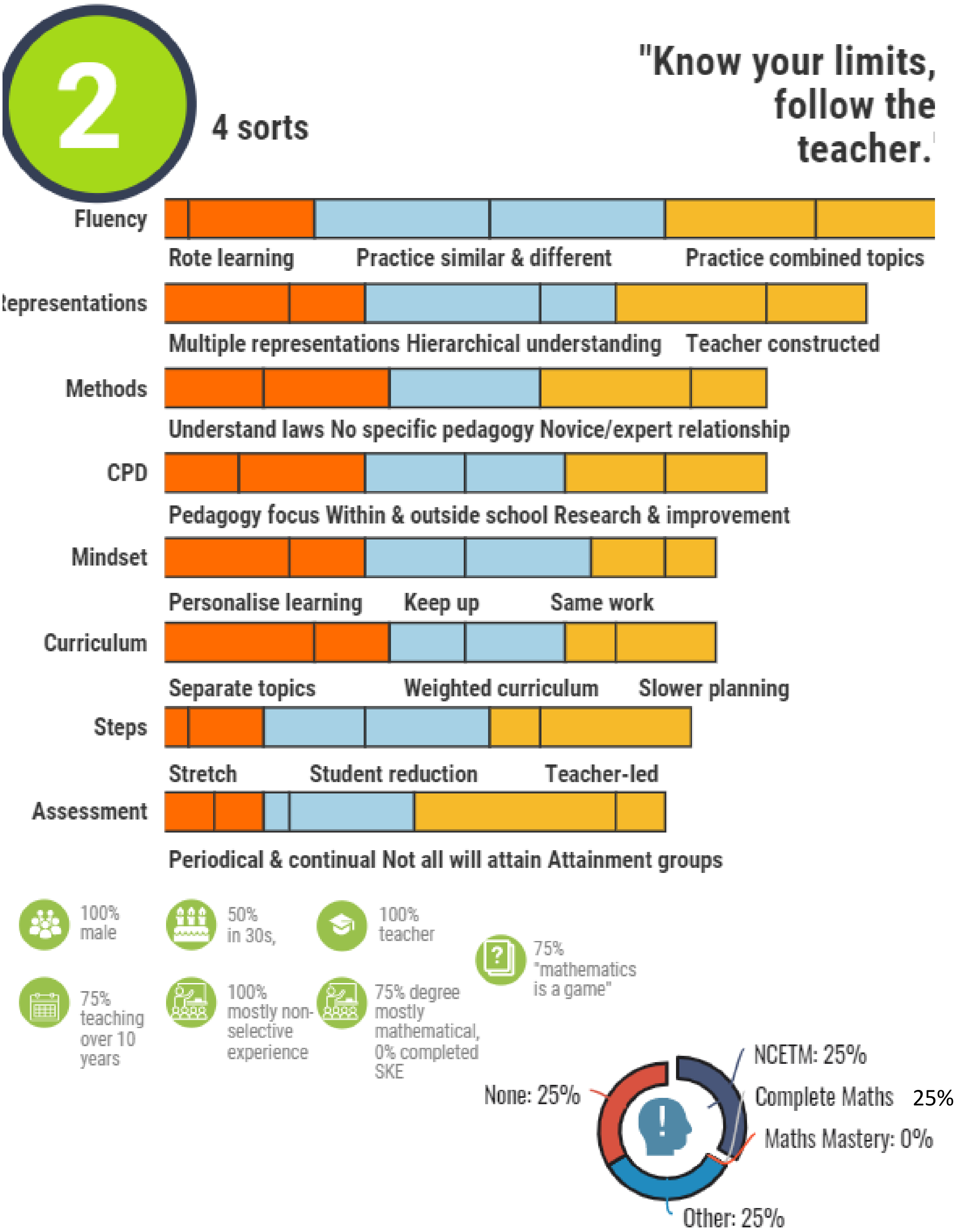
Green cells = consensus statements, Yellow cells = positive distinguishing statement, Red cells = negative distinguishing statement. Grey cells = other distinguishing statement.

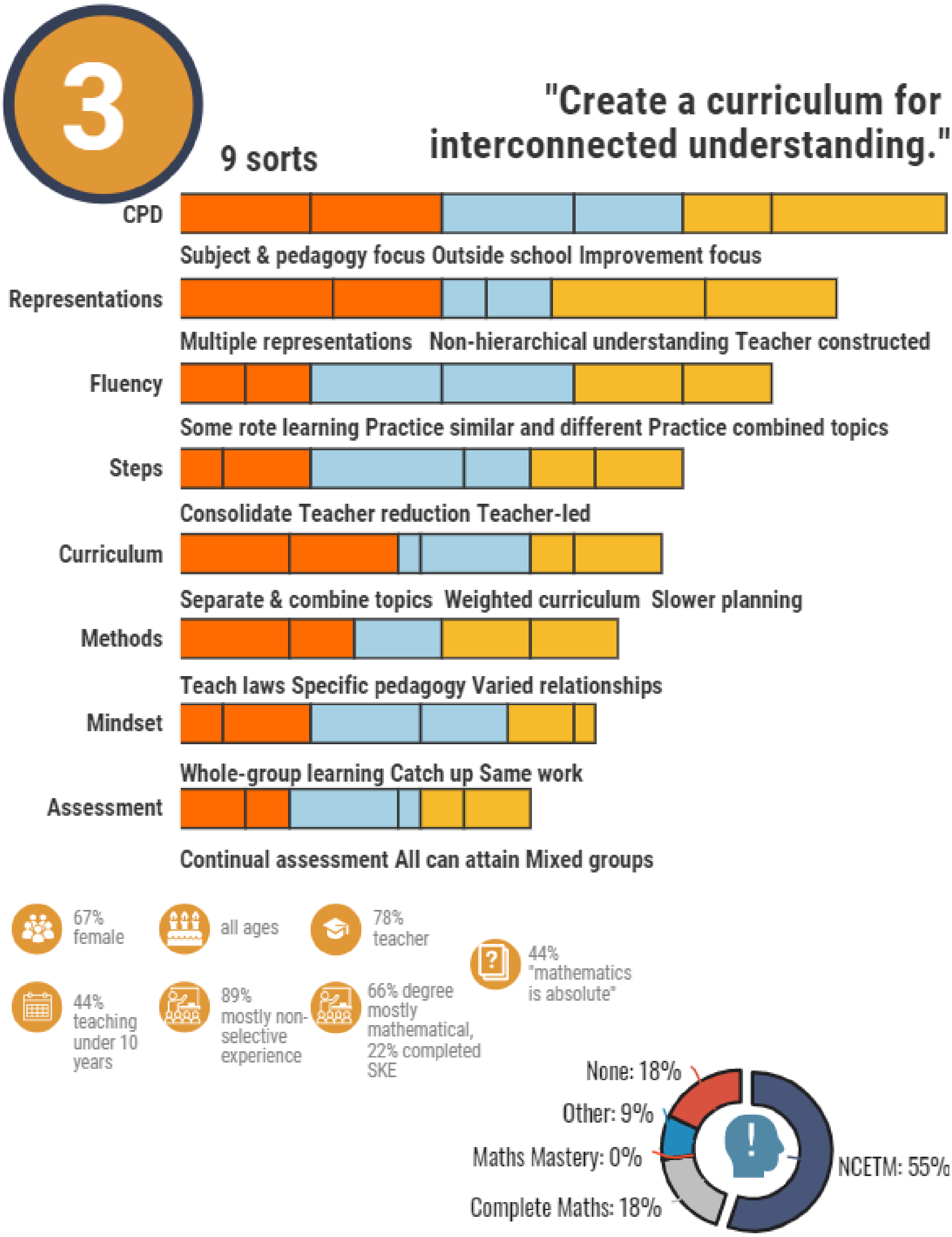
	Factor 1		Factor 2		Factor 3		Factor 4	
Statement No.	Z-score	Score	Z-score	Score	Z-score	Score	Z-score	Score
1	-0.011	0	-1.16	-2	-0.415	-1	0.974	2
2	-1.427	-2	-1.363	-2	-1.331	-2	-1.2	-2
3	1.987	4	-1.414	-3	0.612	1	0.644	1
4	-1.752	-3	0.168	1	-1.743	-3	-1.138	-2
5	-0.711	-1	2.597	4	-1.117	-2	0.478	1
6	-0.3	-1	-1.27	-2	-0.744	-1	-1.113	-2
7	-0.522	-1	0.592	1	-1.263	-2	0.339	0
8	0.867	2	-0.627	-1	0.085	0	-1.225	-2
9	0.395	1	-0.054	0	0.188	1	-0.662	-1
10	1.191	2	0.695	1	-0.081	0	-0.803	-1
11	0.023	0	-0.681	-1	-0.276	-1	-1.842	-4
12	-1.735	-3	-1.359	-2	-1.429	-3	-1.228	-3
13	0.757	1	0.811	2	0.59	1	1.471	3
14	0.52	1	-0.455	-1	0.726	1	1.513	3
15	-1.033	-2	-0.303	-1	-1.608	-3	0.408	1
16	-0.138	0	-0.198	0	0.435	1	-0.386	0
17	-1.086	-2	-0.823	-2	-0.965	-2	-0.325	0
18	0.393	1	-0.13	0	-0.101	0	-0.316	0
19	-0.336	-1	-0.031	0	0.818	1	-0.615	-1
20	0.278	0	0.228	1	-0.924	-1	0.333	0
21	-0.585	-1	-1.945	-4	-0.131	0	-0.549	-1
22	-0.584	-1	1.339	2	-2.047	-4	-0.55	-1
23	-0.062	0	0.845	2	0.183	0	-0.809	-2
24	0.288	0	-0.735	-1	-0.256	0	-0.712	-1
25	-1.219	-2	-1.504	-3	-0.938	-2	-0.912	-2
26	-1.403	-2	-0.811	-1	-0.085	0	-0.565	-1
27	-0.823	-1	0.116	0	1.594	3	0.519	1
28	0.005	0	0.459	1	-0.281	-1	0.973	2
29	1.487	3	-1.266	-2	-0.516	-1	-0.19	0
30	-1.611	-3	1.245	2	0.036	0	0.616	1
31	1.47	3	0.653	1	1.593	3	2.481	4
32	0.802	1	-0.338	-1	0.849	1	0.734	1
33	-0.402	-1	1.394	3	-1.17	-2	-1.679	-3
34	-0.717	-1	-0.45	-1	-0.464	-1	-0.594	-1
35	0.666	1	1.276	2	1.945	3	1.403	2

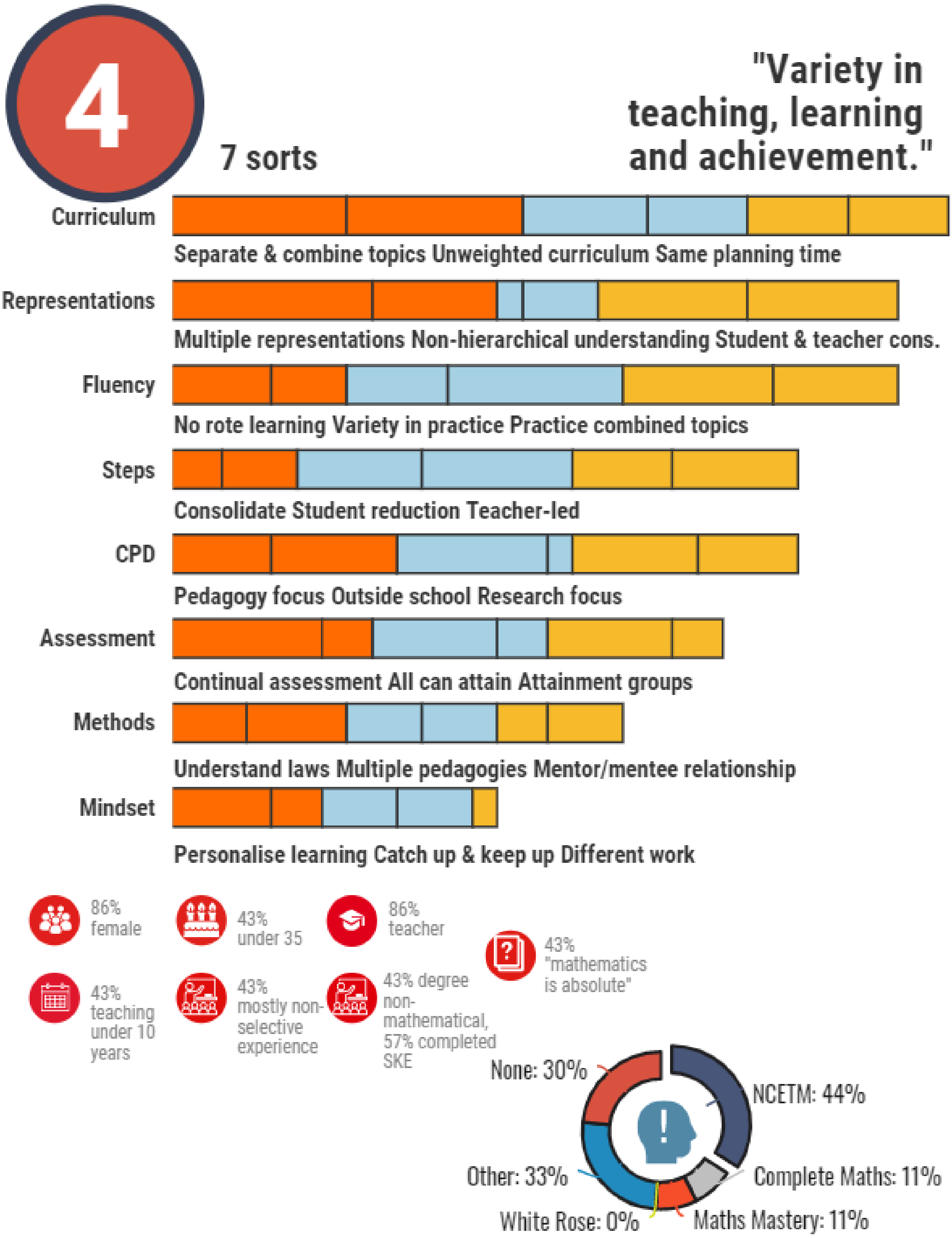
36	1.699	3	-0.295	0	1.043	2	1.227	2
37	1.364	2	-1.465	-3	-0.844	-1	-0.442	0
38	-2.083	-4	0.736	1	-0.586	-1	-0.594	-1
39	0.111	0	1.616	3	1.14	2	0.314	0
40	1.127	2	1.778	3	1.109	2	1.777	3
41	0.287	0	0.976	2	0.96	1	0.995	2
42	0.452	1	0.731	1	0.06	0	0.645	1
43	0.015	0	-0.343	-1	0.971	2	-0.102	0
44	1.232	2	0.702	1	1.305	2	0.5	1
45	1.008	2	0.115	0	1.056	2	1.358	2
46	-0.922	-2	0.083	0	0.215	1	-1.549	-3
47	0.317	1	0	0	-0.157	0	0.784	1
48	0.719	1	-0.134	0	1.956	4	-0.387	0

Appendix E Factor 1-4 'One-pagers'




2
4 sorts
"Know your limits, follow the teacher.
Fluency
Rote learning
Practice similar & different
Practice combined topics
Representations
Multiple representations
Hierarchical understanding
Teacher constructed
Methods
Understand laws
No specific pedagogy
Novice/expert relationship
CPD
Pedagogy focus
Within & outside school
Research & improvement
Mindset
Personalise learning
Keep up
Same work
Curriculum
Separate topics
Weighted curriculum
Slower planning
Steps
Stretch
Student reduction
Teacher-led
Assessment
Periodical & continual
Not all will attain
Attainment groups
100% male
50% in 30s,
100% teacher
75% "mathematics is a game"
75% teaching over 10 years
100% mostly non-selective experience
75% degree mostly mathematical, 0% completed SKE
NCETM: 25%
Complete Maths 25%
Maths Mastery: 0%
None: 25%
Other: 25%


3
9 sorts
"Create a curriculum for interconnected understanding."
CPD
Subject & pedagogy focus Outside school Improvement focus
Representations
Multiple representations Non-hierarchical understanding Teacher constructed
Fluency
Some rote learning Practice similar and different Practice combined topics
Steps
Consolidate Teacher reduction Teacher-led
Curriculum
Separate & combine topics Weighted curriculum Slower planning
Methods
Teach laws Specific pedagogy Varied relationships
Mindset
Whole-group learning Catch up Same work
Assessment
Continual assessment All can attain Mixed groups
67% female
all ages
78% teacher
44% "mathematics is absolute"
44% teaching under 10 years
89% mostly non-selective experience
66% degree mostly mathematical, 22% completed SKE
None: 18%
Other: 9%
Maths Mastery: 0%
Complete Maths: 18%
NCETM: 55%


4
7 sorts
"Variety in teaching, learning and achievement."
Curriculum
Separate & combine topics Unweighted curriculum Same planning time
Representations
Multiple representations Non-hierarchical understanding Student & teacher cons.
Fluency
No rote learning Variety in practice Practice combined topics
Steps
Consolidate Student reduction Teacher-led
CPD
Pedagogy focus Outside school Research focus
Assessment
Continual assessment All can attain Attainment groups
Methods
Understand laws Multiple pedagogies Mentor/mentee relationship
Mindset
Personalise learning Catch up & keep up Different work
86% female
43% under 35
86% teacher
43% "mathematics is absolute"
43% teaching under 10 years
43% mostly non-selective experience
43% degree non-mathematical, 57% completed SKE
None: 30%
NCETM: 44%
Other: 33%
Complete Maths: 11%
White Rose: 0%
Maths Mastery: 11%