Out-of-Field Teachers Knowing, Learning and Doing Mathematics

by

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Abstract

This thesis considers how out-of-field teachers of mathematics conceptualise knowing and learning mathematics and why they value doing it. Out-of-field teachers are those whose degree and professional qualifications are in subjects other than the one they are teaching. This thesis addresses the gap in the knowledge of how they conceptualise the subject they are teaching, paying particular attention to the complex work they undertake recontextualising knowledge as they cross boundaries. A two-dimensional framework was developed to analyse how out-of-field teachers conceptualise mathematics. The philosophical dimension considers how they know, learn and value doing mathematics, respectively their ontology, epistemology and axiology. Mathematics, defined as a discipline of patterns and connections used to model our complex world, is treated as heterogeneous, with four forms forming the second dimension (school, academic, everyday and pedagogical mathematics). The bricolage as methodology provides a critical complex perspective. Data was collected through semi-structured interviews, employing a range of tools, including images and mathematical objects. Data collection and analysis was designed to adopt an opportunity model, give the out-of-field teacher a voice and acknowledge the complexity of their work as they adapt flexibly to take advantage of their context.

Out-of-field teachers of mathematics participating in this study tended to conceptualise mathematics as school mathematics through the lens of learners. The boundaries where they recontextualised their knowledge were generally between school and pedagogical mathematics. Participants' work was both complex and critical, privileging students above mathematics and blurring their own experiences as learners of school mathematics with that of their students.

The conceptual framework developed for this thesis provides a tool for researchers seeking to understand conceptualisations of out-of-field teachers of any subject. The findings of this study have implications for out-of-field teachers and school leaders in recognising the complexity of the work of out-of-field teachers and the opportunities offered for and by them.

This thesis is dedicated to seventeen- and twenty-seven-year-old Fiona.

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Declaration

I declare that:

- The work presented in this thesis is my own and embodies the results of my research during my period of registration.
- I have read and followed the University's Academic Integrity Policy and that the thesis does not breach copyright or other intellectual property rights of a third party. Where necessary I have gained permission to reproduce copyright materials.
- Any material which has been previously presented and accepted for the award of an academic qualification at this University or elsewhere is clearly identified in the thesis.
- Where work is the product of collaboration the extent of the collaboration has been indicated.

Signature:  Date: 20/09/23

Abbreviations

A-Level	Advanced Level (UK academic school leaver qualification at age 18)
BA	Bachelor of Arts
BERA	British Education Research Association
BSc	Bachelor of Science
CSE	Certificate of Secondary Education (UK public examination at age 16 until 1988)
DfE	Department for Education (UK government department)
GCSE	General Certificate in Secondary Education (UK public examination at age 16)
KS	Key Stage
M_4T	Mathematics for teaching (Davis and Renert, 2014; Davis and Simmt, 2006)
MA	Master of Arts
MKT	Mathematical Knowledge for Teaching
NCETM	National Centre for Excellence in the Teaching of Mathematics
O-Level	UK public examination at age 16 until 1988
OOFTAS	Out-of-field teaching across subjects
PE	Physical Education
PGCE	Post Graduate Certificate in Education (UK teaching qualification)
RS	Religious Studies
UK	United Kingdom

1. Out-of-field teachers knowing, learning and doing mathematics: Introduction

This thesis considers teachers who teach mathematics whose degree and professional qualifications are in subjects other than the one they are teaching. It focuses on how they know and learn mathematics and why they value doing it. It considers what these out-of-field teachers of mathematics bring with them in terms of epistemological, ontological and axiological conceptualisations of mathematics. The out-of-field teacher is a bricoleur (Levi-Strauss, 1966), using the tools at hand to traverse boundaries between and within disciplines. Yet disciplinary rather than pedagogical knowledges are rarely considered in the teaching-out-of-field literature. In considering the philosophical basis of out-of-field teachers' conceptualisation of mathematics, this thesis addresses the gap in the knowledge of out-of-field teachers' experience of the subject they are teaching (Darby, 2009b, 2009a).

This introductory chapter begins by providing the rationale for the research focus: identifying the gap in knowledge that it addresses, and its relevance to me. It then introduces the conceptual and theoretical frameworks employed by this thesis, which will be explored in depth in chapters 2, 3 and 4 and form the structure for analysis in chapters 5, 6, 7 and 8 and the conclusion in chapter 9. The chapter closes with an outline of the structure and some notes on style.

1.1 Research Focus

The aim of this thesis is to explore the knowledges of out-of-field teachers of mathematics, employing a methodological approach that facilitates an opportunity model which seeks to hear the voices of out-of-field teachers and acknowledge the complexity of their work. It has a research focus, rather than the more traditional research question to meet these aims. The bricolage enables the researcher to explore a complex phenomenon, responding to the data as it is collected and analysed, sometimes moving in a different direction as a consequence. A research question could predetermine the focus, and so limit the flexibility to explore new ideas as they arise. A predetermined focus may not pre-empt opportunities and so they may go unnoticed. Similarly, a question may assume what participants will want to talk about rather than allowing the open exploration of their voice that is possible through using the bricoleur's data collection methods. The bricoleur uses the tools to hand to explore a complex phenomenon, and a research focus facilitates this flexibility.

My research focus, how out-of-field teachers of mathematics know, learn and do mathematics, developed as I explored the out-of-field literature and encountered lived-experience and boundary-crossing perspectives. These perspectives acknowledge the complex challenges faced by out-of-field teachers, focusing on their professional, pedagogical experiences and knowledges. Earlier reflexive work (Yardley, 2022) explored the complexity of how individuals might conceptualise the discipline of mathematics, but that complexity was not reflected in the literature. The focus of this thesis was directly influenced by my professional journey from out-of-field to in-field mathematics teacher and my personal relationship with mathematics. Here I provide the rationale for my research focus by identifying the gap in knowledge (1.1.1) and my reflexive positionality (1.1.2).

1.1.1 The Gap in Out-of-field Knowledge

The term out-of-field is used in this thesis in preference to the term 'non-specialist' commonly used in the UK (Price *et al.*, 2019). 'Non-specialist' focuses on what a teacher is lacking. It is a deficit model (Dudley-Marling, 2015). To describe someone as teaching-out-of-field is to adopt an opportunity model (Hobbs and Törner, 2019a). Whichever term it uses, the out-of-field literature often adopts a deficit model for what is commonly defined as teachers 'assigned to teach subjects for which they have little training or education' (Ingersoll, 1999, p. 26). Section 3.1.2 problematises how in the out-of-field literature a deficit model is often adopted, out-of-field teachers' voices are not always heard, and the complexity of the phenomena (how out-of-field teachers respond flexibly to their context) is not always recognised. Two recent approaches in out-of-field research adopt an opportunity model, acknowledge the complexity of teaching-out-of-field and give the out-of-field teacher a voice by recognising their agency, the capacity an individual has to make choices and act upon them (Lu, Leung and Li, 2021). These are the lived-experience and boundary-crossing literature.

In the lived-experience literature we hear the voice of out-of-field teachers. In a remarkable study spanning two continents, du Plessis (2013) used a Gadamerian phenomenological theoretical framework to capture the complexity of the lived experience of people impacted by out-of-field teaching. She takes a holistic approach, listening to the complex, intwined voices of in- and out-of-field teachers, school and local authority leaders and parents. More recently Hobbs and Quinn's (2020) three-year longitudinal qualitative case study examined the experience of teaching in out-of-field contexts and gathered data from teachers, their mentors and school leaders. In contrast, this thesis magnifies the detail within the big picture, employing a close-up microphone, to foreground out-of-field teachers' voices and amplify the detail.

This study is not the first to amplify out-of-field teachers' voices. For instance, Mea *et al*. (2019) interviewed and observed forty out-of-field social sciences teachers in the Philippines, and Bugwark (2021) conducted in-depth qualitative interviews and focus group discussions with ten out-of-field tertiary teachers in the same country. Their findings include issues in preparing and administering lessons (Mea *et al.*, 2019), and difficulties in addressing students' queries, establishing authority and employing appropriate teaching approaches (Bugwark, 2021). In these examples, lived experience is conceptualised within a pedagogical context. These examples amplify out-of-field teachers' voices about their teaching rather than about the subject they are teaching out-of-field itself.

It is understandable that studies about *teaching*-out-of-field focus on teaching. However, this focus can mean that other types of knowledge are overlooked, such as teachers' beliefs about the nature of the subject and their cumulative experiences of learning it, which influence how they reconceptualise and reconcile their various beliefs in the classroom (Beswick, 2012). Focussing on the views of mathematics that underpin teaching, Beswick concludes, is 'a crucial and largely missing element in current professional learning efforts' (*ibid.* p. 146). White-Fredette (2009) agrees that 'research is needed that focuses on … how teachers view the mathematics that they teach' (p. 28). These views from the general mathematics education literature align with the call by Hobbs (née Darby) for further research that develops 'rich descriptions of those knowledge, skills and attitudes that teachers bring into their out-of-field teaching from their in-field subjects' (Darby, 2009a, p. 223). To listen to the voices of out-of-field teachers it is insufficient to hear only what they say about teaching. Hobbs argues that it is also necessary to explore their understanding of their in- and out-of-field subjects and how they recontextualise their knowledge when crossing these boundaries (Hobbs, 2013a, 2013b).

Teaching out-of-field exists because there are boundaries between school subjects (Hobbs, 2013a). In their literature review of boundary crossing, widely cited in the out-of-field boundary-crossing literature, Akkerman and Bakker (2011) define a boundary as 'a socio-cultural difference leading to discontinuity in action or interaction' and boundary crossing as 'a person's transitions and interactions across different sites' (p. 133). They cite Bernstein (1973) as an early example of a scholar using the term boundary. Although now several decades old, two of Bernstein's concepts relating to boundaries are used in this thesis: recontextualisation and classification of knowledge. Section 3.2.1 critically examines these two theories and justifies their use. Here they are introduced descriptively.

Recontextualisation is where the text of one field (the primary field) is relocated into another field (the secondary field) through a process of decontextualisation, transformation and repositioning (Bernstein, 1990). Bernstein's (1973) attempts to understand the classification of knowledge give rise to the concept of collection codes. He argues that the classification of subjects is informed not by the knowledge being classified, but the relationships between the content, and so it is the boundaries that give subjects their distinctiveness, and that these boundaries are created by power relationships (Bernstein, 2000). The phrase 'collection code' refers to the strength of boundaries between subjects: an integrated curriculum would have a weak collection code. Writing as the national curriculum was being introduced to England, Bernstein (2000) identified it as a strong collection code with clear demarcation between subjects. This remains an accurate representation of the secondary curriculum in England. Out-of-field teachers in England cross strong boundaries, and so undertake challenging work in the recontextualisation field.

Research into the boundary-crossing work of out-of-field teachers includes Bosse and Törner's (2013) exploration of pedagogical beliefs about teaching mathematics, Goos' (2015) study of the professional identity of out-of-field teachers, and Vale *et al*.'s (2021) opportunity model study of pedagogical epistemology. As with the lived-experience literature, the boundary-crossing literature tends to focus on professional and pedagogical aspects. Bernstein's (1990) original conception of recontextualisation referred to pre-service in-field teachers, and the transformation of disciplinary into pedagogical knowledge (section 2.2). The transformation of knowledge into pedagogical knowledge is important but is only one aspect of the complex work that an out-of-field teacher must undertake in a strong collection code context. I found little literature that took up Darby's (2009b, 2009a) call to research the knowledge and beliefs teachers bring with them, and none that focussed solely on conceptualisations of teachers' out-of-field disciplines.

This thesis seeks to address this gap in the knowledge by adopting an opportunity perspective to consider what knowledge and beliefs about mathematics out-of-field teachers bring with them. It is important to take my own knowledge and beliefs about mathematics into account. How they affect the design and conduct of this study is explored further in chapter 4. Here I consider how my positionality has motivated this study.

1.1.2 Reflexive Positionality

I share here my understanding of why I am working in this field and attempt reflexively to illuminate the ideological imperatives and epistemological presuppositions that inform my research (Telles, 2000; Kincheloe *et al.*, 2017). I present a truthful fiction, which Denzin (2013) characterises as being a narrative faithful to the facts and facticities. Facts are events that occurred and facticities are how these facts were lived and experienced. I begin with a paragraph of facts.

I was born in 1974 in Peterborough, UK, and lived there throughout my childhood. I attended state primary and secondary schools, gaining top grades in ten GCSEs at age sixteen, and the top grade in A-level mathematics, history and German at age eighteen. After school I went to Oxford University, achieving an upper second-class bachelor of the arts (BA) degree in history and a Post Graduate Certificate in Education (PGCE) in history. I taught history and geography in outer London for three years and took further A-levels in politics and geography. In my second job, in inner London, I taught mathematics and history and studied for a masters in curriculum studies at University College London. After two years I was appointed head of mathematics and have since taught only mathematics. After the birth of my second child, I commenced work as a local authority consultant before being self-employed for five years. Contracts included consultancy in diverse subjects and teaching mathematics initial teacher education in several universities. I also studied mathematics with the Open University, gaining an upper second-class Bachelor of Science (BSc) degree in 2014. By this time, I was living back in Peterborough and working as a specialist leader in mathematics in a secondary school. In 2017 I took up a university post as senior lecturer in mathematics education.

In discussing how these facts were lived, I will focus on my experiences as a knower, learner and doer of mathematics. Mathematics has always given me a sense of awe, such as the fascination I had in my teens for different sizes of infinity (Krátká, Eisenmann and Cihlář, 2021) and this early memory:

> *One morning in infants the teacher told us to write down our counting numbers starting at 1 and seeing how high we could go. By playtime I was in the 90s and both anxious and excited – what would happen after 99? I asked Mrs Warriner, a dinner lady. She explained what happened next in a way that made me realise that I could now carry on counting forever. I can still remember how mind-blowing and exciting this realisation was.* (Yardley, 2022, p. 162)

Mathematics was my favourite subject going into A-levels, but the instrumental (Skemp, 1978) pedagogy employed by my mathematics teachers contrasted with the dialogic challenge (Alexander, 2017) of history lessons. Mathematics became a process of 'put that number in there, get an A-level out the other end.' It appeared to be about universal truths, whereas I wanted to use my power to question, challenge and disrupt as I could in history. This is why I studied history for my first degree.

Another frustration with A-level mathematics was how my teachers would express surprise if a student was struggling, saying 'but it's easy': it felt that they considered some people to be naturally good at maths and others not. My strongest memories as a learner of mathematics centre on exploration of concepts, and usually involve other people, although an early memory doesn't. It wasn't until I studied number theory in my thirties that I realised I had a mathematical pastime on car journeys as a child (**Error! Reference source not found.**). As a teacher of mathematics, I

Car hand game

How many times can I count to 3 before I get back to the starting finger?

Interesting, it takes me 3 runs through both of my hands. Let's try 5.

Five is boring. I wonder what happens with 7.

Wow, 7 is brilliant. It took me 7 pairs of hands, and 3 took me 3 pairs. But 5 only took me 1. Why?

Figure 1.1 My childhood car hand game

remember ferocious debates about whether there are infinite base Pythagorean triples. With a year 8 class I once spent several weeks of starter activities calculating how deep the Thames would be if all the water was removed and replaced with the blood of every human. When doing mathematics, I have always been more interested in *why* than *how*. This is exemplified in the reasons that led me to study for a degree in mathematics later in life. One was a job interview in which the interviewer expressed confusion about my identity, saying 'will the real Fiona Yardley please stand up?' (Yardley, 2022, p. 154). Another was hearing a fellow delegate at a conference query the meaning of a symbol which I had not had the confidence to admit I did not recognise. Finally, I read that only seventeen wallpaper patterns exist, but could not understand the proof. My desire to understand this mind-blowing information sustained me through years of studying. I discovered that my passion was for pure and decision mathematics. The dislike for mechanics which began at school continued throughout my degree. We were not given a choice at A-level, being told that we must study mechanics as it was useful for A-level physics, even though more of us were studying humanities and so statistics would have been more useful. Studying mathematics because I wanted to rather than needed to, allowed me to indulge in the abstract and to revel in its aesthetic value rather than its utility (Hardy, 1967; Sinclair, 2004).

Teaching mathematics out-of-field was a positive opportunity for me. I do not remember feeling alienation or dislocation. I embedded myself in the mathematics education community, attending conferences and reading popular mathematics books, especially the works of Eastaway and Singh (Eastaway, 2008; Eastaway and Wyndham, 2003, 2005; Singh, 1997, 1999, 2013). As head of mathematics, I was keen to integrate mathematics across the curriculum and celebrated the diverse pedagogies and expertise that out-of-field teachers brought to my thriving department. With a degree in the subject and two decades of teaching mathematics and training mathematics teachers I am no longer an out-of-field teacher.

Four aspects of the facticities explored in this section particularly influence my positionality:

- Mathematics to me is creative, social, beautiful, surprising and messy.
- I experienced alienation by instrumental teaching that focused on procedure and expert teachers who did not empathise with struggling students.
- I get excited by mathematics when exploring it for its own sake, not for examinations or functional use.
- My experiences as an out-of-field teacher and leader and trainer of out-of-field teachers have been overwhelmingly positive.

This highly personal account reflexively positions me with respect to teaching out-of-field (Day, 2004). It also affects the design of this study, including the collection, analysis and interpretation of data, and will be explored further in chapter 4.

1.2 Theoretical and Conceptual Frameworks

The focus of this thesis, how out-of-field teachers know, learn and value doing mathematics, is a complex phenomenon (Hobbs *et al.,* 2019). Therefore, in this thesis I employ a critical complex theoretical framework, defined and justified in section 1.2.1. Two conceptual frameworks are employed in this thesis and will be introduced in later chapters. A conceptual framework of mathematics which structures this thesis and informs analysis and interpretation of data is introduced in chapter 2, and to conceptualise out-of-field teaching I use the framework developed by Hobbs *et al*. (2020), which is introduced in section 3.1.1.

1.2.1 Critical Complex Theoretical Framework

Critical research takes the theoretical perspective that society is unequal and seeks to reveal, understand, and challenge the imbalance of power. Complex research takes the theoretical perspective that interprets phenomena and their interrelationships in context, which is understood to be dynamic and multi-dimensional. This thesis synthesises these research approaches through the lens of Kincheloe's (2004, 2017) critical complex philosophy. This theoretical framework is critical complex, not complex critical, focusing on the complex phenomena of teaching-out-of-field through a critical lens rather than vice-versa.

I adopt a critical perspective derived from the Frankfurt School's assumptions that society is unequal, and that unequal society is made to appear neutral through the application of rational science (Horkheimer, 1972). Mathematics is often represented as the purest example of scientific rationality (Kincheloe, 2008) by those who hold an absolutist philosophy of mathematics (Ernest, 1991), and so made to appear neutral. Kincheloe uses the term Western to refer to European and Anglophone predominantly White global minority cultures (Kincheloe, 2005), and argues that Western colonial dominance has meant universalism, a single way of seeing the world (Kincheloe, 2008). Even in the context of English governmental policy to integrate Asian pedagogy into mathematics (DfE, 2010), two-way policy borrowing and transfer has maintained a focus on a defined canon of mathematical knowledge with distinct subject boundaries (Forestier and Crossley, 2015). Elites establish what constitutes official mathematical knowledge, using economic and business needs to justify the curriculum (Apple, 1982; Davis, 2014), even though assessment of mathematical knowledge in twenty-first century England celebrates abstract hyper-rationalism and not functional application (Jorgensen *et al*., 2014; Ernest, 2020). Out-of-field teachers are amongst those marginalised by mathematics as they are perceived to be non-specialists (Alderton, 2020). By taking a critical perspective, I seek to reveal and understand such imbalances of power, and to challenge them.

Colloquially the term complex is often erroneously used synonymously with complicated (Davis and Sengupta, 2020). A system is a phenomenon that involves the interaction of many sub-components (Davis and Sumara, 2006), and can be described as simple, complicated or complex. It is possible to delineate the separate elements of a complicated system, although it may be difficult and not worth the effort. This is not possible for complex systems because of the symbiotic nature of their elements. The phenomenon of out-of-field teaching is complex (Hobbs *et al*., 2019; Hobbs and

Quinn, 2021). A complex perspective on out-of-field teaching acknowledges that strands of and influences on knowledge cannot be isolated because their existence and nature is dependent on their context.

Kincheloe brings criticality and complexity together in his critical complex philosophy, which forms the theoretical framework for this thesis (Berry and Kincheloe, 2004; Kincheloe, 2004, 2017). A critical complex theoretical framework seeks to interpret complex phenomena through a critical lens. In the context of this study, it will allow access to otherwise hidden aspects of the knowledges of out-of-field teachers of mathematics (Davis, 2008).

1.3 About this thesis

This introductory chapter ends with some brief notes about stylistic choices and the structure of this thesis.

1.3.1 Style and meaning

The meaning of words is contextual and dynamic (Kincheloe, 2008): 'the universe changes when a thought changes' (Davis and Sumara, 2006, p. 4). In appendix A I provide a glossary to share how I understand terms in this thesis. Its purpose is not to set meanings in stone or require a shared conceptualisation. It is to share my current, contextual understanding. Terms are subject to myriad interpretations which will interact deliciously with mine, sometimes in harmony, sometimes jarring, and sometimes working symbiotically with mine to generate new meaning.

In this thesis I consciously use the terms field, discipline and subject synonymously. Researchers often equate discipline and subject (Doig *et al.*, 2019), rather than identifying the former as referring to an academic study with distinct epistemology and ontology, and a subject as what is taught in schools relating to that discipline (Hobbs, 2012). Field serves as a general term that could refer to either, both, or a subset of these. I use these terms synonymously to avoid connotations of discipline equating with the concept of academic mathematics and subject with school mathematics and to retain the open, heterogeneous conceptualisation of mathematics knowledge introduced in the previous section (1.2.2). This choice is also stylistic.

I adopt a standard academic structure. It is my goal for this thesis to be accessible to academics outside the fields of mathematics education and teaching out-of-field and to a lay audience, while retaining academic rigour. To this end I have endeavoured to make it readable by adopting a simple structure and making stylistic choices to aid fluency (see Appendix B). The structure is mathematical: it uses patterns to simplify access to a complex argument.

1.3.2 Structure

The conceptual framework introduced in section 1.2.2 is developed in chapter 2 through a review of literature exploring conceptualisations of mathematics. Chapter 3 problematises teaching-out-of-field. Chapters 2 and 3 are both in two sections. The first section reviews literature relevant to conceptualisations of mathematics and teaching-out-of-field respectively. The second section explores the knowledge from the first section using the theoretical framework of boundaries and recontextualisation of knowledge. Chapter 4 develops a methodological approach to access out-of-field teachers of mathematics' conceptualisations of the subject.

Out-of-field teachers' conceptualisations of forms of mathematics is the focus of chapter 5, and the philosophical perspectives of ontology, epistemology and axiology structure chapters 6, 7 and 8 respectively. Each chapter begins by introducing relevant data before discussing it. Chapter 6 focuses on knowing mathematics (ontology), exploring how out-of-field teachers conceptualise the nature of mathematics, using Ernest's (1991) categorisation of mathematics as absolutist or fallibilist. Chapter 7 is about learning mathematics (epistemology), considering how out-of-field teachers conceptualise how we come to know mathematics. Key foci in this chapter are notions of ability and the role of emotions in learning mathematics. Chapter 8 considers doing mathematics (axiology), exploring how out-of-field teachers conceptualise purposes for doing mathematics. The main themes in this chapter are perceptions of the utility of mathematics, the role of examinations and mathematics for personal growth. Chapter 9 draws the thesis together to offer conclusions, contributing new knowledge from the evidence of this study of the heterogeneous and complex ways that out-of-field teachers of mathematics know, learn and do mathematics.

1.4 Conclusion

This chapter has introduced the frameworks and theories on which this thesis is constructed. The critical complex theoretical framework informs the methodology and interpretation of literature and data. Two conceptual frameworks were introduced and are explored in more depth in subsequent chapters. A framework to support the conceptualisation of mathematics was introduced and is constructed using mathematics and mathematics education literature in the next chapter (section 2.1). Teaching-out-of-field is conceptualised using the framework developed by Hobbs *et al*. (2020, section 3.1.1). Two theories relating to boundary crossing were also introduced, Bernstein's recontextualisation and classification of knowledge (1973, 1990).

This chapter has also explained the focus of this thesis and identified the gap in knowledge that it addresses: how out-of-field teachers conceptualise the subject they are teaching, rather than its pedagogy. My personal motivation is derived from my conceptualisation of the subject as creative and collaborative and positive experiences as an out-of-field teacher of mathematics, which can be in tension with how others conceptualise mathematics and teaching out-of-field. This directly informs the content of this thesis, and the opportunity model approach which aims to give voice to those undertaking the complex work of teaching mathematics out-of-field and acknowledge their flexible response to context. This thesis considers the recontextualisation work that out-of-field teachers do within mathematics, understanding it to be a complex, heterogenous subject and exploring how they know, learn and value doing mathematics.

2. A framework for conceptualising mathematics

Someone’s conceptualisation of mathematics is what they believe mathematics to be and how they understand it as a subject (Andrews and Hatch, 1999). This chapter begins by introducing a mathematics conceptual framework, which is then populated using mathematics and mathematics education literature in section 2.2. Section 2.3 considers the boundaries between forms of mathematics (Bernstein, 1973), where power operates (Bernstein, 2000). This enables a critical complex exploration of mathematics and the boundary zones where out-of-field teachers undertake recontextualisation work (Hobbs, 2013b).

2.1 Mathematics Conceptual Framework

Mathematics is rarely defined in the literature; instead culturally based assumptions construct situated meanings (Brown and McNamara, 2011). It has no precise, universal definition (Ernest, 2015). Drawing on Lockhart’s (2009) understanding of mathematics as a discipline that creatively seeks patterns and to simplify the complex, in this thesis I understand mathematics to be a discipline of patterns and connections used to model a complex world. Modelling is a fundamental part of mathematics and means to simplify or idealise by focusing on relevant features and disregarding others. Modelling is also useful in research, and has been used to develop a framework for conceptualising mathematics for this study.

The framework has two dimensions: philosophical perspectives and forms of mathematics. Philosophical perspectives are used because disciplines are differentiated from one another in the western tradition by their ontological, epistemological and axiological foundations (Hobbs, 2013a). Using Mertens’ (2007) philosophical transformative paradigm, I define ontology as the nature of the phenomena being investigated, epistemology as how people acquire and communicate knowledge and axiology as the ethical value attributed to knowledge. Knowledge is defined here as our response to things that brings forth new worlds (Osberg *et al.*, 2008). Mertens’ transformative paradigm is appropriate as knowledge is the phenomenon being studied here, and so distinctions are blurred. As human constructs created to aid understanding of a complex world, the meanings of these terms flex as they are recontextualised. In the context of teaching mathematics out-of-field, I am taking ontology to be the teacher's beliefs about the nature of mathematics. The epistemology of out-of-field mathematics teachers is taken to be their understanding of how people come to know mathematics, and their axiology as their conceptualisation of reasons for doing mathematics (Figure 2.1)**Error! Reference source not found.**.

Mathematics is usually wrongly used as a homogenous term (Brown and McNamara, 2011; Burton, 2001; Dörfler, 2003), relatively fixed and pre-given (Davis and Sumara, 2006). The forms of mathematics that comprise the rows of the conceptual framework make no claims to be comprehensive and are inevitably reductionist, suggesting boundaries where there is blurring and overlap. The forms of mathematics used in this thesis have been iteratively constructed from the literature (section 2.2), personal experience (including section 1.1.2 and Yardley, 2022) and data collected and analysed for this study. The purpose of employing this taxonomy is to allow a recognition of the complexity of conceptualisations of mathematics and to facilitate the concept of boundaries and zones for the recontextualisation of knowledge. The taxonomy's purpose is not to assume a universal understanding.

In the context of such complexity, the naming and conceptualisation of the forms of mathematics is challenging. The extensive research into the mathematics associated with compulsory education has been termed Mathematical Knowledge for Teaching (MKT) (Ball *et al*., 2008) or Mathematics for teaching (M_4T) (Davis and Renert, 2014; Davis and Simmt, 2006). Both differentiate between the mathematical knowledge that constitutes the content of the curriculum as well as the knowledge needed to help students learn the content. However, both are limited to the parameters of the mathematics taught in primary and secondary schools, viewing knowledge from a teachers' perspective. Golding's (2017) phrase 'school mathematics' refers specifically to the body of mathematical knowledge that makes up the primary and secondary curriculum, a body of mathematical knowledge structured to be taught in these settings (Davis and Renert, 2014). This phrase is adopted to refer to the content of the school curriculum. The knowledge required for teaching mathematics, such as MKT and M_4T (*Ops. Cit.*), is referred to here as pedagogical mathematics, borrowing from Shulman (1986).

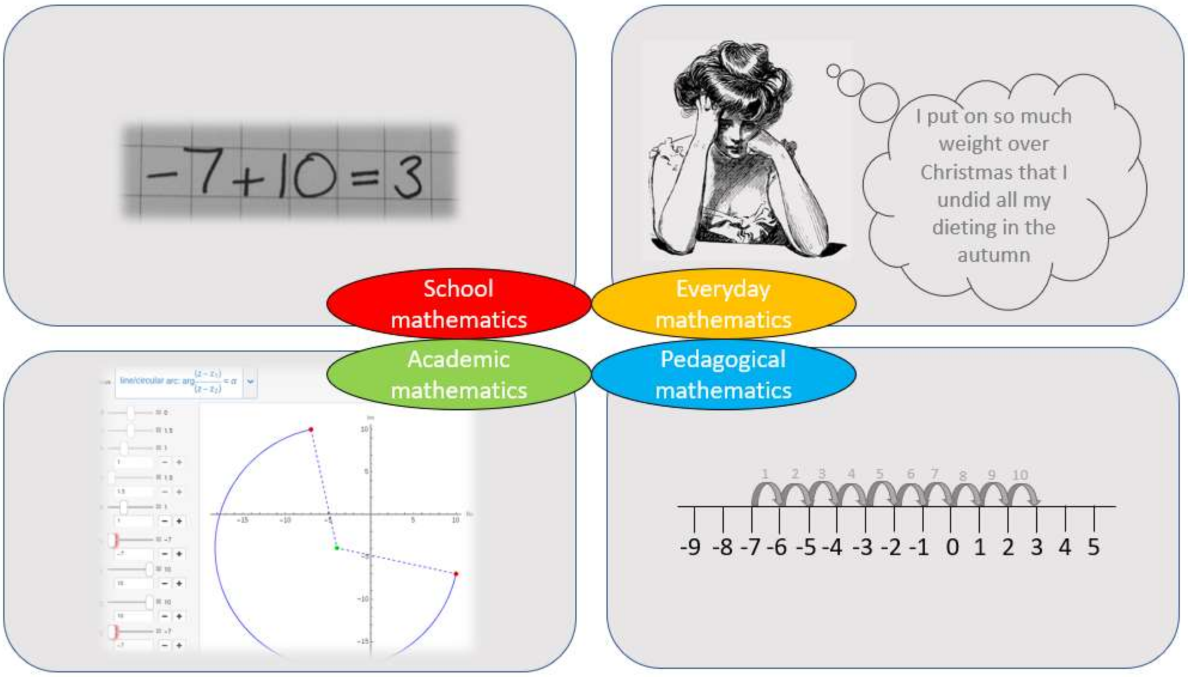
Academic and mathematics everyday were more problematic to name. An alternative to 'academic mathematics' is Crisan's 'advanced mathematics' (2022), derived from the concept of Advanced Mathematics Knowledge, defined as 'knowledge of the subject matter acquired in mathematics courses taken as part of a degree from a university or college' (Zazkis and Leikin, 2010, p. 264). This differentiates a body of knowledge from school and pedagogical mathematics, but it is still mathematics structured to be taught or learnt. The term 'academic mathematics' pushes beyond the content of university curricula to include knowledge explored and created by research mathematicians, referred to by Watson as 'the activities that advance mathematical knowledge' (2008, p.3). The final category refers to mathematics that takes place outside of mathematics education at primary, secondary or higher educational institutions. Possible terms include ethnomathematics (Marchand, 2018; Vidal Alangui, 2019), folk mathematics (Bruner, 1996; Claxton, 2021; Walkerdine, 1988), metamathematics (Lakatos, 1976), cultural mathematics (Davis and Renert, 2014), common sense (Keogh *et al*., 2018), indigenous mathematics (Bernales and Powell, 2018), or sort-of-right mathematics (Armstrong, 2017). Each of these terms carries implications which limit the reach of the term so that it does not encompass the full range of non-educational and non-research mathematics, mathematical knowledge informally or contextually structured as it is used (Davis and Renert, 2014). The term everyday was chosen as it goes further towards the generality required for this form of mathematics.

Difficult to define, the boundaries between these four forms of mathematics overlap and blur complexly. The definitions in Table 2.1 draw out key ideas from the discussion above. Figure 2.2a provides an alternative approach to conceptualising the four forms by diagrammatically suggesting how each may represent the same idea relating to negative numbers, while Figure 2.2b takes the example of two types of numbers and suggests how users of the different forms of mathematics may interact with them.

Table 2.1 Definitions of forms of mathematics

Everyday mathematics	Mathematical knowledge informally or contextually structured as it is used (Davis and Renert 2014)
School mathematics	The mathematics learnt at school (Golding, 2017)
Academic mathematics	The activities that advance mathematical knowledge (Watson, 2008)
Pedagogical mathematics	Mathematical knowledge for teaching (Ball *et al.*, 2008)

Figure 2.2a Diagrammatic representation of how each of the forms of mathematics used in this thesis may approach the same idea relating to negative numbers



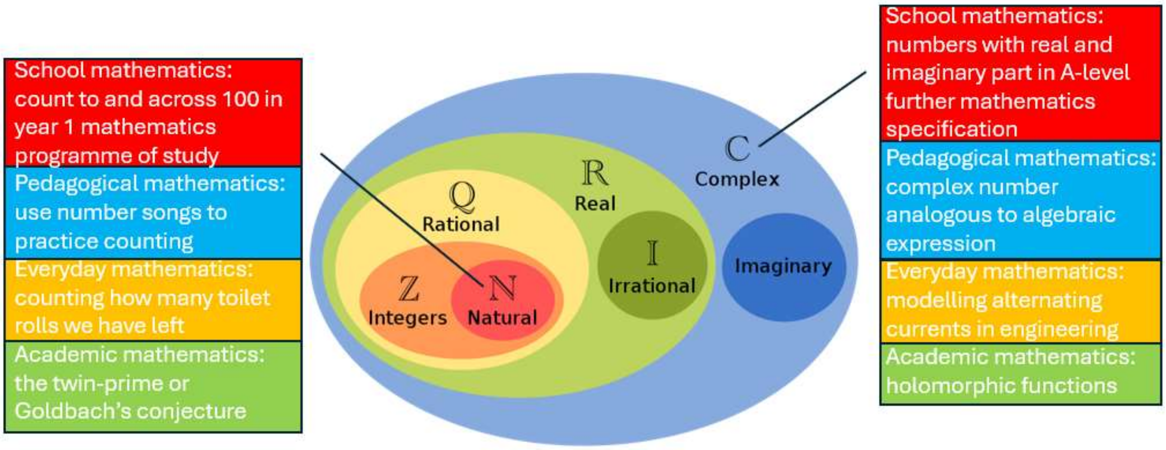


Figure 2.2b Suggestion of how the forms of mathematics may interact with different types of number

The philosophical dimension and mathematical forms of the conceptual framework are brought together in a two-way table (Table 2.2). Section 2.2 uses mathematics and mathematics education literature to populate the table.

Table 2.2 Mathematics Conceptual Framework

		Knowing (ontology)	Learning (epistemology)	Doing (axiology)
		Teachers' beliefs about the nature of mathematics	*Understanding of how we come to know mathematics*	*Conceptualisations of purposes for doing mathematics*
School mathematics	*The mathematics learnt at school* (Golding, 2017)			
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)			
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)			
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)			

2.2 Knowing, learning and doing mathematics

The philosophical dimension of the mathematics conceptual framework provides the structure of this section. Each subsection begins with a general exploration before considering how this is perceived through the lenses of the four forms of mathematics.

2.2.1 Knowing mathematics

The ontological question that has concerned people for millennia is whether mathematics is discovered or created, whether it exists independently of the human mind. Ernest (1991) differentiates between absolutist (existing independently of the human mind) and fallibilist (an evolving human construct) mathematics in his seminal 'Philosophy of Mathematics Education'.

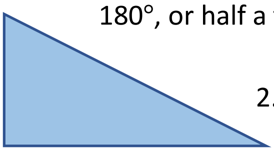
Absolutist ontologies of mathematics, which can be traced back to Ancient Greece, hold that mathematics is a universal truth, fixed and waiting to be discovered (Gordon, 2019). The Stanford Encyclopaedia of Philosophy (Horsten, 2022) suggests that mathematics can be Platonist or non-Platonist. Plato's theory of Forms (Plato, 2007) establishes the ontological basis of absolutist philosophies of mathematics: if there were no humans, mathematics would still exist (Bloor, 1973; Hardy, 1967; Peck, 2018). In Platonism mathematics has logical completeness and purity. This historical absolutist ontology of mathematics has been shown to have had a significant impact on contemporary conceptualisations (Gordon, 2019).

Fallibilists, such as Shapiro (2000) and Ernest (1991), argue unequivocally that mathematics is a human construct which is evolving and culturally and historically situated. Even the quote attributed to Kronecker and borrowed by Hawking (2006), 'God made the integers; all else is the work of man' is inaccurate: integers are a human invention. The Babylonians had a place value system, but the only way to distinguish between, for example 24, 204 and 240 was the context (Bloor, 1973), and the Romans had no concept of zero (de Cruz, Neth and Schlimm, 2010). Anthropologists have identified societies with different counting systems based on the properties of objects (Valério and Ferrara, 2022) whereas the Western counting system has the abstraction principle, that any set is countable even if it contains different items (Gelman and Gallistel, 1978). Mathematics is constructed through human interaction with the natural world and with each other (Bloor, 1973).

My definition of mathematics includes the phrase *used to model a complex world*. Euclid's geometry is a good example of using mathematics to model – to simplify – a complex world (Andrade-Molina and Valero, 2015). Euclidean geometry is based on an idealised two-dimensional world. Figure 2.3 shows two ways in which it fails to describe the world (Lerman, 1983). Additionally, both examples include measures (degrees and kilometres): human constructs that make it simpler to interact with the world. Mathematics is a human construct, a social product with meaning located in practice (Lerman, 2000).

Activity

1. Draw a triangle and measure its angles. Euclidean geometry tells us that these will sum to 180°, or half a full turn.



2. Now do the same but on an inflated balloon.

Figure 2. 3 Two examples of how Euclidean geometry does not describe the world

Ontologically we can understand mathematics as discovered (absolutist) or constructed (fallibilist). Ernest (1991) uses absolutism and fallibilism as a binary when defining five educational ideologies which combine school and pedagogical mathematics. Ernest defines four of the five ideologies as absolutist and only public educators as fallibilist. For example, progressive educators view 'mathematical truth as absolute and certain' (1991, p. 182) even though they see mathematics as creative, human and connected. Ernest's use of absolutism and fallibilism as binaries is reductionist. It oversimplifies the complex conceptualisations of mathematics that this thesis explores.

Ontologies can be a complex mix of absolutist and fallibilist, where socio-mathematical norms conflict with social norms (Stephan, 2020). This complexity is increased as the mathematical products of human activity are reified and built into a canon of traditions labelled as mathematics school curricula, and so fallibilist mathematics can be mistaken as absolutist (Burton, 2004; Lundin, 2012). Combined with pressures on schools to deliver examination results (Ball, 2003) mathematics teachers can experience conflict between an absolutist transmission model of mathematics in schools and a fallibilist connectionist understanding from their training (Bamber, 2015). In many mathematics classrooms, there is a complex relationship between procedural knowledge of mathematics (knowledge of the language, symbols, rules and algorithms for finding solutions) often privileged above conceptual knowledge of mathematics (a rich connected web of relational knowledge) (Hiebert, 2013).

A similar complexity of ontologies is found in academic mathematics. While compelling arguments have been made for the fallibilist nature of academic mathematics (Shapiro, 2000), noting its complexity (Dreyfus, 2014), creativity (Halmos, 1968) and messiness (Golding, 2017, 2018), Burton's empirical finding is that professional mathematicians hold absolutist conceptualisations (Burton, 2001). The literature suggests that academic, school and pedagogical mathematics are fallibilist, human constructs conceptualised widely as absolutist with a complex mix of fallibilism.

Everyday mathematics is defined in this thesis as *mathematical knowledge informally structured as it is used* after Davis and Renert (2014). The definition does not give a context – this form of mathematics is a catch-all for any mathematical activity that does not sit in school, academic or pedagogical mathematics. This makes it a broad category which could include domestic (DIY, personal finances), leisure (music, sport, puzzles), and professional contexts (engineering, accountancy). What these contexts have in common is that in each one mathematics is a tool, a means to an end rather than the focus of the activity itself. The user of everyday mathematics is a bricoleur, using the mathematical tools to hand to carry out a task (Levi-Strauss, 1966). Ontologically, mathematics in this context could be understood as created by the user to meet their needs, or a naturally existing tool which they selected – it could be fallibilist or absolutist. As with the other three forms of mathematics, everyday mathematics is understood here to be fallibilist in nature but with people's conceptualisations of it being a complex mix of dominant absolutism and elements of fallibilism.

2.2.2 Learning mathematics

Starting from the conceptualisation of mathematics as a complex mix of absolutism and fallibilism, this section considers what this means for learning mathematics, its epistemology. Firstly, I argue that absolutism leads to ideas of fixed mathematical ability and suggest this shapes school and pedagogical mathematics, especially by privileging speed and accuracy, and emotional aspects of learning mathematics. I then consider how these features are elements of other forms of mathematics.

A fallibilist ontology understands mathematical knowledge to be a human construct which is constantly evolving. Epistemologically people come to know by constructing meaning individually and collaboratively. In contrast, an absolutist ontology understands mathematical knowledge to be naturally occurring. Epistemologically we come to know facts that are waiting to be discovered (Ernest, 1991). Similarly, mathematical ability can be constantly evolving, individually and collaboratively constructed (fallibilist), or naturally occurring and waiting to be discovered (absolutist) (Bartholomew *et al.*, 2011).

Embedded in the fallibilist conceptualisation of ability is the idea of agency, that the individual has the capacity to make choices and act on them (Lu *et al.* 2021). Having agency does not mean having complete control: there are parameters. But working with society, agentic individuals are complicit in the evolution of mathematics and of their own capacity to learn mathematics, their ability. In the absolutist conceptualisation knowledge is fixed. The individual and society believe that they have no control over, nor the capacity to construct, mathematical knowledge. They do not have agency. No-one does. The capacity of an individual to learn mathematics is assumed to be similarly fixed (Boaler and Dweck, 2015): we have no control over how we learn mathematical knowledge. Mathematical ability is often taken to be inherited (Ernest, 1991), although some research suggests that it is attitudes rather than ability that is fixed (Itter and Meyers, 2017).

School and pedagogical mathematics in England are shaped by notions of fixed ability (Boaler and Dweck, 2015), and curricula are organised around incontestable answers rather than mathematical thinking (Cuoco, Goldenberg and Mark, 1996). Privileging of answers is derived from the belief that there is one correct answer, one superior procedure for finding the answer and a clearly defined set of rules, which can (and should) be learnt by rote and memorised (Boaler, 2002). Belbase's (2019) comprehensive literature review of teacher beliefs about mathematics learning identifies three categories of beliefs: traditional, constructivist and integral. The traditional teacher believes that 'mathematics is absolute, objective, formal, axiomatic, structured, and independent of the human cognition; it is a collection of rules and procedures, and it is a tool to solve problems; mathematics knowledge is fixed' (*ibid*., p. 7), consistent with an absolutist conceptualisation. Such teachers put 'stress on speed and accuracy,' and 'learning means memorisation of facts, rules, and procedures.' (*ibid*. p. 10). By contrast, teachers with constructivist or integral beliefs help students to construct conceptual meanings. That Belbase's traditional beliefs are embedded in school mathematics is reinforced by studies into school children's conceptualisations of what it means to be a successful learner of mathematics. In one such study Bütüner and Baki (2020) reported that prior to their intervention Turkish 8th grade students saw mathematics as being closed to development. In another, Darragh (2014) quotes a 13-year-old Australian participant describing someone who is good at maths as 'someone that can pick up answers really fast and … can answer questions really fast, so it's just like (clicks fingers) answer that question super-fast' (*ibid*., p. 83).

Darragh (*ibid*.) argues that learners exhibit negative mathematical identities and exclude themselves from mathematics. Mathematical identities are just one area of affect in mathematics education, which also includes attitude, anxiety, beliefs, meaning, self-concept, emotion, interest, motivation, needs, goals, norms and values (Hannula *et al.,* 2019). All of these are notoriously difficult to measure (Leder, 2019), and can result in ‘overlapping constructs, partially commensurate methods, and somewhat contradictory findings’ (Schoenfeld, 2015, p. 395). This thesis focuses on one area of affect, emotions. Emotions are an intrinsic element of learning and teaching mathematics (Boylan, 2009; Boylan and Povey, 2009), and important for challenging absolutist conceptualisations of mathematics which wrongly hold the subject to be neutral and value-free (Kincheloe, 2008b; Luitel and Taylor, 2007).

Walkerdine’s (1988) feminist critique of mathematics argues that the female voice is Other in mathematics, where the dominant voice is a masculine fantasy of reason, Ernest’s absolutism (1991). Ernest (2004) describes absolutist mathematics as cold, abstract and inhuman, which denies access to, amongst others, women in Western countries. Recent studies consider intersectionality (Rubel *et al.*, 2022) and identify other groups marginalised by mathematics, including women of colour with disabilities (Lambert *et al.*, 2022), and those perceived to be non-mathematicians (Alderton, 2020). Mendick and Francis (2012) use media representations to suggest that the language of ability has gender and social class connotations and is complex, with both East Asian and Black heritage learners being Othered, through their inclusion and exclusion respectively as boffins and geeks. Absolutist conceptualisations of mathematics and resulting notions of fixed ability mean that learning school mathematics is emotional.

Learners’ emotional responses could be categorised as positive or negative, exemplified by bestselling mathematical fiction writer Doxiadis’ (2003) confession to having been a fanatical hater of mathematics until becoming a passionate lover of the subject at age fourteen. Positive emotional responses to mathematics include Crisan’s (2021) autobiographical description of the pleasure she found in mathematical symbols, and Mlangeni’s (2019) excitement at learning to add. Nardi and Steward’s (2003) rigorous study of 1500 English 16-year-olds identified characteristics of negative attitudes towards mathematics including: Tedium, Isolation, Rote learning, Elitism and

Depersonalisation, leading to reported feelings of boredom and depression, including one assertion that they describe as chilling:

> If I have a maths lesson I have no negative nor positive emotions. You just sit there and do it – it's like a null period. (*ibid*., p. 361)

This suggests that a binary of positive and negative emotions is insufficient. Emotional responses to mathematics reflect human emotional complexity. For example, Andersson and Wagner (2019) found caring, antagonism and reclusion intertwined in their analysis of the figured worlds of Canadian high school learners, and Lutovac's (2019, 2020) discourse analysis suggests that failure can be a positive motivating factor, as can shame (Bibby, 2002). Complex emotions are associated with learning school mathematics.

Pedagogical mathematics has an emotional dimension in response to the complex emotions associated with school mathematics, such as Watson's (2021) *care in mathematics* which describes transformative educational spaces which promote care for mathematics, learners and their communities. The emotional dimension of teaching out-of-field has been a focus of several studies (Hobbs and Quinn, 2020). Bosse and Törner (2015) propose a theoretical framework which accounts for systemic (emotions, agencies, contexts) and situational elements of the out-of-field teacher of mathematics' dynamic and complex personal and professional identities. These elements include the beliefs, self-image, motivations and emotions teachers bring with them and develop. Two of these elements relate to out-of-field teachers of mathematics: the aesthetic (Darby, 2007; Hobbs, 2012) and the *pedagogy of support* culture of mathematics classrooms (Darby, 2010) where mathematics classroom teachers offer support to learners to protect them from the perceived challenges of mathematics.

Competitiveness in the mathematics classroom brings together the themes discussed in this section. Competition constructs and is informed by notions of fixed ability. For example, an important contribution of *Masculinities in Mathematics* (Mendick, 2006), is its identification of binary oppositions, including ‘Maths people/Non-maths people’ and ‘Competitive/Collaborative’ (Boylan and Yackel, 2009, p. 88). Mendick’s (2006) case study of Mrs Sawyer’s A-level lessons illustrates the complex interplay of competition and comparison with gender, notions of natural ability and public examination. Locating the elite (Nardi and Steward, 2003), identifying the victors of mathematical competitiveness, often involves privileging of speed and accuracy. Ocean relates this to 200-year-old military practice which emphasises commands, obedience, rules, silence, speed and competition (Ocean and Skourdoumbis, 2016; Ocean, Ersozlu and Hobbs, 2021; Ocean, Sawatzki and Ersozlu, 2021). Although Ocean’s research took place in Australian primary schools with an American historical perspective, her focus of times tables drilling is familiar in English classrooms (Field, 2020), as is the environment of testing and ranking (Jones and Ball, 2023). Gamification of mathematics learning aims to address the challenges learners have in engaging positively (Tokac, Novak and Thompson, 2019), an example of a pedagogy of mathematics defined by competitiveness.

So far, I have only considered the learning of school and pedagogical mathematics. I now turn to notions of ability and emotional dimensions of academic and everyday mathematics. Notions of ability are prevalent in society, as epitomised during the Prime Minister’s call to improve attainment in mathematics, in which he refers to mathematics being used in the film industry, healthcare, retail and daily lives – everyday mathematics:

> We make jokes about not being able to do maths. It’s socially acceptable. We say things like: “Oh, maths, I can’t do that, it’s not for me” – and everyone laughs. (Gov.uk, 2023)

The notions of ability expressed in everyday ‘cocktail-party type conversations’ (Burton, 2004 p. 3) were also reflected in Burton’s academic participants’ reliance on intuition, the idea that you either have mathematical ability or you do not. However, accuracy and speed are not shared by the academic discipline (*ibid*.) and rarely feature in everyday mathematics (Boaler, 2009). As everyday mathematics uses mathematics as a tool to other ends, it is the desired outcome that determines what is privileged. Accuracy is privileged in engineering not because it reflects an individual’s ability, but because of its importance in output. In her study of seventy professional mathematicians working in twenty-two universities across the UK, Burton (2001) found that participants readily talked about feelings associated with learning mathematics which they appeared to consider appropriate to academic mathematics, 'but quite inappropriate to the classroom where mathematics is presented as a supreme example of the triumph of reason over emotion' (*ibid*., p. 596). The strong negative emotions expressed about mathematics by adult learners in Part's (2016) research in the UK and Whitten's (2018) in New Zealand were found to be predominantly related to their own school experience. Keogh *et al.* (2018) found similar results among adults talking about their use of mathematics in the workplace.

Learning mathematics, like knowing mathematics (section 2.2.1), is a complex mix of dominant absolutist thinking and fallibilist ideas. Absolutism leads to notions of ability and the privileging of speed and accuracy in school and pedagogical mathematics, but learning academic and everyday mathematics are more about intuition and the focus is on the output. Emotions are a central dimension in learning all forms of mathematics and are complex.

2.2.3 Doing mathematics

Everyone who does mathematics of any form has a reason for doing it, consciously or not (Goos, 2006). Their reason for doing mathematics is defined here as their axiology: how they value it, what they think the purpose is for doing mathematics. A person's axiology of a subject affects how they teach it (Brooks, 2016).

The programme of study published by the Department for Education sets out the statutory content for school mathematics. It states the following reasons for studying mathematics:

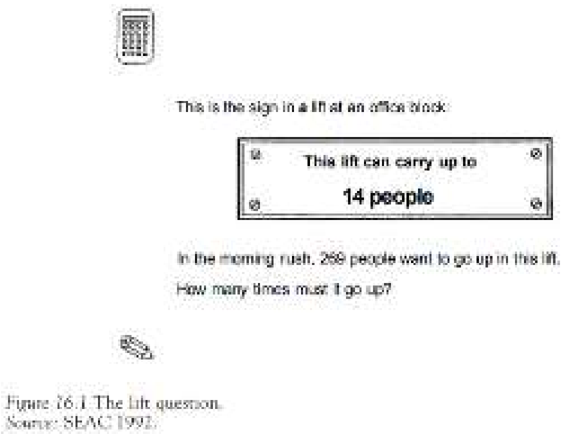
> Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education

> therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (DfE, 2021, p. 3)

This section begins by considering the purposes suggested by this extract: because it is useful, because it has intrinsic value, and to gain a qualification. Exploration of each of the three purposes begins with what it might mean in the context of school mathematics before considering other forms of mathematics.

Giving function or utility as a reason for learning mathematics in school is global (see for example empirical studies from England (Andrews, 2007), the USA (Rubel and McCloskey, 2021) and Nepal (Lamichhane and Belbase, 2017)), and is encapsulated in the focus on STEM learning (Colucci-Gray *et al.*, 2017) and transferable skills (Bertrand and Namukasa, 2020). The ability to transfer mathematical skills into other contexts is believed to be important in employment (Sulak *et al.*, 2020) and the developments of skills such as perseverance and adaptability (Bertrand and Namukasa, 2020). In this thesis, mathematics that is useful is referred to as everyday mathematics, mathematical knowledge informally structured as it is used (Davis & Renert, 2014). Political rhetoric, such as the DfE's stated purpose for learning mathematics or the Prime Minister's speech (section 2.2.2), positions school mathematics as preparation for everyday mathematics and develops the imperative for school mathematics to involve 'relevant' problem solving (Darby-Hobbs, 2011). Figure 2.4 presents two school mathematics questions that appear to apply mathematics to real-world contexts, chosen because they have both been subject to convincing analysis by education researchers (Boaler, 1994; Cooper, 2002). Cooper (2002) demonstrates how the lift question (Figure 2.4a) is not designed to apply mathematics in a functional context but to reduce it to a single calculation, producing a meaningless answer. The fashion question (Figure 2.4b) is open-ended but has carefully designed parameters, and Boaler (1994) shows that only by ignoring the context can learners reach the 'correct' answer. Learners who approach these questions in a functional way are penalised: even when teachers desire to bring ‘real’ mathematics into the classroom, they do not achieve it (Blanton, Westbrook and Carter, 2005; Howley, 2013). Although they have not been subject to the same analysis, Figure 2.4c and d are similar examples from recent GCSE examinations. The ‘relevance imperative’ in education rhetoric (Darby-Hobbs, 2011), is a ‘compelling fiction’ (Lundin, 2012, p. 76). Mathematics learnt in school serves little functional purpose (Ernest, 2000, 2010, 2020; Wolfram, 2020).

a)


This is the sign in a lift at an office block.

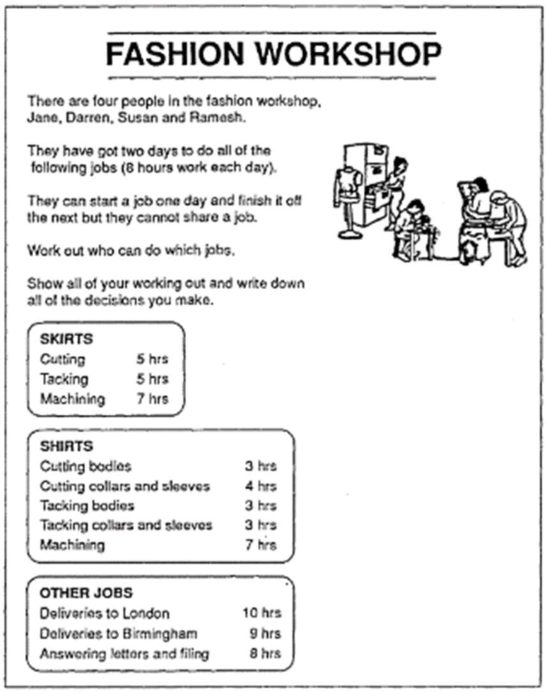
This lift can carry up to
14 people

In the morning rush, 269 people want to go up in this lift.
How many times must it go up?

Figure 16.1 The lift question.
Source: SEAC 1992.

©Taylor and Francis (Cooper, 2002, p. 246)

b)


FASHION WORKSHOP

There are four people in the fashion workshop, Jane, Darren, Susan and Ramesh.

They have got two days to do all of the following jobs (8 hours work each day).

They can start a job one day and finish it off the next but they cannot share a job.

Work out who can do which jobs.

Show all of your working out and write down all of the decisions you make.

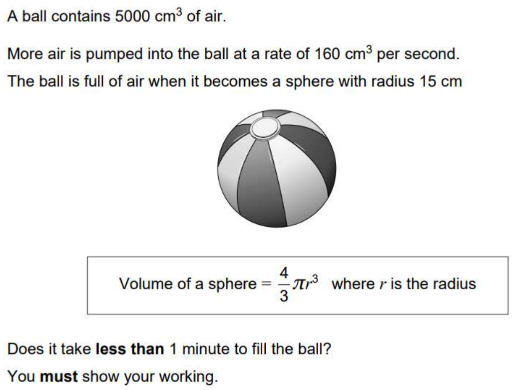
SKIRTS	
Cutting	5 hrs
Tacking	5 hrs
Machining	7 hrs

SHIRTS	
Cutting bodies	3 hrs
Cutting collars and sleeves	4 hrs
Tacking bodies	3 hrs
Tacking collars and sleeves	3 hrs
Machining	7 hrs

OTHER JOBS	
Deliveries to London	10 hrs
Deliveries to Birmingham	9 hrs
Answering letters and filing	8 hrs

©John Wiley & Sons (Boaler, 1994, p. 559)

c)


A ball contains 5000 cm^3 of air.

More air is pumped into the ball at a rate of 160 cm^3 per second.

The ball is full of air when it becomes a sphere with radius 15 cm

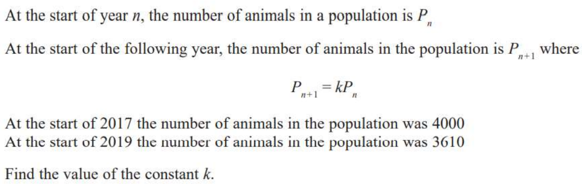
Volume of a sphere = $\frac{4}{3}\pi r^3$ where r is the radius

Does it take **less than** 1 minute to fill the ball?

You **must** show your working.

©AQA 2021 (AQA, 2021, p. 21)

d)


At the start of year n, the number of animals in a population is P_n

At the start of the following year, the number of animals in the population is P_{n+1} where

$$P_{n+1} = kP_n$$

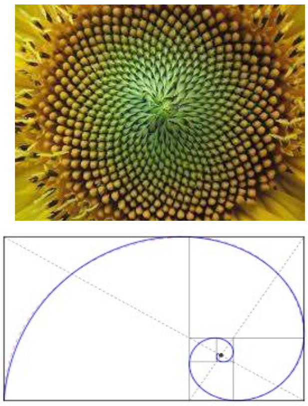
At the start of 2017 the number of animals in the population was 4000
At the start of 2019 the number of animals in the population was 3610

Find the value of the constant k.

© Pearson 2021 (Pearson 2021, p. 20)

Figure 2.4 School mathematics questions applying mathematics in real-world contexts

Another reason given by the DfE for learning mathematics is to develop ‘an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject’ (DfE, 2021, p. 3), to meet basic human desires essential for flourishing (Su, 2020). To an absolutist like the early twentieth century mathematician Hardy (1967), the beauty of mathematics is revealed in nature. To a fallibilist, mathematics is a manifestation of human playfulness, seeking patterns and making connections (Ernest, 2015). These axiologies align with what is being referred to here as academic mathematics. Figure 2.5 provides some examples of the beauty of mathematics.



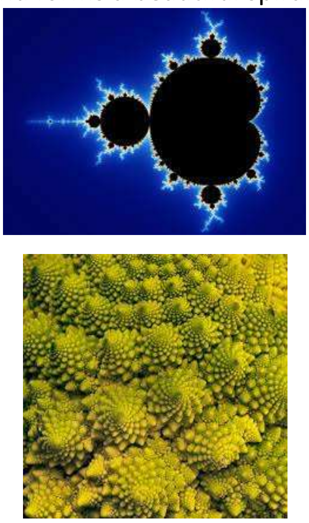
a) Fibonacci
A simple number sequence made by adding the previous two numbers contains numbers found in nature and forms a beautiful spiral.

A	B	C
C	A	B
B	C	A

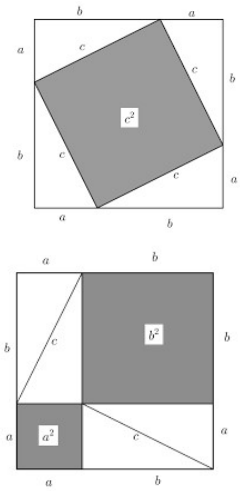
A	B	C
B	C	A
C	A	B

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

b) Latin square & Sudoku
Every row and column in a Latin square contains each symbol exactly once and has many symmetries and patterns.



c) Fractals
A fractal is an infinitely complex pattern that repeats itself as you zoom in and out and has finite area but infinite perimeter.



d) Proof
Mathematical proof is based on logical assumptions and is beyond doubt. They can be elegant and efficient such as these two visual proofs of Pythagoras’ theorem.

Figure 2.5 Some examples of the intrinsic value of mathematics

Enjoyment of mathematics has been shown empirically to be a reason for studying mathematics given by some teenagers (Brown *et al.,* 2008), professional mathematicians (Burton, 2004) and prospective teachers (Goos, 2006), potentially suggesting that the intrinsic value of mathematics as a motivator across school, academic and pedagogical mathematics. However, case study data suggests that learners tend to perceive school mathematics for mathematics' sake as abstract and that it can have a negative impact on learners (Aguirre, 2009). In his passionate treatise, Lockhart (2009) decries the Western societal approach to mathematics education:

> If I had to design a mechanism for the express purpose of destroying a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done— I simply wouldn't have the imagination to come up with the kind of senseless, soul-crushing ideas that constitute contemporary mathematics education. (*ibid*. pp. 20–21)

Despite the functional and intrinsic value of mathematics being the stated purpose for the study of mathematics to age 16 in England (DfE, 2021), most people are unlikely ever to use most of the mathematics they learnt at school (Wolfram, 2020) and few people need or want to appreciate mathematics for its own sake (Kessler, 2019).

Mathematics is nevertheless a compulsory school subject, universally seen as an unqualified force for good (Ernest, 2018, 2020). Employers and further and higher education seek people with mathematics ability (Brown and McNamara, 2011), and governments seek prominence in the global league tables (Hillman, 2014; Greany *et al.*, 2016; Colucci-Gray *et al.*, 2017). Given that the previous paragraphs have suggested that school mathematics does not have a strong functional nor intrinsic value, the question remains why mathematics is nevertheless presumed to be indispensable for being human (Dörfler, 2003).

School mathematics is a gateway to opportunities (Ernest, 2020). It plays a key role in the distribution of life chances (Ernest, 1994), identifying who deserves membership of the elite 'Maths Club' (Bartholomew *et al.*, 2011). A benign justification is that it is easy to measure and a good indicator of other abilities: Bjälkebring and Peters (2021) report that success in mathematics correlates to higher income and increased satisfaction with life. Similarly, it is argued to be a way of sorting people: those who have shown success in mathematics examinations are better suited to certain tasks. Sulak *et al.* (2020) found that higher numeracy scores predicted employment while 'soft skills' (collaborative problem solving, critical thinking) did not. The empirical (usually quantitative) data used in these studies fails to account for the complexity of learning and learners, the multiple factors that impact examination outcomes (Kaspersen, Pepin and Sikko, 2017). It makes assumptions about the validity of assessment data and the value of what it claims to assess and suggests valuing what we can measure rather than measuring what we value (Goodson, 1983): pedagogy changes the nature of mathematics itself so that it can be more easily assessed (Brown and McNamara, 2011).

The critical interpretation is that school mathematics is a gate*keeper*, not a gate*way* (Stinson, 2004; Bryk, 2010). It is a critical filter in the same way that classics once was (Ernest, 2020). It is a way of maintaining privileges of race (Moses and Cobb, 2001), gender (Alderton, 2020), culture (Leonard, 2019) and class (Jorgensen, Gates and Roper, 2014), exerting control through standardised behaviours which accept the voice of authority (Gordon, 2019). Absolutism reinforces this control by justifying a body of knowledge and set of behaviours (Kollosche, 2018), a status quo which cannot be questioned (Aikenhead, 2021b, 2021a), as does the consequent use of relevance 'as a camouflage for the struggles between maintaining an elite curriculum or standards' (Drake, 2009, p. 80). Boaler and Greeno (2000) found learners who claimed that obedience and compliance played a central role in success in school mathematics.

Just as mathematics models a complex world, school mathematics can turn complex social beings into predictable individuals (Beisiegel and Simmt, 2012), regulating thoughts (de Freitas, 2008) and manufacturing the mathematical child (Llewellyn, 2018).

2.2.4 Conceptual framework revisited

Knowledge explored through the literature in section 2.2 is summarised in **Error! Reference source not found.**, which updates the framework of conceptualisations of mathematics.

Table 2.3 How knowing, learning and doing mathematics is conceptualised in this study

		Knowing (ontology)	Learning (epistemology)	Doing (axiology)
		Teachers' beliefs about the nature of mathematics	*Understanding of how we come to know mathematics*	*Conceptualisations of purposes for doing mathematics*
School mathematics	*The mathematics learnt at school* (Golding, 2017)	Complex mix of dominant absolutism, with elements of fallibilism.	Learn according to fixed ability, privileging of accuracy and speed. Complex emotional element.	Acts as gateway/ gatekeeper, qualifications valued.
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.	Intuition important and recognised as emotional work.	Mathematics valued intrinsically.
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.	Learners require support because of challenge and emotional element.	Acts as gateway/ gatekeeper, qualifications valued.
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)	Complex mix of dominant absolutism, with elements of fallibilism.	Mathematics used as tool appropriate to context. Individual and societal emotional element.	Mathematics valued as a tool to other ends.

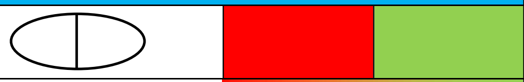
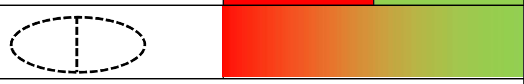
2.3 The boundaries of the conceptual framework

When we distinguish between and classify school subjects, we are referring 'not to what is classified, but to the relationships between contents' (Bernstein, 1973, p. 366). What happens at the boundaries is how we know what is going on in a subject. These boundary zones are the space in which out-of-field teachers are working (Hobbs, 2013a, 2014). The approach taken by this thesis is unusual in that it considers boundaries *within* subjects, unlike Bernstein (*Op. Cit*.) who was considering the boundaries *between* subjects. It also contrasts with the out-of-field literature, which often focuses on the boundaries between in- and out-of-field knowledge (for example, Hobbs, 2013b; Vale, Campbell and White, 2021). Mathematics education literature does explore boundaries within mathematics, but this is usually restricted to the boundaries between pedagogical mathematics and a generic (often undefined) mathematics (for example, Alderton, 2020). With a focus on the boundaries within mathematics, the subsections that follow begin by focusing on the relationship that school mathematics has with the other three forms of mathematics, with the last subsection considering the boundaries between non-school forms of mathematics. Two influential and dominant theories inform this section, Bernstein's recontextualisation (Bernstein, 1990) and Shulman's (1986) pedagogical content knowledge. These theories are introduced and critiqued throughout this section.

Boundaries are complex (Hobbs and Quinn, 2021). Diagrammatic representation is used in this section to model these complex contexts. I acknowledge these to be reductionist metaphors and employ them with the purpose of allowing access to otherwise inaccessible concepts. Table 2.4 provides a key to the diagrams.

Table 2.4 Key to diagrams used in section 2.2

(red)	School mathematics
(amber)	Everyday mathematics
(green)	Academic mathematics

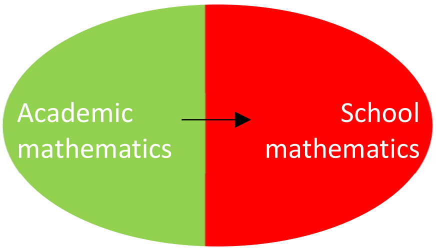
	Pedagogic mathematics
	Distinct boundaries
	Blurred boundaries
	Recontextualisation in direction of arrow
ε	The hypothetical universal set within which all mathematical knowledge lies

2.3.1 Boundary of academic with mathematics school mathematics

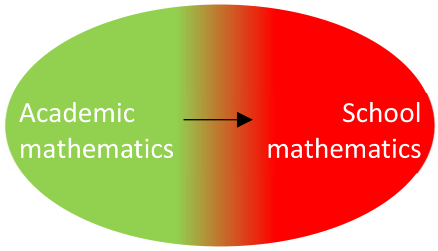
In Bernstein's (1990) theory, the knowledge that is recontextualised comes from the field of production: knowledge produced by researchers. This knowledge must first be decontextualised, mentally removed from the context in which it was produced. In Bernstein's model, professional bodies and state regulators in the official recontextualisation field convert academic knowledge into the school curriculum. In Figure 2.6a I suggest he conceptualises this as a one-way process of recontextualisation between two distinct bodies of knowledge.

Shulman (1986), responding to the then current trend in teacher education to focus on generalised pedagogy, identified different types of knowledge teachers require, including content knowledge: 'we expect that the subject matter content understanding of the teacher be at least equal to that of his or her lay colleague, the mere subject matter major' (*ibid*. p.9). In this statement, suggesting academic mathematics as a starting point, Shulman implies the same direction of recontextualisation as Bernstein (Figure 2.6b). In including it as one amongst his types of knowledge that teachers require, he implies more blurred boundaries.

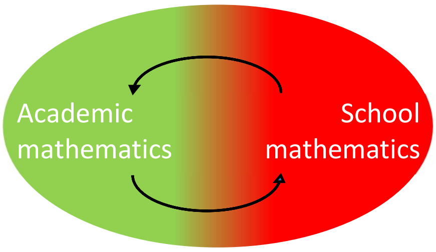
The recontextualisation of academic mathematics into school mathematics has been a focus for many high-profile mathematics education researchers, three of whose models have been selected as an illustration of the range of possible models of recontextualisation between academic and school mathematics (Figure 2.6). The first of these, Figure 2.6c, models Ernest's (1991) 'Old Humanist' philosophy of mathematics education in which talent-spotting for future academic mathematicians is the purpose of teaching mathematics. From this perspective, school mathematics develops into academic mathematics and so the boundary is a gradient. The next illustrative model (Figure 2.6d) represents the idea that school mathematics is a recontextualised selection of academic mathematics, situated within the complex world of the mathematical classroom (Hodgen, 2011). Finally, Golding (2017) argues that there is a distinct boundary between school and academic mathematics, although with overlapping cultures as they both sit within the discipline of mathematics (figure 2.6e).



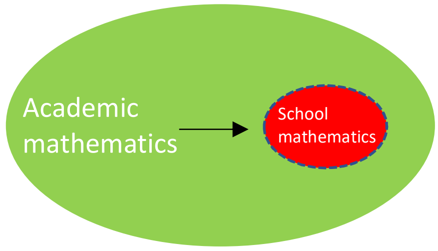
a. Bernstein's (1990) field of production into official field of recontextualisation



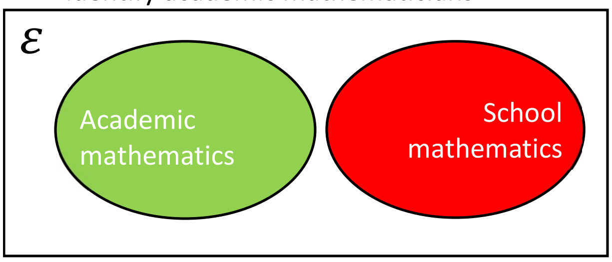
b. Shulman's (1986) content knowledge into curricular knowledge



c. Ernest's (1991) Old Humanist where the purpose of school mathematics is to identify academic mathematicians



d. Hodgen's (2011) school mathematics as a recontextualised selection of academic mathematics



e. Golding's (2017) distinct forms with shared overlapping cultures

Figure 2.6 Diagrammatic models of possible relationships between school and academic mathematics

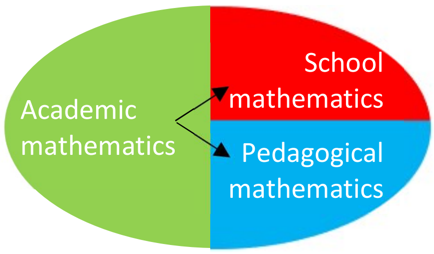
Out-of-field teachers of mathematics can only traverse this boundary between academic and school mathematics in one direction: they cannot move from academic mathematics as they have not experienced it. The two models in Figure 2.6 in which out-of-field teachers could potentially participate are Golding's (2017) (Figure 2.6e), where school mathematics is held to be distinct from academic mathematics and Bernstein's (1990) (Figure 2.6a), where the recontextualisation is undertaken by the state. Here, power is removed from the teacher in both the production of new mathematical knowledge and curricula selection.

2.3.2 Boundary of pedagogical mathematics with school mathematics

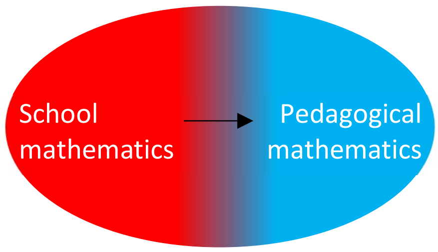
The nature of the boundaries between pedagogical and school mathematics is similar in Shulman's (1986) model to that between academic and school mathematics. He defines pedagogical content knowledge as that 'which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for* teaching' (*ibid*. p. 9, emphasis in original). This gives the direction: subject matter knowledge is recontextualised into pedagogical content knowledge. The relationship between Bernstein's conceptualisation of pedagogical and school mathematics is harder to define. The pedagogical recontextualisation field sits alongside the official pedagogic field, both created by the de- and re-contextualisation of knowledge from the field of production. Figure 2.7 builds on Figure 2.6, showing a three-way relationship.

Both Bernstein's and Shulman's model put the direction of transformation as recontextualising the content of the curriculum into pedagogical knowledge. This is Future 1 in sociologists Young and Muller's three educational scenarios (2016), in which elite subject boundaries are maintained with universities as knowledge producers (Figure 2.7c). Future 2 ends subject boundaries, with the result that schools swing from content-based to general skills-based priorities and so pedagogical knowledge drives the curriculum (Figure 2.7d). Future 3 recognises the importance of the boundaries between different forms of knowledge (Figure 2.7e). From a critical perspective, this scenario empowers learners and their teachers by explicitly recognising boundaries, enabling Epistemic Insight (Billingsley, 2017). This is similar to Figure 2.7din that pedagogy informs the curriculum, but with a stronger boundary around the concepts and skills of school mathematics.

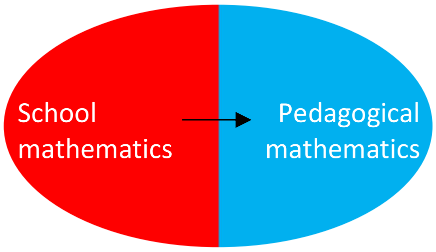
Out-of-field teachers are excluded from Future 1, risk working in Future 2 and potentially have a lot to offer in Future 3.



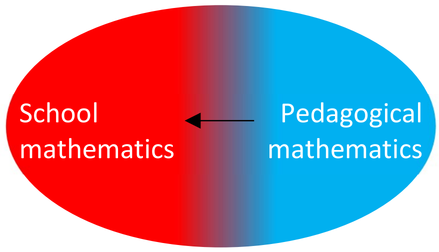
a. Bernstein's (1990) field of production into official and pedagogic fields of recontextualisation



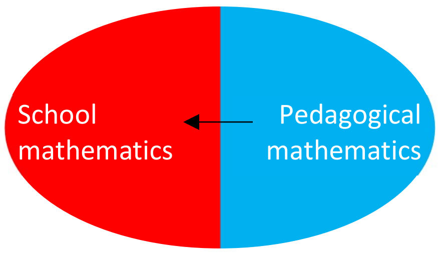
b. Shulman's (1986) content knowledge into pedagogical content knowledge



c. Young and Muller's (2016) Future 1 – boundaries as given and fixed



d. Young and Muller's (2016) Future 2 – the end of boundaries



e. Young and Muller's (2016) Future 3 – boundary maintenance prior to boundary crossing

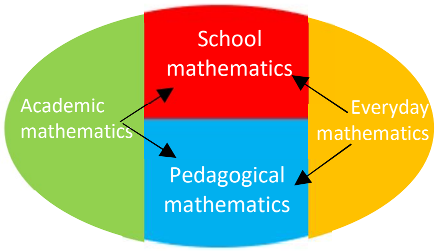
Figure 2.7 Diagrammatic representations of possible relationships between school and pedagogical mathematics

2.3.3 Boundary of everyday mathematics with school mathematics

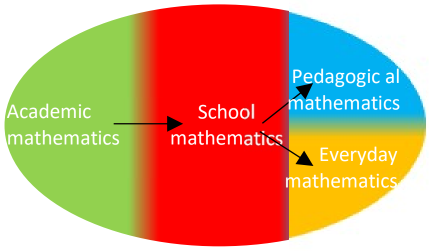
The conceptualisation of everyday mathematics used in this thesis (mathematical knowledge informally structured as it is used) overlaps with Bernstein's (1990) 'field of production' and his 'field of reproduction'. Figure 2.a positioned Bernstein's 'field of production' as academic mathematics. However, Bernstein included professional institutions and corporate research as producers of mathematical knowledge. Figure 2.8a therefore shows some aspects of everyday mathematics being recontextualised as school mathematics. If the 'field of reproduction' is where recontextualised knowledge is transmitted, and everyday mathematics contributes to the production of mathematical knowledge, then pedagogical mathematics will contain elements of recontextualised everyday mathematics. The conceptualisation of everyday mathematics in this thesis is potentially too broad to capture these nuances (see section 9.2).

Subject knowledge in Shulman's conceptualisation appears to be restricted to academic, pedagogic and school mathematics. In his 1986 article he does refer to a theoretical teacher drawing on knowledge of drugs to inform their pedagogy, but this is within the context of teaching pharmacology and so is pedagogical knowledge rather than everyday informal knowledge. The justification for his research and subsequent theories is to understand how teachers enable their students to learn the intended outcomes. This informs the development of Figure 2.6a into Figure 2.8a.

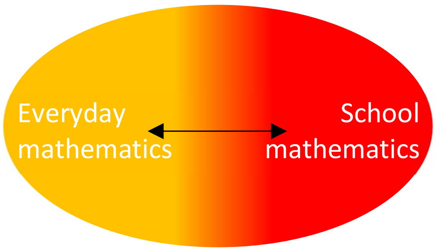
As in section 2.3.1, a further model is provided by Ernest's (1991) five educational ideologies. Of these, only the Old Humanist is concerned with the maintenance of the body of elite, academic knowledge. Although different politically, the other four ideologies have everyday mathematics as their aim: for example the aim of technological pragmatists is mathematics useful in an industrial context. In Figure 2.8c, then, everyday mathematics is recontextualised as school mathematics. The boundary is blurred and weak, and school and everyday mathematics are almost integrated in Ernest's model (*ibid*.) Arguing from a very different perspective to Ernest's critical pedagogy perspective (Ernest, 2020), Wolfram (2020) considers the needs of capitalism and concludes that the content of school mathematics serves little functional purpose for its learners and instead both the content and how it is taught should be informed by mathematics they will use in their future lives, everyday mathematics (Figure 2.8d). Out-of-field teachers bring with them their experience of everyday mathematics, and so unlike in section 2.3.1, these models do relate to them, especially Figure 2.8c and 2.8d, which do not include academic mathematics.



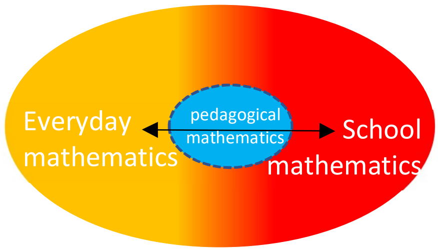
a. Bernstein's field of production into official and pedagogic fields of recontextualisation (1990)



b. Development of Shulman's model of knowledge (1986)



c. Ernest's other four educational mathematicians (Ernest, 1991)



d. School mathematics informed by mathematics learners will use (Ernest, 2020, Wolfram, 2020)

Figure 2.8 Diagrammatic representations of possible relationships between school and everyday mathematics

2.3.4 Boundaries between academic, everyday and pedagogical mathematics

It is testament to Shulman's (1986) impact that curricular knowledge now dominates the literature. In mathematics education there is now a plethora of models of the mathematical knowledge that teachers require. Most of this research responds to Shulman's (*ibid.*) call to theorise the process as expert graduates transform their expertise to that of novice teacher, the recontextualisation of academic into pedagogic knowledge. Mathematical Knowledge for Teaching (MKT) (Ball *et al.*, 2008), the Knowledge Quartet (Rowland *et al.,* 2004) and Mathematics for Teaching (M_4T) (Davis and Renert, 2014) all focus on the recontextualisation of academic knowledge into pedagogical knowledge. These excellent models are less relevant to out-of-field teachers because they assume academic knowledge (MKT & M_4T) or professional training (Knowledge Quartet) or both.

2.4 Conclusion

This chapter has considered conceptualisations of mathematics. It began by reviewing the literature on different philosophical perspectives people have for different forms of mathematics, summarised in Table 2.1. The second part of this chapter considered the boundaries between different forms of mathematics, where Bernstein (1973) argues power lies, and where out-of-field teachers undertake their complex boundary-crossing work (Hobbs, 2013a).

The term mathematics is rarely defined in the literature (Davis and Renert, 2014). This chapter challenges assumptions that the term is unproblematic or universally understood. The framework constructed in this chapter considers different forms of mathematics and different philosophical beliefs about how we know, learn and value doing mathematics. Consideration of the framework's internal boundaries in section 2.3 further challenges assumptions about the homogeneity of mathematics, demonstrating that there is complex overlap and blurring between forms and philosophies.

The research focus of this thesis is out-of-field teachers knowing, learning and doing mathematics. This chapter has used a critical complex approach to challenge understandings of what mathematics is and what it means to know and learn it, and why we do it. The next chapter critically analyses the complexity of the other element of the research focus: out-of-field teachers.

3. Out-of-field teachers' conceptualisation of mathematics

The previous chapter explored complex conceptualisations of mathematics. This chapter begins by exploring the complexity of teaching out-of-field (section 3.1). Following the same structure as the previous chapter, section 3.2 explores the concepts of boundary crossing and recontextualisation in the context of the teaching-out-of-field literature.

3.1 Conceptualising teaching-out-of-field

The first part of this section (3.1.1) uses Hobbs *et al*.'s (2020) recent work defining out-of-field teaching to adopt a conceptual framework for out-of-field teaching. The second part (3.1.2) reviews the out-of-field literature to further justify the opportunity model which recognises the agency of out-of-field teachers and the complexity of teaching-out-of-field adopted in this thesis.

3.1.1 Defining teaching-out-of-field

The phenomenon of teaching out-of-field is complex (Hobbs *et al.,* 2019) and difficult to define (Hobbs *et al*., 2020, 2021; Porsch and Whannell, 2019). Most researchers use Ingersoll's (1999) definition of teachers 'assigned to teach subjects for which they have little training or education' (*ibid*., p. 26) (for example, du Plessis, 2015a; Singh, Luft and Napier, 2021). This definition is problematic: it is a deficit model (Hobbs, 2013a), does not recognise the complexity of teaching-out-of-field, and diminishes out-of-field teachers' agency by using the word *assigned*. These issues will be explored in the next section. Recent work to define teaching-out-of-field (du Plessis *et al.*, 2019; Hobbs *et al.*, 2020, 2021) builds on Hobbs' (2013a, 2014) previous work to avoid a deficit model and recognise the complexity of the phenomenon. The framework developed by Hobbs *et al.* (2020) is adopted in this thesis to provide a conceptual framework for teaching-out-of-field and is reproduced in full in Appendix D. This section critically evaluates it to justify its central role in the selection of participants and analysis of data.

Hobbs *et al*.'s (*ibid*.) model provides a clear framework while representing the complexity of teaching-out-of-field. It identifies three clusters of criteria: measurable criteria, self-report criteria and longitudinal criteria. Each criterion is further subdivided into dimensions. For each dimension three bands are described. In each case band 1 describes what this dimension would look like for a teacher who is in-field, band 2 describes a teacher who is partially in-field, partially out-of-field, and band 3 describes the context in which a teacher for this dimension would be fully out-of-field.

The first cluster, measurable criteria, is subdivided into qualification, workload and capability. The first of these recognises that there are two dimensions to a teacher's qualifications to teach: their subject specialism and teacher education. For example, a teacher could have a degree in economics and PGCE in mathematics, partial technical alignment (Band 2 for dimension 1.1). Recognising these nuances is important as while basic subject knowledge is assumed to be important (Hobbs and Törner, 2019a), it does not guarantee quality teaching (Askew *et al.*, 1997; Hobbs *et al.*, 2020). The workload criterion considers the current proportion (dimension 2.1) of teaching hours on the out-of-field teachers' timetable, whether they have been teaching out-of-field for years or as a one-off (dimension 2.2 *Longitudinal proportion*), and whether this is predictable from one year to another (dimension 2.3 *Stability*). The third measurable criterion, capability, assumes that a teacher's suitability for teaching a subject is based on their teaching experience or demonstrable teaching qualities (Hobbs and Quinn, 2021). This recognises that an early career teacher's capability to teach effectively out-of-field is not as great as that of an experienced teacher, as they are establishing their professional practice (Hobbs, *et al.,* 2019). Expertise (dimension 3.1) is related to the workload criteria of longitudinal proportion and stability, adding the extra layer of professional learning, formal and informal, as the teacher becomes experienced in their new field.

The second cluster in Hobbs *et al*.'s (2020) framework is self-report criteria. The next section (3.1.2) suggests that research exploring the incidence and impact of teaching-out-of-field has tended to rely on measurable criteria rather than listen to the voice of the out-of-field teacher. Yet different teachers with the same measurable qualifications and experiences can feel more in- or out-of-field than each other (Hobbs, 2013b; Porsch, 2016). The self-report criterion is divided into identity and structures. The identity criterion considers teachers' commitment (dimension 4.1) to the subject. Its differentiation between teachers who have high, limited or no 'personal interest in the subject' (*Op. Cit.* p. 20–21) is important for this study's focus on conceptualisations of mathematics rather than of teaching mathematics, although each band also includes a component of professional commitment to students' learning experience. The dimension of self-concept (dimension 4.2) considers how closely the teacher sees themselves to the out-of-field subject and whether they label themselves as in- or out-of-field, and whether they feel confident in their out-of-field practice (dimension 4.3). The latter is of interest to the learning element of this study (section 2.1.2) as the band descriptors consider self-concept of ability and emotional response to the out-of-field subject. While the identity criterion focuses on the out-of-field teachers' perspective, the structures criterion considers the context they are working in. This includes opportunities for professional development to meet out-of-field teaching needs (dimension 5.1 *School context*), and the culture of support in the school (dimension 5.2).

The final cluster in Hobbs *et al*.'s framework (*Op. Cit*.) is longitudinal criteria. These criteria question what trajectories (dimension 6.1) and opportunities for role expansion (dimension 6.2) are available to enable an out-of-field teacher to become in-field, combining elements from the measurable criteria cluster, especially longitudinal proportion (dimension 2.2) and stability (dimension 2.3) with the school context (dimension 5.1) and school support culture (dimension 5.2). This is necessary as individuals can respond to the same set of circumstances in different ways:

> Teachers can accept an out-of-field subject as part of their teaching role and this may be evident as increased teaching expertise in the subject, commitment to undertaking professional learning and increased confidence. On the other hand, teachers can accept that teaching a subject out-of-field will be part of their ongoing load but without seeing

themselves as suitably skilled or qualified to teach, despite years of experience. (*Op. Cit.* p. 15)

In Table 3.1 I use Hobbs *et al.*'s (2020) framework to position myself at different stages. Placing myself in this model as an out-of-field, transitioning, and in-field teacher of mathematics tests its flexibility and the complexity it encompasses. The framework represents my experiences effectively, provides clear positionality, and presents a useful tool for positioning participants.

Table 3.1 Positioning myself on Hobbs et al.'s (2020) framework

Cluster	Criterion	Dimension	First teaching mathematics, fourth year of teaching.	Head of mathematics, sixth year of teaching.	Specialist leader of mathematics and senior leader, twenty years after qualifying.
1. Measurable Criteria	1. Qualification	1.1 Technical Alignment	Band 3: Misalignment	Band 2: Partial alignment	Band 1: Full alignment
		1.2 Specialism Alignment	Band 3: Far misalignment	Band 2: Near misalignment	Band 1: Full alignment
		1.3 Phase Alignment	Band 1: Full alignment	Band 1: Full alignment	Band 1: Full alignment
	2. Workload	2.1 Current proportion	Band 2: Low partial	Band 1: Whole	Band 1: Whole
		2.2 Longitudinal proportion	Band 2: Low partial	Band 2: High partial	Band 1: Whole
		2.3 Stability	Band 3: Temporary	Band 1: Stable	Band 1: Stable
	3. Capability	3.1 Expertise	Band 3: Beginning	Band 2: Practiced	Band 1: Capable
		3.2 Career stage	Band 2: Early career teacher	Band 1: Experienced teacher	Band 1: Experienced teacher
2. Self-report criteria	4. Identity	4.1 Commitment	Band 2: Professional commitment	Band 1: Personal and professional commitment	Band 1: Personal and professional commitment
		4.2 Self-concept	Band 2: Peripheral	Band 1: Close	Band 1: Close
		4.3 Confidence	Band 2: Medium	Band 1: High	Band 1: High
	5. Structure	5.1 School context	Band 1: Opportunities created	Band 1: Opportunities created	Band 1: Opportunities created
		5.2 School support culture	Band 1: Fully supported	Band 1: Fully supported	Band 1: Fully supported
3. Longitudinal criteria	6. Pathways	6.1 Trajectories	Band 2: Professional development concentration	Band 1: Qualification upgrade	Band 1: Qualification upgrade
		6.2 Role expansion	Band 1: Acceptance with extended identity	Band 1: Acceptance with extended identity	Band 1: Acceptance with extended identity

Positioning myself on Hobbs *et al*.'s (2020) framework did not provide the opportunity to state explicitly why I taught out-of-field. Role expansion (dimension 6.2) came closest, where I was consistently in Band 1:

> Teacher accepts the out-of-field subject as part of their ongoing teaching load and identity is extended to include the subject. (*ibid*. p. 24)

This is inaccurate. I did not *accept* teaching out-of-field, I *chose* it, like the scenario of Seral, 'a graduate teacher who chose to teach mathematics even though it is technically out-of-field' (Hobbs *et al*., 2020, p. 33).

The literature that seeks to explain why out-of-field teaching occurs is under-theorised (section 3.1.2) and tends to adopt a deficit model which diminishes the agency of out-of-field teachers. Instead, it considers out-of-field teachers to be subject to external, institutional forces such as school timetabling, curriculum decisions or governmental teacher supply policies. Two studies hint at the agency of teachers in taking on out-of-field assignments, suggesting that leaders allocating teachers to classes should listen to them (du Plessis *et al.*, 2019) and allow them to follow their interests (Campbell *et al.,* 2019). There are also hints about the type of professionals who accept out-of-field assignments in the professional development literature, with some being more disposed to it (Donitsa-Schmidt and Zuzovsky, 2021). I only found one study in which teachers' voices are heard explaining why they teach out-of-field (Barańska and Zambrowska, 2022). There is no explicit reference in Hobbs *et al.*'s otherwise excellent framework (*Op. Cit*.).

In the context of this thesis, Hobbs *et al.*'s (2020) framework nevertheless sufficiently encapsulates the complexity of out-of-field teaching and provides a conceptual framework of teaching-out-of-field. Hobbs *et al*. (2019) caution that research should focus on selected criteria and minimise variety in dimensions that may blur the landscape. Following this guidance, this study uses participants who fall into Bands 2 or 3 for dimensions 1.1 and 1.2 and Band 1 for dimension 3.2. That is, that their broad or narrow in-field specialist subject is not mathematics, and that they are an established rather than beginning teacher (Hobbs, *et al*., 2019). Participants in this study are out-of-field teachers who have taught for six or more years and have a degree and teaching qualification in a subject other than mathematics.

3.1.2 Problematising conceptions of teaching-out-of-field

This thesis builds on Hobbs *et al*.'s (2020) framework and recent approaches to researching teaching-out-of-field that acknowledge its complexity and give the out-of-field teacher a voice, especially the boundary-crossing literature (section 3.2.1). This section critically analyses out-of-field literature to problematise the predominance of a deficit model, the diminished agency of out-of-field teachers and reductionist perspectives that do not recognise the complexity of the phenomenon.

Teaching-out-of-field is often presented as a problem. Ingersoll's prolific and widely cited output includes titles such as *The Problem of Out-of-Field Teaching* (Ingersoll, 1998), *The Wrong Solution to the Teacher Shortage* (Ingersoll and Smith, 2003), and *Misdiagnosing the Teacher Quality Problem* (Ingersoll, 2020). Using rigorous quantitative methods and robust data, Ingersoll convincingly demonstrated that one third of US secondary teachers of mathematics were out-of-field at the turn of the century (Ingersoll, 1999, 2001b; Ingersoll and Gruber, 1996). Ingersoll's positionality contrasts starkly with mine: we both experienced teaching out-of-field throughout our careers, but in his case unwillingly and unhappily (Ingersoll, 1998, 2001a, 2001b). This may explain why he sees it as a problem, and he is not alone. Darling-Hammond found a similarly stark situation (Adamson and Darling-Hammond, 2012; Darling-Hammond, 1999; Darling-Hammond and Ball, 1998). A Department for Education report (DfE, 2016) identified that 11.1% of all subjects and 10.8% of mathematics in England was taught by non-specialists. Other recent studies suggest that teaching out-of-field is still prevalent globally (Costa *et al.,* 2018; Porsch, 2016; Price *et al.,* 2019; Qin and Brown, 2019; Shah *et al.,* 2019), and that disadvantaged students are more likely to experience out-of-field teaching (Adamson and Darling-Hammond, 2012; Allen and Sims, 2018b, 2018a; Qin and Brown, 2019).

The incidence literature suggests that teaching-out-of-field is widespread, but it does not follow that it is a problem. For it to be a problem, it must have a negative impact. One way to measure impact is on student achievement. Since Begle (1979) studies have attempted to measure the impact on student outcomes (see for example, Costa *et al.,* 2018; Darling-Hammond, 1999; DfE, 2016; Goldhaber and Brewer, 2000; Porsch and Whannell, 2019; Sheppard *et al*., 2020; Shirbagi *et al.,* 2018; Tomas-Fulgado, 2020). And since Begle (*Op. Cit.*), studies have repeatedly found small, if any, positive impact of in-field teaching, usually only in certain subjects or contexts (for example, Askew et al., 1997; Dee and Cohodes, 2008). Despite there being no evidence of a negative impact of teaching-out-of-field (Friedman, 2000), and occasional positives (Becker, 2000), the student impact literature widely treats the phenomenon as a problem requiring a solution, a deficit model.

Some studies identify the eradication of teaching-out-of-field as a solution to this perceived problem (for example Ingersoll, 1998; Luft *et al.,* 2020). This is impractical (Crisan and Hobbs, 2019). Professional development for out-of-field teachers is more commonly presented as a solution (for example Sani and Burghes, (2021)). While there are many examples of effective professional development (such as Crisan and Rodd (2011)), it can perpetuate the deficit discourse by regarding the out-of-field teacher as a professional object requiring 'upskilling' (Goos and Guerin, 2021) and assuming they require a specific body of knowledge (de Souza Pereira Grilo and Cerqueira Barbosa, 2022). Programmes are designed to develop participants' relationship with (Adler *et al.*, 2013), and expand their conceptualisations of, mathematics. An example of the latter is Crisan and Rodd's (2011) decision to avoid national curriculum classifications and instead arrange their curriculum around four mathematical themes: infinities, uncertainties, structures and spaces. Yet de Souza Pereira Grilo and Cerquira Barbosa (2022) identify and problematise a discourse in mathematics education which they summarise as 'to teach mathematics it is necessary to know a specific mathematics for teaching’ (p. 136). They believe this comes from an assumption of the existence of a specific mathematics to teach, the idea of expected knowledge. This is consistent with the discourse that Davis and Simmt (2006) recognise in professional development courses that are framed in terms of studying formal, school mathematics. This is understandable as government funding is linked to academic outcomes (Goos and Guerin, 2021). Governments explicitly state that funding is to improve out-of-field teachers’ knowledge of mathematics and its pedagogy (see for example NCETM (2021)). Despite the attempts of courses such as Goos and Guerin’s (2021) and Crisan and Rodd’s (2011), professional development for out-of-field teachers adopts a deficit model.

A deficit model diminishes the agency of out-of-field teachers. It is reductionist, not recognising the complexity of the field. Research into incidence and impact of teaching-out-of-field, for example, uses isolatable quantifiable measures such as the teacher’s highest public qualification in a subject and students’ high stakes assessment outcomes. As Ingersoll admits, ‘how one chooses to define and measure out-of-field teaching makes a difference to the amount of out-of-field teaching one finds’ (2019, p. 21). Goldhaber and Brewer (2000) claim that the inconclusive findings regarding the impact of out-of-field teaching on student achievement in their longitudinal study involving over 7000 students and 3000 teachers should ‘cast doubt on the claims of the educational establishment that standard certification should be required of all teachers’ (*ibid.*, p. 141). The blunt tool of high-stakes exam results fails to take into account the range of knowledge and experiences recognised in Hobbs *et al*.’s framework (2020, section 3.1.1). Additionally, the impact of out-of-field teaching reaches beyond student exam results as explored by du Plessis (2013) in her study of the lived experiences of in- and out-of-field teachers, leaders and parents.

The lack of understanding in the literature of why out-of-field teaching exists (Ingersoll, 2003) involves all three of these issues, the deficit model, the failure to acknowledge agency and reductionism. Whenever teaching-out-of-field is timetabled, a leadership decision has been made and a teacher has agreed, or vice versa a teacher has requested and leaders have agreed. The agency of the teacher in choosing to teach out-of-field is mostly overlooked in the literature (see section 3.1.1), being seen instead as the decision of leaders at national, regional and school level. For example, Qin and Brown (2019) identify US government policy as the cause of inequitable distribution of suitably qualified teachers, and Allen and Sims suggest US government policy causes shortages of teachers of particular subjects (2018a). Shah *et al*. (2020) identify curriculum design and recruitment strategies in university faculties, and Ingersoll (1999, 2002) and Goos and Guerin (2021) point towards school principals' recruitment behaviours. Other studies suggest out-of-field-teaching to be the result of abstract factors, such as the nature of individual subjects (Luft *et al.*, 2020), the low status of teaching (Ingersoll, 1998, 2002) and an education system that creates and privileges discrete subjects (Hobbs and Törner, 2019b; Hobbs and Porsch, 2021). The only research I found exploring the reasons given by teachers for teaching out-of-field was Barańska and Zambrowska's (2022) study which asked Polish teachers why they had studied for a qualification to teach mathematics as a second subject. Further research is needed, research that makes no assumptions about whether out-of-field teaching was a positive or negative choice and acknowledges that it is likely a complex mix of both. While such research is beyond the scope of this study, participants' reasons for teaching mathematics out-of-field are considered in section 8.1.2, to allow further insight into participants' axiology of mathematics.

3.2 Conceptualising teaching mathematics out-of-field

This thesis uses two of Bernstein's theories to explore the boundary work undertaken by out-of-field teachers, recontextualisation (1990) and classification of knowledge (1973). Recontextualisation was explored in depth with respect to mathematics in section 2.2 and will be briefly summarised here. I will then introduce Bernstein's theory of classification of knowledge, particularly collection codes, before considering them in the context of teaching-out-of-field.

3.2.1 Using Bernstein to theorise boundary-crossing

Recontextualisation is the term used by Bernstein to describe the transformation of knowledge between different sites and groups of people (Ivinson and Duveen, 2006). In Bernstein's (1990) theory, knowledge is transformed from the field of production, where it is created, into the field of reproduction, where it is shared. As introduced in section 1.1.1, he considers the former to be the creation of new knowledge in academic and professional settings and the latter to be in educational contexts, especially schools. Section 2.2 used this theory to consider how knowledge could be recontextualised at the boundaries within and beyond mathematics, reinforcing Moore *et al*.'s (2006) celebration of Bernstein's theories as having the power to explore, analyse and engage with a broad range of contemporary contexts and debates.

Bernstein (1973) distinguishes between two broad types of curriculum: collection and integrated. In the latter, subjects have an open relationship to one another. The alternative is where there is a bounded, insulated collection of favoured content which needs to be collected in order to satisfy criteria of evaluation. Bernstein analyses these two curriculum types. One way he does so is framing. Framing is about the transmission of knowledge, focusing on pedagogical knowledge and so of less relevance here. The other way he analyses the two curriculum types is by classification. Classification refers not to the content being classified, but to the strength of the boundaries between the concepts. In the visual language of the figures in section 2.2, a weak boundary was represented by a dotted line and blurring between colours, while a strong boundary was represented by a solid line. Bernstein (1973) identified the English system as having exceptionally strong classification, exemplified through the nuances of pure and applied content and ability grouping. In one of his last publications, written as the National Curriculum was being introduced, Bernstein (2000) confirmed that the English system maintained its exceptionally strong collection code. In my experience, this remains an accurate description of secondary education in England.

There is insufficient space to consider the critiques of his high-profile body of work in full. Here I explore relevant critiques of Bernstein's work and theories. In his edited collection of essays responding to major concerns about Bernstein's work, Sadovnik (1995) noted that the problematising of his work was testament to the importance of Bernstein's contribution. The most pervasive critiques of Bernstein's work relate to his work on language use and elaborated and restricted codes (Labov, 1972). Although these theories are not used in this thesis, it is relevant that Labov was accusing Bernstein of using a deficit model. As this thesis eschews a deficit model in favour of an opportunity model, this accusation requires attention. Dimitriadis and Kamberelis (2006) argue that it is a misinterpretation, and that what Bernstein did was to expose the tension between working towards equitable education and simultaneously respecting the home culture of learners. The misinterpretation runs deep. Bolander and Watts (2009) quote the German Wikipedia entry for Bernstein describing him as having established 'die Defizithypothese in der Soziolinguistik' (the deficit hypothesis in sociolinguistics). They suggest that differences arise because of the social-cultural context of the reader, rather than what Bernstein wrote. In later life Bernstein (2000) distanced himself from sociolinguistics, refuted working from a deficit perspective, and noted that he had not worked in the field since the 1970s. He also noted that while the deficit debate was 'of little theoretical significance', it did lead 'to educationalists and teachers having to re-examine their value assumptions, expectations and methods' (*ibid*. p. 148-9). This is more opportunity- than deficit-model.

Encountering Bernstein had that effect on me. Reading his works challenged my thinking and changed my perspective. For example, it puzzles me when people express surprise that I have apparently incongruent degrees in history and mathematics. From reading Bernstein I came to understand this perspective as being the product of a culturally constructed strong collection code. Bernstein's theory of collection and integrated curricula empowered me to question the existence of discrete disciplines, and to articulate why I am not surprised that someone could have passion for both history and mathematics (Yardley, 2022). My response reflects two features of his work that make it influential. Not only does he challenge people to think about a particular agenda, but his abstract writing style means that his work had the potential to link to a broad range of educational and social processes (Dimitriadis and Kamberelis, 2006). Bernstein's general reach and application can make his work difficult to read, but means that it has the capacity to be applied in remarkable variety (Moore *et al.*, 2006).

Bernstein's use of the term 'code' has also been critiqued. Bolander and Watts (2009) understand Bernstein to be using it as a general way of describing social behaviour as understood semiotically, instead of the normative labelling his critics portrayed it as (for example Labov, 1972). Atkinson (1995) suggests viewing the concept of codes within the structuralist tradition, as a structuralist after Levi-Strauss. Drawing on an analogy created by Levi-Strauss, Atkinson suggests that Bernstein was not a structuralist looking at how the jigsaw pieces fit together, but instead was looking at the mechanism which drove the jig. Reflexively this appeals to me. As well as it being pleasing and reassuring for Levi-Strauss to have influenced Bernstein's codes, the jigsaw analogy resonates with how I conceptualise mathematics: the images in Figure 2.5 which I find intriguing and beautiful, and my reflections in section 1.1.2 on always wanting to know *why* rather than *how* when learning mathematics. Bernstein's theories of recontextualisation and classification of knowledge provide appropriate structure and freedom through which to explore how out-of-field teachers conceptualise mathematics.

3.2.2 Boundary crossing by out-of-field teachers into and within mathematics

Teaching out-of-field exists because there are boundaries between school subjects (Hobbs, 2013a, 2013a; Hobbs and Porsch, 2021; Hobbs and Törner, 2019c). Crossing these boundaries, the act of recontextualisation, is challenging in a context with an exceptionally strong collection code, as Bernstein (2000) characterised the English system. Bernstein's focus is on how knowledge is distributed and organised (Sadovnik, 1995), critically locating power and control in the boundaries between subjects which are cultural constructs (Bernstein, 2000). It is this cultural construction which determines the challenge that out-of-field teachers face when working at the boundaries between and within subjects. Akkerman and Bakker (2011) identify that a cultural context with a strong collection code focuses on differences rather than samenesses. A by-product of this is a strong sense of subject identity (Hobbs and Törner, 2019b) with individuals internalising the idea of difference. This makes it harder to move between disciplines.

Out-of-field teachers' work moving between disciplines could be described as multidisciplinary, interdisciplinary or transdisciplinary. The definitions used here draw on the work of Drake and Burns, and Helmane and Briška (2004; 2017). Multidisciplinary refers to multiple, distinct subjects or disciplines. In a strong collection code out-of-field teachers' work is multidisciplinary: they are moving between discrete subjects with clear boundaries. An interdisciplinary approach would recognise common concepts and skills between the disciplines. In the context of teaching out-of-field this could refer to a shared pedagogy, using a similar teaching and learning approach across subjects. Transdisciplinary approaches interconnect and integrate all knowledge. This would be a feature of the curriculum type that Bernstein (2000) contrasts with the collection code, the integrated code.

The opportunity model adopted by the boundary-crossing literature seeks to identify the similarities between a teacher's in- and out-of-field pedagogical knowledge, conceptualising their work as interdisciplinary. This enables a positive, constructive perspective on teaching out-of-field. For example, Bosse and Törner (2013) explore pedagogical beliefs about teaching mathematics, Goos (2015) the professional identity of teachers, and Vale *et al.* (2021) study the pedagogical epistemological beliefs of out-of-field teachers. These studies all focus on interdisciplinary pedagogical knowledge, the idea that pedagogy is transferrable between subjects, often expressed colloquially as 'I teach *children*, not a subject'. I concur with Ingersoll that this misrepresents the complexity of teaching (Ingersoll, 1998, 2001b, 2003). It could be seen as a throwback to the generic professional studies trend that Shulman (1986) decried, and whose work influenced the now widespread appreciation of the importance of pedagogical content knowledge. In Bernstein's (1973) language, the recontextualisation work of the out-of-field teacher is made easier by focusing on the weaker framing between subjects (pedagogy, or transmission of knowledge), rather than the stronger boundaries between disciplines.

Out-of-field teachers also cross boundaries within their out-of-field subject. Teachers will never come to their out-of-field subject completely new. They are knowers, learners and doers of everyday and school forms of it and have experience as observers of the pedagogical forms of it. This is especially the case with a compulsory subject such as mathematics. To qualify as a secondary teacher in England, candidates must have GCSE or equivalent qualifications in mathematics and English, but a GCSE in the subject they are intending to teach is not mandatory (DfE, 2023a). I did not find any research that considers the recontextualisation of out-of-field teachers' knowledge of the subject they are teaching out-of-field.

3.3 Conclusion

This chapter has considered how teaching-out-of-field is conceptualised. It began by introducing Hobbs *et al*.'s (2020) conceptual framework before reviewing the out-of-field literature, critically analysing it to inform an opportunity approach which acknowledges the complexity of teaching out-of-field and gives those teachers voice. The second part of the chapter problematised Bernstein's theories of recontextualisation and classification of knowledge before using them to review the out-of-field boundary crossing literature.

The complexity and agency embedded in Hobbs *et al.*'s (*ibid*.) conceptual framework makes it appropriate for an opportunity-model study. It also makes it a suitable complementary partner for the complex framework of conceptualisations of mathematics developed in chapter 2 and summarised in Table 2.3.

Using these two frameworks together will be greater than the sum of its parts, a classic definition of complexity (Holland, 2014). When interpreting data of out-of-field teachers' thoughts about mathematics, the multiple and varied dimensions of mathematics can be explored alongside the multiple and varied dimensions of teaching out-of-field.

These first three chapters have established theoretical and conceptual frameworks for considering how out-of-field teachers of mathematics conceptualise knowing, learning and doing mathematics. The next chapter introduces and justifies a methodology for accessing the symbiotic, creative, messy work that out-of-field teachers are doing at the boundaries of mathematics, a methodological approach that enables an opportunity model, acknowledges the complexity of teaching out-of-field and gives teachers voice.

4. Accessing out-of-field teachers' conceptualisations of mathematics: Methodology

Methodology and research design is the focus of this chapter. Section 4.1 theorises research design, considers how other researchers have attempted to access conceptualisations of mathematics, and justifies the bricolage as appropriate for this study. The second section, 4.2, describes the research design of this study and introduces the participants.

My positionality is threaded throughout this chapter and thesis. Working in a complex field, I have a moral and ethical imperative to attend to how I am implicated in this research (Davis, 2008). Critical research acknowledges that the researcher inhabits a position of power (Telles, 2000). As a doctoral researcher and university lecturer, I magnify the academic and political power implicit in my white, middle-class status (Denzin and Lincoln, 2000; Kara *et al.,* 2021). Being a former out-of-field teacher who is now in-field I also have professional power with respect to my field. My relationship as researcher with the researched, my participants, is complex (Paradis, 2013; Sharp, 2019), and the meaning of data is not independent of location in time and space (Luitel and Taylor, 2011). In Table 3.1 I positioned myself using Hobbs *et al.*'s (2020) definition of out-of-field teaching. I was an out-of-field teacher, through personal choice, and am now an in-field teacher of mathematics.

Table 4.1 records some thoughts about how I currently conceptualise mathematics, using the mathematics conceptual framework (Table 2.3) to locate me as researcher in time and space.

Table 4.1 How I currently conceptualise mathematics

	Knowing (ontology)	Learning (epistemology)	Doing (axiology)
My conceptualisation of mathematics	Mathematics is fallibilist, a culturally created way of describing our world.	All can learn and enjoy mathematics. Important to empathise with learners.	Is intrinsically beautiful and creative. Should only be done if valued by the doer.

The complexity of the field makes accessing participants' conceptualisations of mathematics challenging (Kaasila, Hannula and Laine, 2012). Section 4.1.1 reviews how other researchers have attempted this task. It is followed by a justification of the bricolage as appropriate for this study (section 4.1.2). Section 4.2 describes the research design of this study and introduces the participants.

4.1 Theorising research design

Research design, methods and methodology are often confused by researchers (Cohen, Manion and Morrison, 2017). Crotty's (1998) four elements of research model (Figure 4.1a) has saved many researchers from throwing these elements together 'in grab-bag style' (*ibid*., p. 3). In Crotty's model, methods (techniques or procedures used to collect and analyse data) are governed by methodology, the thought behind the choice and use of methods of data collection and analysis. A theoretical perspective lies behind the methodology which is informed by the researchers' epistemological stance.

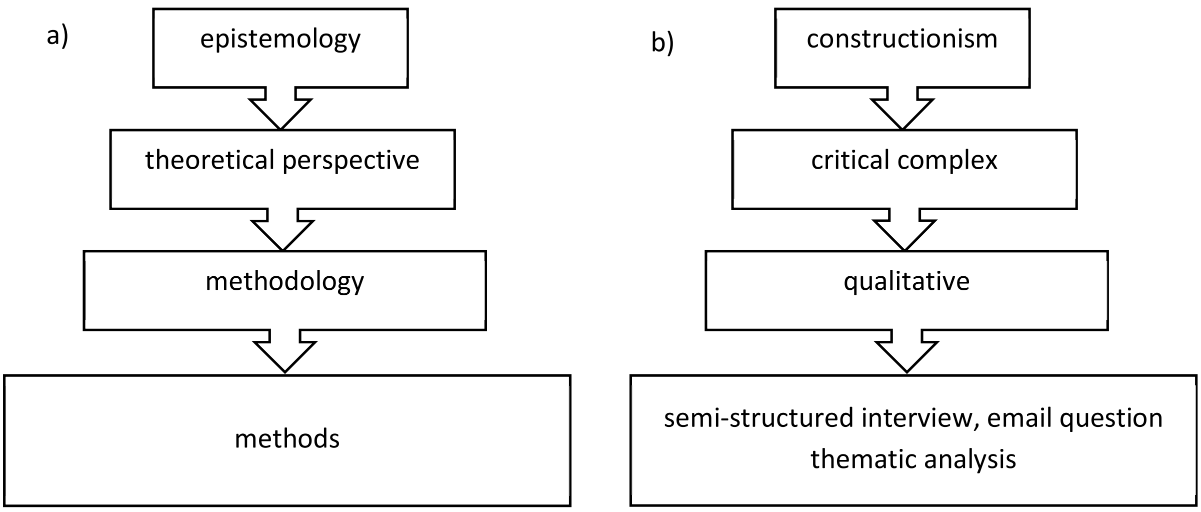


Figure 4.1 Crotty's four elements of research (Crotty, 1998)

Figure 4.1b populates Crotty's model for this study. Consistent with a fallibilist epistemology of mathematics (Ernest, 1991), the epistemological foundation of this thesis is constructionism: meaning is not discovered but constructed (Crotty, 1998). The critical complex theoretical perspective was introduced in section 1.2.1. This study is a bricolage, and so might be expected to appear in Figure 4.1b. However, the bricolage is a philosophical approach, not a methodology. As section 4.1.2 will explore, the bricolage is open to all methodologies and methods, and the final two elements of Figure 4.1b summarise the methodology and methods I selected using the rigour and flexibility of the bricolage as the study developed.

The methodology is described in Figure 4.1b as qualitative. The contents of the qualitative methodologies chapter in Cohen *et al*.'s (2017) comprehensive handbook of educational research relates most closely to the thinking that informed my methods of data collection and analysis. As a critical researcher I wanted to avoid being shackled by a closely defined, rule-bound methodology (Yardley, 2023b). Cohen *et al*.'s description of qualitative research as 'a loosely defined term that includes a vast range of kinds of research, has a wide range of meanings and covers a heterogeneity of fields' (*Op. Cit*. p. 287) allows the flexibility and responsiveness this study requires. Hammersley (2013) concludes that he does 'not believe that "qualitative research" is a genuine or useful category' (*ibid*., p. 99), suggesting a weak collection code, a field which is poorly insulated from other research fields. This makes it consistent with the theoretical framework and the research focus of this study. The choice of semi-structured interviews and email questions as data-collection method and a data analysis approach based on thematic analysis will be described and justified in sections 4.1.1, 4.2.1 and 4.2.2. The first of these sections considers how researchers have accessed conceptualisations of mathematics; the other two justify the selection of the methods of data collection and analysis used in this study.

4.1.1 How conceptualisations of mathematics have been accessed

Accessing conceptualisations of mathematics is not simple. Brown and McNamara (2011) noted the paucity of data they collected when directly asking 'what is maths?'. This section reviews methods employed in the field of mathematics education to access conceptualisations of mathematics. Data collection found in the literature can be roughly categorised as questionnaire, observation, interview and creative approaches, sometimes combined as mixed methods.

Questionnaires or surveys have been used to test mathematics skills and invite self-report about attitudes to mathematics. A significant example of a study testing mathematics skills is the Learning Mathematics for Teaching project (Ball *et al.,* 2008), a long-term project testing American elementary school practitioners' knowledge of school mathematics. Questionnaire design is complex (Hobbs *et al.*, 2012), as is the administration, analysis and interpretation. Even carefully designed questions reflect the researchers' own conceptualisations. In the case of mathematics skills, questions often test participants' capacity to replicate established approaches to school mathematics, measuring replication of processes rather than accessing conceptualisation (Fauskanger, 2015). Attitude self-report Likert scale and multiple-choice questions assume that participants share the researchers' understanding of the item under scrutiny. For this reason, the mathematics education literature tends to avoid positivist analysis of questionnaire data, instead using mixed methods approaches to back up questionnaires with observations or interviews.

Studies using observation attempt to access how teachers and learners conceptualise mathematics through how they explain, do and assess mathematics. These vary in scale from Borko and Eisenhart's (1992) study of part of a lesson where an elementary teacher failed to justify fraction division to the extensive data collected by Rowland *et al.* (2004) to devise the Knowledge Quartet. Lesson study, a cycle of joint planning, mutual observation, and analysis, has been used effectively to unpack conceptualisations informing pedagogical actions (Khalil, Lake and Johnson, 2019). Concept study (Davis and Simmt, 2006) combines lesson study with concept analysis, allowing a focus on mathematical concepts, not just its pedagogy (Davis and Renert, 2009). Studies using observation often take a grounded theory approach, using thematic analysis within interpretivist theoretical frameworks. Most studies observe mathematical activity in the classroom, and so focuses more on pedagogical mathematics than other forms.

Interview as a data collection method has the capacity to focus on and explore aspects of how participants conceptualise mathematics and how they came to those conceptualisations. Many focus on professional experiences and pedagogical conceptualisations, often through a lens within the affective domain, such as anxiety (García González and Sierra, 2020) or agency (Lu, Leung and Li, 2021). By using narrative approaches such as life history or lived experience, participants' conceptualisations are recognised as dynamic and reflective of the complexity of personal disciplinary knowledge and critically acknowledge the personal interpretations of both researcher and researched. Du Plessis' study of the lived-experience of out-of-field teachers is an example of this (du Plessis, 2013, 2015b, 2020b; du Plessis *et al.,* 2015; Hobbs *et al.,* 2019). Theoretical perspectives employed by researchers using interview tend to acknowledge the impact of the researcher on data collection and analysis. They seek to illuminate, not generalise.

Creative approaches such as researcher and participant jointly doing, observing or analysing mathematical work have been used to access conceptualisations of mathematics (Armstrong, 2017; Buerk, 1982; Davis *et al.,* 2008; Hodgen, 2011; Hughes and Greenhough, 1998). Researchers have used tools to indirectly reveal conceptualisations of mathematics, such as fortune line (Hobbs and Quinn, 2020), word association (Arzi and White, 2008), and vignettes (Boylan, 2016), as well as artifacts from the classroom (Chan and Yung, 2018; Hodges and Cady, 2012) and beyond (Hottinger, 2016), and can take both ethnographic and autoethnographic approaches to analyse (auto)biographical accounts of mathematics (Hekimoglu, 2010; Luitel, 2020). Studies have analysed objects in the public domain for conceptualisations of mathematics, such as magazines (Hall, 2021), stamps (Hottinger, 2016) and the American sitcom Big Bang Theory (Ausman, 2018). The bricolage has been used to access understandings of learning (Armstrong, 2017), functional mathematics in the classroom (Howley, 2013), identity (Gine, 2011), and the cultural context of learning mathematics (Luitel and Taylor, 2007). Some of these studies access conceptualisations of everyday mathematics as well as school and pedagogical mathematics.

Of these approaches, questionnaires are not appropriate as they do not allow detailed exploration of conceptualisations, and observation is not appropriate as it would privilege pedagogical mathematics. Creative and flexible data collection methods have the capacity to access the evolving, complex conceptualisations of out-of-field teachers who use their existing conceptualisations of mathematics and their own subject(s) creatively and flexibly (Ní Ríordáin, Paolucci and Lyons, 2019). Doing so in the context of a semi-structured interview provides a context which allows flexibility and creativity in exploring participants' conceptualisations of mathematics.

4.1.2 The Bricolage

I define the bricolage as a philosophical approach to research that is open to all methodologies, continuously and rigorously selecting and adapting methodologies and methods to suit the focus of the research. I begin by introducing the conceptualisation of the bricolage used in this thesis and then explain why it is an approach to research consistent with the critical complex principles used in the thesis: an opportunity model that acknowledges the complexity of the work of out-of-field teachers and allows their voice to be heard.

Levi-Strauss (1966) was the first to use the word bricolage in an academic context, developing the concept of intellectual bricolage in what Derrida (1978) describes as a remarkable endeavour. In Levi-Strauss' bricolage the individual uses their own forms of knowledge for the task in hand because there is nothing else available (Lincoln, 2001). The focus of intellectual bricolage is knowledge, and out-of-field teachers are bricoleurs in this sense (Kincheloe, 2004). Out-of-field teachers use the knowledge to hand, are adept at performing many diverse tasks, and are always putting something of themselves into their work (Levi-Strauss, 1966). Levi-Strauss contrasts bricoleurs with engineers who subordinate tasks to the tools available, tools with only one use. The forms of mathematics that out-of-field teachers are likely to have to hand are school and everyday mathematics, as well as the pedagogical mathematics that they have observed as learner and colleague.

The potential of Levi-Strauss' bricolage as a research methodology was first identified by Lincoln and Denzin in The SAGE Handbook of Qualitative Research (2000). Kincheloe (2001) was fascinated by their use of the term, leading to its iterative development through further interactions and subsequent SAGE Handbooks (Kincheloe, 2001, 2005; Denzin and Lincoln, 2005, 2011, 2017). This thesis uses Berry and Kincheloe's conceptualisation (2004), over Denzin and Lincoln's (2017), as it allows the adaptation of methods and methodologies to suit the research, just as out-of-field teachers use and adapt their existing conceptualisation of mathematics. Rather than taking a uni-disciplinary approach (Kincheloe, 2001) or designing research around a particular methodology (Denton, 2022; Guba and Lincoln, 1982), bricoleurs select methods and methodologies as appropriate to reflect the heterogeneity of the human experience, in this context the heterogeneity of ways of knowing, learning and doing different forms of mathematics.

The bricolage supports an opportunity model. A bricoleur is a magpie (Yardley, A. 2019), finding methodologies in expected and unexpected places, adapting their approach as new threads and themes develop, closely observing and listening, looking for what is missing as well as what is there (Paradis, 2013). Methods and methodologies can be selected as appropriate for the data rather than the methodology informing the method of data collection (Denton, 2022), and develop with the research rather than rigidly sticking to one methodology (Pinar, 2001). No methodology is off-limits (although there may be methodologies inappropriate or unethical in some contexts). This interdisciplinarity does not mean the absence of disciplines (Pinar, 2001). On the contrary, it heightens the awareness of the researcher to the blurred (*ibid*., 2001) and fragile (Lincoln, 2001) boundaries between methodologies. Like out-of-field teachers, bricoleurs approach new fields from multiple and competing perspectives with fresh eyes (McLaren, 2001; Rogers, 2012), being aware of differences and open to complexity and developing a sophisticated understanding of the nature of knowledge (Kincheloe, 2001; Paradis, 2013), making visible the invisible (Luitel and Taylor, 2011).

As an opportunity model, open to all methodologies and methods, it is crucial that appropriate methodologies are selected with rigour and knowledge (Berry and Kincheloe, 2004; Denzin and Lincoln, 2011; Sharp, 2019). This rigour is reflected in the definition of the bricolage used in this thesis. The bricoleur is often defined as a handyman or Jack-of-all-trades, using the tools to hand.

This definition is too simplistic (Rogers, 2012; Paradis, 2013; Sharp, 2019). Kincheloe never used the term handyman (Paradis, 2013): his bricoleur is 'far more skilled than merely a handyman' (Lincoln, 2001, p. 693). I have chosen not to use handyman or Jack-of-all-trades in my definition, because of their inherent sexism, and because they fail to communicate the rigour with which tools are selected, adapted, and developed over time in an eclectic, incremental, instinctive progression (Berry and Kincheloe, 2004; Campbell, 2018). The bricolage is not a separate or distinct methodology: it is an openness to all methodologies and readiness to adapt them. It is an opportunity model.

The bricolage acknowledges the complexity of teaching-out-of-field. Like out-of-field teachers, bricoleurs undertake challenging boundary work, working at and across intersections between disciplines (Kincheloe, 2001; Hobbs, 2013b, 2013a). This raises concerns of superficial understanding of methodologies (Kincheloe, 2001) and postmodern eclecticism (Lincoln, 2001; Paradis, 2013). Avoidance of superficiality and postmodern eclecticism is dependent on the bricoleur selecting methodologies and methods appropriately (Sharp, 2019), relating the methodology to the context of the field of study (Tobin and Kincheloe, 2015). The bricolage is more than a mixed methods approach. Mixed methods research uses a single methodology and multiple methods. The bricolage uses multiple methodologies, and so could potentially have multiple theoretical perspectives and epistemological roots, which Crotty (1998) warns against. Through its critical complex theoretical perspective (Berry and Kincheloe, 2004), the bricolage acknowledges that knowledge produced within any paradigm reflects its historical and geographical context and is equally open to interpretation, even knowledge produced in an empirical epistemology of certainty (Berry and Kincheloe, 2004; Morrison, 2008; Rogers, 2012; Paradis, 2013). The bricolage recognises that all contexts are unique and fluid and does not seek to summarise or generalise (Berry and Kincheloe, 2004).

Potentially working with diverse data, the bricoleur must be aware of the blindnesses and limitations of the methodologies and methods they employ (Kincheloe, 2008), and that it is not possible to pre-empt all limitations. Some limitations will become apparent as knowledge is produced (Mclaren, 2001; Denton, 2022), and the bricolage enables the researcher to shift between, adapt and use multiple methods and methodologies as the study develops (Paradis, 2013). This increases the bricolage's capacity to explore complexity critically, allowing the researcher to access dimensions that may otherwise be overlooked (Vandenbussche, Edelenbos and Eshuis, 2019). Kincheloe refers specifically to the cosmos and radical love as dimensions often overlooked in the Western academic tradition (Berry and Kincheloe, 2004; Kincheloe, 2001, 2005, 2008b). Paradis' bricolage uses tarot readings as data (Paradis, 2013) and mine draws on personal spiritual experiences (Yardley, 2022). By its openness to such dimensions, the bricolage increases critical access to complex, heterogenous conceptualisations of mathematics.

The bricolage attempts to give participants a voice, but also acknowledges that the voice of the researcher cannot be eliminated. The assumptions of the researcher and the time and space within which they are researching influence every aspect of the research (Kincheloe, 2008). From the inception of my doctoral studies, I have been aware of my positionality. It directly informed my focus and how I selected, read and used the literature. It informs every aspect of research design, data collection, analysis and discussion. I believe it to be morally wrong to claim a study has been designed to diminish or eradicate the impact of the researcher's positionality and have used a methodology that, instead of mitigating against the impact of the researcher's positionality, enabled me to take ownership of and embrace my positionality.

4.2 Research design

The bricolage is a philosophical approach that is open to all methodologies, continuously and rigorously selecting and adapting methodologies and methods to suit the focus of the research. In this section I outline the qualitative methods used to collect and analyse data.

4.2.1 Methods of data collection

The main method of data collection used in this study is semi-structured interviews with creative tools on hand to be used as appropriate. In the light of section 4.1.2, semi-structured interviews and creative methods appear to be the methods of data collection most appropriate to this study. Semi-structured interviews allow flexibility to respond to new layers and knowledge.

The structure of the interviews follows the philosophical dimension of the mathematics conceptual framework (Table 2.3). There were three elements to the semi-structured interview, bullet-pointed rather than numbered as they were taken in the order they arose:

- You as a learner of mathematics
- You as a knower of mathematics
- You as a doer of mathematics

You as a learner of mathematics

The aim of this section of the interview was to explore participants' historical and current lived experiences as learners of mathematics. It was often the first element covered because of its autobiographical nature. An initial prompt was for participants to share their timeline as a learner of mathematics. This is an 'innovate method of graphic elicitation' (Sheridan *et al.,* 2011 p. 552) which builds on the concept of lifecourse as a tool for understanding how people experience transitions through their life (Hogg, 2013). Sheridan *et al*. (2011) use a timeline pre-populated with the participants' personal data: I adapted the tool to ask participants to construct a timeline of their confidence in learning mathematics (y-axis) through their lives (x-axis).

You as a knower of mathematics

Ontological assumptions are difficult to access. By their nature they are embedded in our unconscious thinking (Kincheloe, 2001). I designed two data collection tools for this element, using photographic images to attempt to elicit participants' deepest thoughts. The choice to use images came from personal experiences which enabled me to explore my own thinking. In one of these I was guided to select from a set of pictures to reflect on metaphors for my identity as a researcher (Durrant, 2022) which reminded me of a similar device used during a Christian retreat (Yardley, 2022). Selecting images enabled me to access unconscious beliefs. This inspired the ‘pick an image’ data collection tool (Figure 4.2). Another experience that led to the use of images as an elicitation tool was reading and re-reading Hottinger (2016), a feminist critique of gendered, racialised mathematics in which she uses a variety of artefacts including portraits and stamps to suggest that Western logico-mathematical reasoning was not the only correct way to engage with mathematical ideas. Finally, I have used a teaching strategy of displaying an image and asking, ‘where’s the maths?’ successfully with school children and adult learners many times.

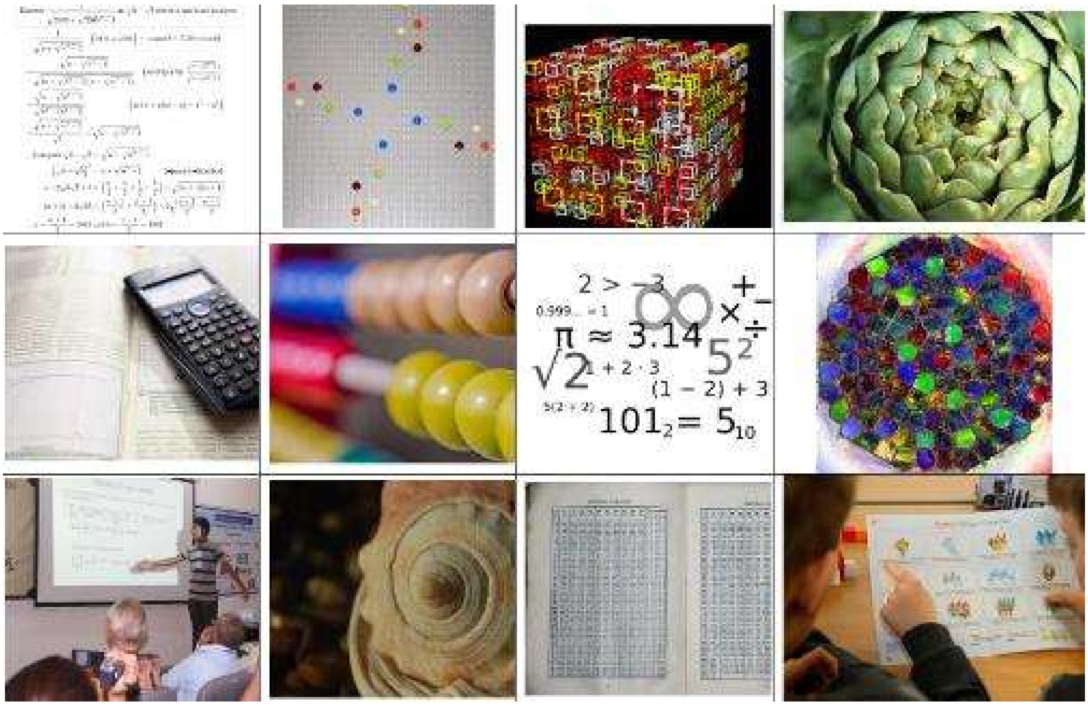


Figure 4.2 'Pick an image that says maths to you' data collection tool

I acknowledge that my positionality influenced the images I selected. I selected an image of Wembley Stadium (Figure 4.3a) as it has elicited rich responses over the years, and the Alhambra in Spain (Figure 4.3b) as its ornate decorations famously contain all seventeen tilings, one of my reasons for studying a degree in mathematics (section 1.1.2). The pictures used in the ‘pick an image’ tool were selected to depict the heterogeneity of mathematics and elicit a variety of ontological beliefs. I searched for “maths” on Google commons with the intention of taking the first twelve images. This did not produce a sample that I felt reflected the complexity of mathematics and so I picked twelve to represent a diverse range of conceptualisations of mathematics.

a) Wembley Stadium image

b) Alhambra image

Figure 4.3'Where's the maths?' data collection tool

You as a doer of mathematics

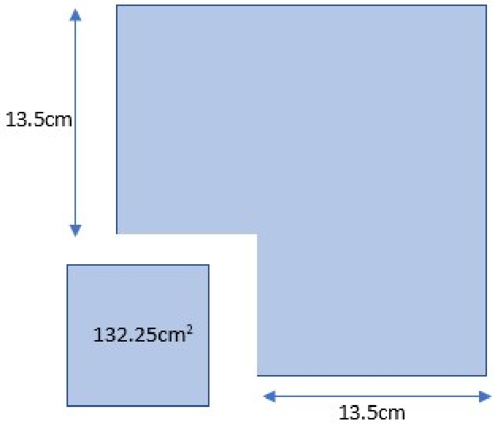
This element of the semi-structured interviews appears to take a prosaic meaning of ‘doing’: performing mathematical calculations. Considering the power imbalance and the unnatural participant interview setting (Brown and Danaher, 2019), asking participants directly what they consider to be the purpose of learning mathematics may elicit responses of what they think they should say (Brown and McNamara, 2011). To access participants’ axiological beliefs about mathematics I chose to use the tools at hand as the moment arose.

The tools used in this element of the semi-structured interview were a set of five quadratics questions (Figure 4.4) and five methods for solving a long multiplication question (Figure 4.5). The use of mathematical tasks as a data-elicitation tool was inspired by Armstrong’s study (2017) which entwines participants as bricoleurs with bricolage as methodology. She notes that participants (8^{th} grade students) defined the boundaries and then pushed them.

1. Solve $x^2 - 27x + 50 = 0$

2. I think of two numbers. I add them together and get 27. I multiply them together and get 50. What were the two numbers I was thinking of?

3. I have a square of paper. I measure 13.5cm along two sides and then cut away the corner. The piece I cut away has an area of 132.25 cm^2. What is the area of the other piece?



4. To find the correct dosage of and adult medication you have to subtract 13.5 from a person's age and then square the result. Ali's dosage is 132.25mg. What is their age?

5. Where does $y = -2x^2 + 54x - 100$ intercept the x axis?

Figure 4.4 Five questions mathematical object data collection tool

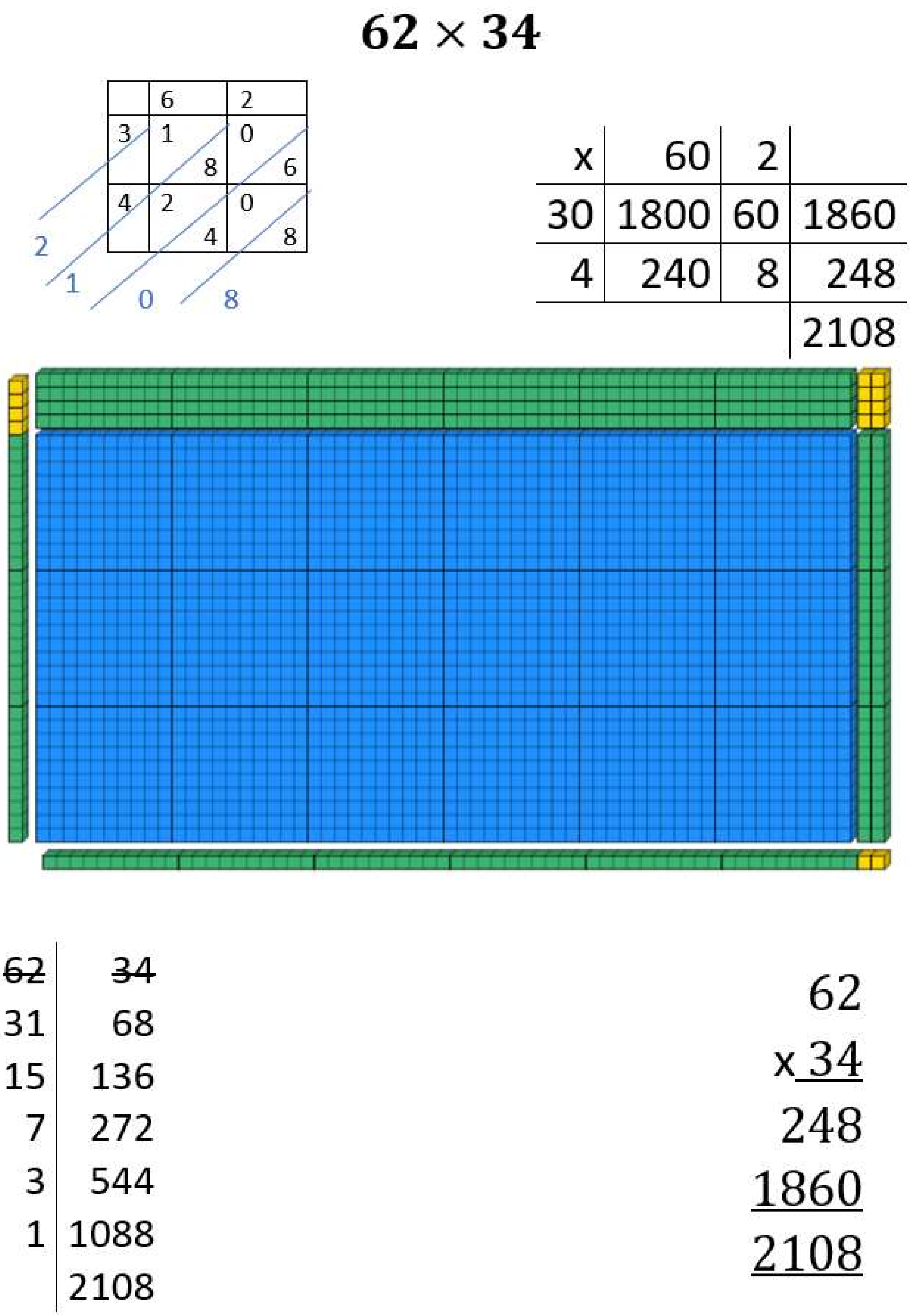


Figure 4.5 Five long multiplication methods mathematical object data collection tool

The power imbalance implicit in the research interview situation (Brown and Danaher, 2019) could potentially compound notions of fixed ability and emotions associated with doing mathematics. I felt that if I asked participants to do the questions in front of or with me, the only data I would collect would be their response to the power imbalance. Consequently, I asked 'if I was to ask you to do some maths, which one of these questions would you pick?'.

The design of the five questions reflects my conceptualisation of mathematics, my sense of awe and fun. The mathematical structure of all five questions is identical, but they are presented differently. I love that five identical questions can be manipulated to appear different. The second set of mathematical artefacts plays with this idea further: if one question can be represented in different ways, so too can one solution. The halving and doubling method, an algorithm used by pre-literate non-Western societies, was included to disrupt participants' thinking and provoke ontological and epistemological reflections about how we know what we know. With this artefact I asked participants what was going on and to talk through the different representations of mathematics.

My positionality influenced the design of data collection tools. It also influences my expression and follow up questions during the interview (Brown and Danaher, 2019). I consciously acknowledged this by annotating each transcript after typing it up, freely journalling my experience of the interview and critically analysing my contributions. These reflections became part of the data.

4.2.2 Methods of analysis

I originally embraced the bricolage's multidisciplinary nature (Berry and Kincheloe, 2004) as an opportunity to feed my hunger for new knowledge and began reading about critical discourse analysis and conversation analysis (Fairclough, 2013; Drew, 2015). Two realisations halted this approach. One was Kincheloe's warning against 'naïve overspecialisation' (2004, p. 53) in which he states that bricoleurs must go beyond studying disciplines in the traditional manner and undertake Foucauldian genealogy, a deep understanding of the discipline's knowledge bases, epistemologies and knowledge production methods. My second realisation was that even if I developed new specialisms, such an approach would be multi-disciplinary rather than inter- or trans-disciplinary (Helmane and Briška, 2017), maintaining strong collection codes (Bernstein, 1973) rather than exploring boundaries and recontextualising. Instead, I adopted and adapted tools available from my existing knowledge, deliberately becoming aware of my statistical, historiographical and education studies knowledge.

Thematic analysis and coding were the first two tools I drew on from my existing knowledge, influenced by my reading of Clare, Braun and Hayfield (2015) as well as my teaching and supervision of postgraduate trainee teachers' academic work. I began by using inductive thematic analysis, categorising data from interview transcripts as relating to knowing, learning or doing mathematics. This provided me with three smaller datasets. I immersed myself in each of these datasets, re-reading, re-listening, coding and recoding. Figure 4.6 gives two illustrative snapshots from the thematic analysis and coding process. Figure 4.6a shows on the left how I used NVivo to inductively analyse data as knowing, learning or doing, analysing it further by the subsets I encountered in the literature review (chapter 2). On the right a sample of the data (for 'doing', 'functional'). A further analysis took a deductive approach, coding responses from similar parts of the semi-structured interviews (Figure 4.6b). Although this did not structure my thesis in the way that the inductive analysis did, it helped me to immerse myself in and explore the data.

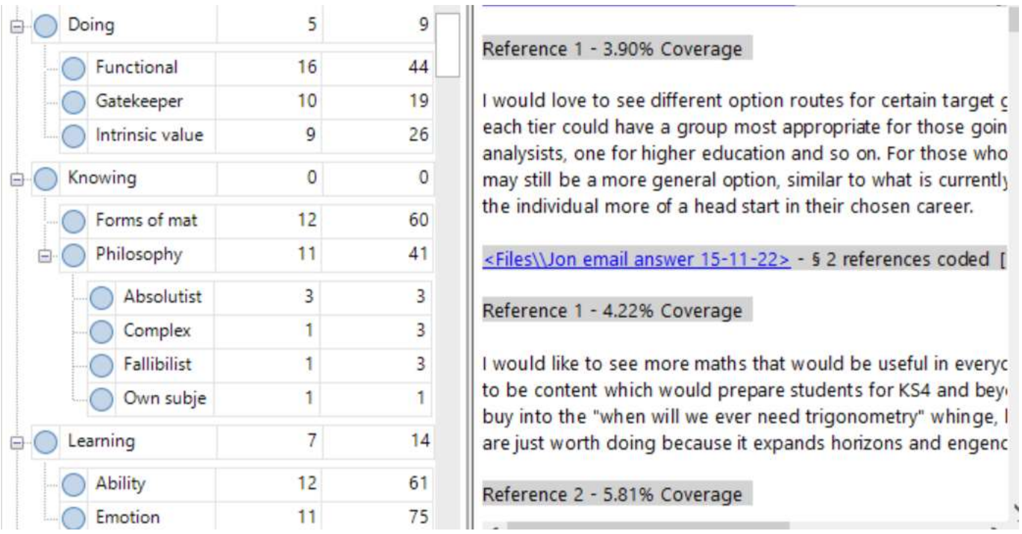


Figure 4.6a Induction thematic analysis

You as learner question	0	0
Algebra	2	2
Brick wall	2	4
Compares to English	1	2
Compares to general other	1	1
Compares to own subject	2	7
Compares to specific peer	2	3
Confidence	2	2
Enjoyment	2	2
Failure	2	3
Geometry	2	3
Imposter syndrome	2	3
Maths is hard	2	3
Natural ability or lack	1	3
Need to understand	1	2
Number	1	3
Othered	2	6
Procedure or algorithm	1	1

Figure 4.6b Deductive analysis of an element of the semi-structured interviews

Thus, the thematic analysis and coding was not a linear process. The flexibility of the bricolage, its rigorous responsiveness to the data, meant that I could work iteratively, adapting my approach and reaching for new tools. One example of the iterative process in action was how, when immersing myself in and coding the 'doing' dataset, I found myself discussing with participants in my head what they thought should be included in the mathematics curriculum. A tool to hand was my ability to get back in contact with participants to ask them directly. I emailed participants to ask them what content they would advise the Education Secretary to include in the key stage 3 mathematics curriculum. Their responses became part of the dataset.

Another example of the iterative process is how I adjusted the structure of chapters in my thesis. My intention in using inductive thematic analysis to form the three smaller datasets was that each smaller dataset would inform each of three chapters, one exploring knowing, one for learning and one for doing mathematics. As I worked with the datasets, taking my non-linear, iterative approach, the idea of chapter 5 evolved. This was to be a chapter that considered the forms of mathematics dimension of the mathematics conceptual framework, to explore how the data could be interpreted as suggesting a conceptualisation of mathematics as school mathematics through a student lens. As

I inhabited the data, re-reading and re-listening, I reached for tools to hand, forming categorisations, spotting patterns. A pattern is where a participant or several participants repeatedly generated similar data. The form of analysis and presentation of these patterns (tabulating, superimposing, counting, proportional measures) depended on which statistical forms of analysis in my toolbox I felt to be most appropriate to the context. I was also interested in ideas relating to only one participant. For example, there were two occasions when participants gave unique, passionate and rich responses to a data elicitation tool. These formed the basis for analysis and discussion as much as patterns, in this case a form of analysis my akin to working with historical primary sources.

I want to make explicit the absence of the word or connotations of 'emergence' and my careful use of the first person. Clarke, Braun and Hayfield (2015) note that to suggest that themes and codes 'emerge' is to assume that they exist within the data waiting to be found. Constructionist analysis and interpretation of data is set in space and time (Kincheloe, 2008; Luitel and Taylor, 2011). If someone else were to work with the same data, or I were to work with it at a different time, they or I would attend to different patterns and ideas (Mason, 2003).

4.2.3 Participants

The decision to work with participants who have taught for six or more years and have a degree and teaching qualification in a subject other than mathematics was made to minimise variety in a complex field (Campbell *et al.,* 2019; Hobbs *et al.,* 2019). As my focus is on personal conceptualisations of mathematics rather than professional or pedagogical, experienced teachers with established professional identities were sought (Hobbs *et al*., 2019). I introduce my participants first by defining their out-of-fieldness using Hobbs *et al.*'s (2020) framework (Table 4.2). I then provide a pen portrait of each participant (using pseudonyms) (Figure 4.7), before reflecting on their similarities and differences.

Table 4.2 Participants positioned on Hobbs et al.'s 2020 framework of the dimensions of teaching out-of-field (all names are pseudonyms)

Cluster	Criterion	Dimension	Zoe	Liz	Clare	Pete	Lee	Ross	Nik	Jon	Ben	Emil
Measurable Criteria	1. Qualification	1.1 Technical Alignment										
		1.2 Specialism Alignment										
		1.3 Phase Alignment										
	2. Workload	2.1 Current proportion										
		2.2 Longitudinal proportion										
		2.3 Stability										
	3. Capability	3.1 Expertise										
		3.2 Career stage										
Self-report criteria	4. Identity	4.1 Commitment										
		4.2 Self-concept										
		4.3 Confidence										
	5. Structure	5.1 School context										
		5.2 School support culture										
Longitudinal criteria	6. Pathways	6.1 Trajectories										
		6.2 Role expansion										

Zoe is a geography specialist who has been a head of humanities in several schools. Her current school has a small fluctuating roll. Consequently, most teachers are required to teach out-of-field. Zoe has previously taught history out-of-field and began participating in this study just before taking on two year 8 mathematics lessons each week and beginning a secondment as associate assistant headteacher.

Liz is a head of physics and associate assistant headteacher at a high achieving state comprehensive. She began teaching single lessons of mathematics to four separate classes two weeks before beginning participation in this study. She has been told that this is a temporary measure while the school addresses a staffing shortage in the mathematics department.

Clare is a recently retired religious studies (RS) teacher. Since her retirement she has returned to her former school to cover lessons and support learners. She has taught mathematics and worked with individual or small groups of learners with special educational needs. As she is a trusted and respected former colleague, teachers often provide little guidance when she takes their lessons.

Pete is a music specialist who discovered a love for teaching in a special needs context. He applied for a job at a school he was so keen to work in that he was not concerned what subject he would be teaching. Still at the same school, Pete has a diploma in mathematics from the Open University, is head of mathematics and is an experienced mentor of trainee mathematics teachers.
Lee is a physical education (PE) specialist and head of year. He was asked about eight years ago to teach some mathematics to bring 'something a bit different' to the subject. He is generally allocated one or two mathematics classes each year which he and his school identify as challenging groups. He is recognised as an outstanding teacher.
Ben is a PE specialist who was unable to find a job after he qualified. He did supply work and short-term contracts until the opportunity came up for a permanent teaching job, teaching mathematics out-of-field alongside Ross and Nik. He is usually timetabled to teach a mix of mathematics and PE.
Ross is a PE specialist who found there were no jobs in the subject when he qualified. He took a job in a school which was committed to supporting out-of-field teachers of mathematics (Ben and Nik started at the same time as him). He taught only mathematics in his first four years of teaching before gradually picking up more PE, which he describes as 'where my love is'. He is generally allocated groups known to exhibit challenging behaviour.
Nik is a geography specialist whose school recognised that he was 'a good teacher of anything' and asked him to teach maths (along with Ross and Ben). He mainly teaches foundation GCSE groups with occasional higher. Throughout his career he has taught more mathematics than geography, and for a long time now has only taught mathematics. He is a senior leader.
Jon is a PE specialist and now vice-principal at the school where Ben, Ross and Nik work. In middle and senior leadership he has taught various subjects at different levels as required. He currently

teaches mainly English. In the past he has taught middle-ability GCSE mathematics students, helping to boost the school's performance.
Emil has a master's degree in neuroscience and intends to pursue an academic career in psychology. He was taking a break working in a café when made redundant because of the pandemic. He heard through a friend that a local university was recruiting a foundation year mathematics tutor, and decided this would be a good way to move towards his intended career. He teaches GCSE and A-Level mathematics content to students who do not have the necessary qualifications to commence undergraduate courses.

Figure 4.7 Pen portraits of the participants

Zoe and Liz differed from the other participants as I interviewed them as they started teaching out-of-field for the first time. I met with them on more than one occasion. Our meetings were conversations about their experiences of mathematics as they began teaching it. Instead of following the structure described in section 4.2.1, we revisited previous conversations, and I only used the data collection tools when appropriate.

Clare and Emil self-identified as not fulfilling the criteria for participation. Clare meets the criteria in that her specialist subject is not mathematics and she is an established teacher, but as a cover teacher she has no timetabled mathematics lessons, and her work does not involve planning or assessment. Clare is an acquaintance who I know to be a critically reflective deep thinker. I interviewed her as a pilot with the understanding that she may not be suitable as a participant. Emil did not fulfil the criterion of being an experienced (or even qualified) teacher. I agreed to interview him for an alternative perspective, he having not been educated in this country and teaching mathematics in a non-school context. Both provided valuable data.

Of the ten participants three are female, which includes the two who were teaching mathematics for the first time, Liz and Zoe. Four are PE specialists by academic background and training (Lee, Ben, Ross and Jon). Zoe and Nik are geographers, and the others came from backgrounds in music (Pete), RS (Clare), physics (Liz) and psychology (Emil). Jon and Nik are experienced senior leaders, while the two who were teaching mathematics for the first time, Liz and Zoe, are associate assistant headteachers, a role giving them experience of senior leadership. Five are or were middle leaders, including Pete who is head of mathematics and Lee who is a head of year. Zoe, Liz and Clare were heads of department in their in-field discipline. Three of the participants were currently or had previously taught other subjects out-of-field (Jon now teaches English, Zoe also teaches history out-of-field, and Clare teaches all subjects).

Eight of the participants teach in mainstream comprehensive schools. Of the other two, Pete teaches in a special school and Emil a university. Pete and Emil were both recruited through colleagues at the university where I work. Four of the participants, Ben, Ross, Jon and Nik, teach in the same school. One of them was introduced through a mutual friend and recruited his colleagues. Zoe is a former teaching colleague, and Lee was recruited through a mutual friend. Liz and Clare both teach at the same school, the school I attended as a student and that my children attended. I explore the ethical implications of this in the next section.

4.2.4 Ethics

The human participants involved in this study were non-vulnerable adults, all volunteers, the subject matter does not relate to protected characteristics (gov.uk, 2010), data collection involved one-to-one interviews in locations chosen by the participants, and a methodological approach was used that made no claims to generalise or represent reality. Ethical clearance was obtained in line with British Education Research Association guidance (BERA, 2018) and university processes. All participants were sent an information sheet and consent form which they submitted electronically before the interview (Appendix D). This included the right to withdraw, and consent for audio recordings to be made of interviews, transcribed, stored on secure password protected hardware and deleted in accordance with guidelines. Transcripts and fieldnotes were shared with participants following interviews, and all participants were invited to read and feed back on a complete draft of the thesis. Pseudonyms were used for all participants and any people or places that they referred to.

Initial challenges in recruitment led to all participants being recruited though personal and professional contacts, which raises potential ethical implications. Recruitment of participants through personal and professional contacts could be ethically problematic because some information cannot be anonymised and because of the potential for power imbalances. I deal with each of these individually before considering the case of the two participants who taught at the school where I studied as a student, which encompasses both potential ethical challenges.

The small number of participants and the diversity this research celebrates means that any participant reading this thesis will be able to identify themselves. I could have avoided this by not identifying individual participants when sharing extracts, but this would have reduced them to a single homogenous entity and lose the complexity and diversity of teaching out-of-field. Instead, I embrace participants' ability to recognise themselves and use it to further explore their voices. We can never hear their voices, only my interpretation, but giving participants opportunities to read and respond to how I have interpreted what they said adds a layer of richness.

There is always an imbalance of power involved in the process of recruiting and collecting data from participants (Ely *et al.*, 2003). As a third party was involved in recruiting 8 out of the 10 participants, the relationship between the participant and the third party, and their perceived relationship between the third party and me, exacerbates the complexity of the power imbalance. Sometimes this distanced the participant, for example where an acquaintance of my husband put me in touch with someone who worked with them. I felt that this participant saw me as holding authority, but also felt freedom to express themselves in a one-off, anonymous interaction. This contrasts with a participant who mentors beginning teachers in our university partnership. I felt that they saw me as a peer but were more measured in their responses. The bricolage embraces the power imbalance. Rather than attempting to mitigate against power imbalance in interviews and interpretation, I actively acknowledge it.

Liz and Clare are the most identifiable participants in the study. They were both aware of this when agreeing to participate. They have both had the opportunity to read a full draft, and with Liz, Clare and I fully aware of their identifiability, we have all worked to make their voices heard as clearly as possible. I was concerned that having two participants who taught at the school I attended as a student would be traumatic for me and lead to impartial, emotional data collection and analysis. All research is emotional. We are never impartial. The bricolage acknowledges this. Nevertheless, I was nervous. Interviews with Liz took place in the school, fifty metres from my GCSE and A-Level mathematics classroom. My experiences in those classes influenced my research focus (section 1.1.2). I worried that these interviews would awake latent feelings. In a conference paper entitled *Thesis or Therapy?* (Yardley, 2023a) I concluded that this is a thesis, not therapy. I argued that every thesis is about the researcher, that there is no amount of actual or metaphorical decontamination that can eliminate the researcher's positionality. When as researcher I used my reflexive positionality to look at myself deeply, I was using a methodology that allowed me to do so consciously and rigorously and to explore a research focus that is relevant to a wide academic and professional audience.

4.3 Conclusion

This chapter has introduced the bricolage as the philosophical approach used in this thesis. It began by exploring how researchers have accessed conceptualisations of mathematics. The bricolage was justified as an appropriate approach before introducing the tools used for data collection and analysis, the participants in this study and ethical considerations.

The bricolage is defined in this thesis as a philosophical approach to research that is open to all methodologies, continuously and rigorously selecting and adapting methodologies and methods to suit the focus of the research. The approach of this thesis is to use an opportunity model which acknowledges the complexity of teaching out-of-field and gives those teachers a voice. This research does not claim to report the 'authentic voice' of out-of-field teachers. It openly and honestly presents my interpretation of how participants responded to my questions, and both interpretation and questions are personal, social and historical products. It allows an opportunity model which acknowledges the complexity of out-of-field teachers' work and attempts to hear it from their perspective. The four chapters that follow are structured to briefly revisit relevant knowledge from the first three chapters before two main sections. The first section consists of narrated extracts selected according to themes I identified in analysing the data. These are discussed in the second section, bringing together the literature and data to consider how out-of-field teachers know, learn and do mathematics.

5. School, academic, everyday and pedagogical mathematics: Out-of-field teachers conceptualising different forms of mathematics

The next four chapters use the data from ten out-of-field teachers to explore their conceptualisations of the subject they are teaching out-of-field, mathematics. In each chapter the introduction recalls the element of the mathematics conceptual framework that the chapter focuses on. There then follow three sections: analysis, discussion and conclusion. In the analysis section, extracts from interviews, field notes and email correspondence are foregrounded to amplify out-of-field teachers' voices (with the caveat that my voice as researcher is never absent – the selection, narration and ordering of extracts is mine). This is then followed by a section which engages the out-of-field teachers' voices in discussion with the literature and with me. Each chapter concludes with a section that brings together the main ideas from the discussion.

5.1 Analysis

When the four forms of mathematics used in the conceptual framework were introduced in section 1.2.2, it was noted that they were iteratively constructed from the literature, my personal experience and data collected and analysed for this study. The aim of this chapter is to offer some insight into the ideas expressed by out-of-field teachers that contributed to the construction of the classification. I make no claims for this classification of four forms to be comprehensive. The forms are not discrete, and the definitions deliberately broad and flexible. Like mathematics, they are used to model a complex world, to simplify it to make it possible to access out-of-field teachers' conceptualisations of mathematics.

5.1.1 School Mathematics

School mathematics is defined in this thesis as the mathematical knowledge that is learnt at school (Golding, 2017). The word 'learnt' is used rather than taught, as this thesis seeks to avoid seeing mathematics solely through a pedagogical lens. The use of 'learnt' acknowledges the hidden curriculum, that learning in school mathematics is not confined to curricular intention (Dewey, 1916) and so encompasses aspects of mathematics such as epistemological ideas of ability explored in chapter 2. This section begins with school mathematics because almost every participant began with school mathematics in their interview. Most interviews started with participants being asked to talk about themselves as learners of mathematics. Table 5.1 graphically represents the periods of their lives that participants mentioned in response to the initial and follow-up questions about themselves as learners of mathematics. Pete was the only participant to begin their story before school-age:

> I was born and then began to understand space, shape and the world. (Pete)

This initial statement is a rare direct reference to everyday mathematics, but Pete swiftly moves to school mathematics. He jumps straight to GCSEs. Although no-one was asked, every participant referred to GCSE or another compulsory mathematics examination at age 16. Four participants started their narrative as learners of GCSE or O-level mathematics, and for Lee and Clare this was the only mathematics learning they referred to. When talking about themselves as a learner of mathematics, all participants spoke predominantly about school mathematics, especially public examinations.

Table 5.1 Periods of own life referred to by participants when reflecting on themselves as learners of mathematics

Age	0 – 4	5 – 7	7 – 11	11 – 14	14 – 16	16 – 18	18+		
Stage					GCSE/ O-level	A-Level	Degree	Teacher Training	In-service training
Pete									
Emil									
Liz									
Ben									
Jon									
Zoe									
Nik									
Ross									
Lee									
Clare									

Most of the mathematical content mentioned by participants was facts and procedures related to number, algebra, geometry and statistics. One participant, Lee, provided an extensive list of the mathematics in the Wembley image which is reproduced in Appendix E to illustrate its richness. Figure 5.1 is a word cloud of the areas of the mathematics curriculum that Lee referred to. As I listened to Lee's extensive response, I identified some areas of mathematics not taught in schools (such as logistics) and in my follow-up questions explored what forms of mathematics he perceived his response to cover. Lee concluded that not only was everything on his list related to school mathematics, but there was also nothing on the school curriculum that could not be found in the image of Wembley Stadium.

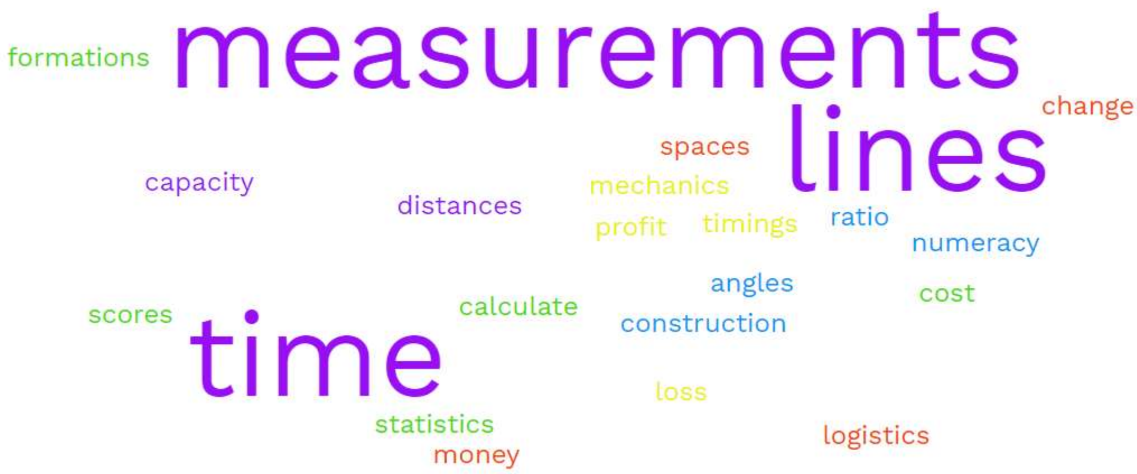


Figure 5.1 Word cloud of areas of mathematics Lee referred to in his Wembley response (Appendix E)

Several participants expressed the idea that mathematics poses questions to which learners are expected to use established processes to find preordained answers. For example:

> I enjoyed the kind of relative recall of GCSE, you know, you follow an algorithm and you do the algorithm and then you succeed at the algorithm. Somebody says, 'well done, you could do the algorithm,' and then you start with a new algorithm. (Pete)

In the first extract below Ben expresses similar satisfaction to Pete's. In the next extract he is talking about what he does to help his students have the same experience. Participants expressed views that finding correct answers was important to them as learners, and to their students.

> When I was doing A-Level, I used to love solving equations. A whole page and there was just an answer, and finding out that it was correct was just excellent. (Ben)

> That's the way I would try and help them. I know that there's other ways to go about it, but I feel that's the easiest way because it is visual. (Ben)

5.1.2 Pedagogical Mathematics

It was not always possible to identify whether participants were referring to pedagogical or school mathematics. Here Ben moves back and forth between pedagogical and subject knowledge perspectives while considering a mathematics question.

> So, I always try and teach the kids … [*reads under breath*] … that is … no … I'm drawing brackets there … so I am thinking about forming an equation from the worded question there. [*pause*] [*reads under breath*] From a teaching point of view I'd probably ask the students to either cut this out or draw it somewhere. [*pause*] So I'd try and encourage them to work backwards on this one. With all of them I would also – I don't know if this is the right way round – I'd get them to annotate what's useful, what's not. I just think I'd draw an axis on that one straight away. (Ben)

I observed that he had approached the question as he would teach it. He remarked that,

> Every time I see a question, I try to think to myself 'how will the students that I'm teaching understand this?' What will they need to look at this as, and that sort of thing. (Ben)

Ben was not alone in looking at questions through his students' eyes. When presented with five questions and asked to select which they would do first, Nik, Emil and Lee related the questions to what they'd taught that day. Clare and Pete analysed the questions' carrier language, and Clare, Pete and Jon tried to work out the level of the questions and where they fit into the curriculum.

Nik drew on his own experiences as a learner of school mathematics and used this to talk about how he would teach differently.

> Without trying to sound old, 20 years ago when I was sitting my GCSE was very different. I think there's a bit more of excitement now. You try and link maths, don't you? When I was taught maths at school it was very, very dry…. Whereas now,… it's always in the back of my mind how maths was taught in school. I'm thinking, how can I get these students… how can I get them interested? I'm coming at that probably from a not-a-maths-specialist point of view. (Nik)

In this last sentence Nik suggests that seeing mathematics questions through students' eyes is possibly an out-of-field teachers' perspective. Clare's analysis of the carrier language comes with a similar suggestion:

> I've got no basis for saying this, but I think maths teachers underestimate the significance of the reading. I think they take for granted that kids can read and that all they're doing is teaching the maths. They don't realise what a barrier it is not being able to actually read with understanding that particular piece of information or those questions. (Clare)

Several participants commented on their empathy with their students because of the similarity in their level of understanding of mathematics:

> I try to put myself in my students' shoes because I was, well, I was essentially in their shoes before. (Emil)

> I think what I enjoy in terms of teaching is that emotional connection, that empathy with what makes kids want to be in the room. (Jon)

> I view myself as being in a better position because I'm not for a minute saying I am 100% expert in this. I'm having a conversation with them, probably from a similar starting point that they are. I think that at this level a lot of students find that quite useful. Lots of our students have made comments that if they get me or Jon, we might explain something in more steps than the head of department would, who's coming from a 100%, pure maths, conceptual approach. (Nik)

Liz and Zoe, the two participants new to teaching mathematics out-of-field, refer in every interview to their desire to develop a holistic understanding of mathematics, expressing frustration that they are not able to see the bigger picture. But in both cases, it is school mathematics that is their framework:

> It's the process of how they do it. Knowing when, what they need to do and how they need to do to get to the end product. Because the way I might do it will probably skip various things. And I think that is the thing or is probably the big sticking point. It makes me very unsure what I'm doing…. I think it's knowing where to go. So, like I said, these resources are apparently available. But when I look at them, it doesn't go key stage three or year seven, year eight. It literally goes, this is algebra, and it will be everything…. I don't know what level they're pitched at, so I don't feel I can even access the resources that are there. (Liz)

> I think probably I need to see the whole of the maths curriculum… but we're all in the process of writing that, so… I'm guessing she [*head of maths*] understands how it… [all fits together]. I'm just looking at what comes next. Enlargements and bearings. Probably … algebra … (Zoe)

School mathematics provides the framework in which out-of-field teachers construct their pedagogical knowledge of mathematics.

5.1.3 Academic mathematics

By definition, out-of-field teachers of mathematics have not experienced academic mathematics. Five participants referred to mathematics at degree level. Of these, only one had studied mathematics as a discrete discipline – Pete had studied for 90 credits with the Open University. This experience led him to dismiss what he had previously thought of as mathematical content in his music degree (calculating hertz), saying,

> it was only much later I realised, actually, that's really not a lot of maths. That's just living, that's just being alive. (Pete)

Similarly, Ben said of his sports science degree:

> we used equations for biomechanics and things, but we didn't do maths, no. (Ben).

How they are categorising the mathematics that was a part of their degree aligns with what is being described in this thesis as everyday mathematics.

The participants who referred to the mathematics content of their degree and own discipline were Liz, Nik and Zoe. Liz's degree is in mechanical engineering, of which she said,

> the maths was great, and I had no problem with that. (Liz)

Zoe uses the mathematical content of her degree to justify teaching mathematics:

> When I was at uni obviously, we did stats, but we also had to pick extra modules and I picked a couple of maths ones because I found it easy, to be honest. It was much better than the sociology and all the rest of it. I mentioned that if I had to teach another subject out of humanities, I'd quite like to have a go at some maths. (Zoe)

Nik also referred to the statistical analysis in his geography degree. I asked Zoe in each subsequent interview if her confidence about mathematics had changed, and she responded each time that it had improved. However, where in her first interview she spoke about higher statistical concepts such as Spearman's rank correlation, in subsequent interviews she connected her confidence to

> things like long multiplication, (Zoe)

relating her confidence to school mathematics rather than academic mathematics.

Pete and Clare reflected more generally on academic mathematics, about which they expressed alienation. To Clare it was a mystery, to Pete a group from which he was excluded.

> I often wonder if you do maths at university or if you become a supersonic lecturer of maths, what do you do? What else is there to discover and do? How? (Clare)

> … maybe that's because I don't have the higher knowledge and upper echelons of the mathematical fraternity to do anything else… (Pete)

Despite feeling alienation toward academic mathematics, Pete and Clare both displayed a sophisticated philosophical conceptualisation of mathematics which will be explored in section 5.2.2.

5.1.4 Everyday mathematics

Everyday mathematics is conceptualised in this thesis as informally structured mathematics as it is used (Davis and Renert, 2014). It is a broad conceptualisation brought together by the idea of using mathematics as a tool. In section 5.1.1 it was noted that most of the mathematical content mentioned by participants was facts and procedures related to the basic areas of knowledge in the GCSE curriculum: number, algebra, geometry and statistics. Less often participants referred to other areas of mathematics. These are summarised in Table 5.2.

Table 5.2 Everyday mathematics referred to by participants

Area of mathematics	Referred to by
Engineering, electronics, lighting	Emil, Clare, Lee
Structure, architecture, construction	Emil, Clare, Lee, Pete
Algorithms	Pete, Emil
T-test	Emil
Modelling	Pete
Logic, Rubik's cube	Pete, Nik

Lee's extensive response to the question 'where's the mathematics?' in the picture of Wembley, (section 5.1.1, Appendix E) included some areas of mathematics not taught directly in schools.

> There's also mathematics for the emergency exits for when there's an issue, such as a fire. You've got to be able to empty the stadium in a certain amount of minutes safely and effectively…. You'd need mathematics for the transport to get the timings right for people to get there on time before the football match or rugby match or whatever starts. So, there's trains that go past Wembley: they'd have to be there at a reasonable time. You'd have to have logistics for the teams to get there. You'd have to have the coaches and the car parks available for enough cars to fit in, so there's enough spaces for the people to go and watch that sport there. (Lee)

Lee is a PE specialist, and it is possible that he is recognising here the mathematics that he uses as a tool professionally or for leisure organising fixtures and tournaments, although he does not refer to this explicitly.

Some participants did make explicit links between the mathematics they teach and everyday mathematics, as in the extracts from Ross and Nik below.

> In my early years of teaching maths, I would do activities outside, which I don't do now. I would have area and perimeter sessions and work outside. They would measure the centre circle on the football pitch and work out the area. They would measure the 18-yard box and find the area of that. And walk round school, work out the area of windows and doors or tables, just like totally random. I think that was more for me to get out of class. If it was sunny, I'd say 'OK, right, let's go.' I'd get loads of tape measures and metre rulers and I'd print this sort of table out and I'd say find things or go looking for things of different shapes, to try and find them outside. (Ross)

> Yesterday I was putting some new slate down in the garden. I had to dig it all out and I had to work out what area it was and how many bags I had to buy. Those sorts of questions which could be on an exam, I don't think the kids actually realise that you do that in real life…. And Jon will probably murder me for saying this, but I always tell the story about when he went into B&Q to buy some paint and he took a tin he already had in the house and said I need some of this colour paint and the guy in B&Q said, 'how much do you need?' and he said, 'I don't know.' And the guy from B&Q said, 'well, if you don't know how am I meant to know?' And I tell that story to our students when we do geometry problems like that. And I say, well a teacher from this school has actually gone into B&Q and looked silly because he doesn't know how many tins of paint he needs…. It's that link. I'm not saying that that makes it exciting, but you can see some of the students go 'hey, he works round here and he's a senior leader and he didn't even know how many tins of paint he needed to paint the wall.' (Nik)

As with pedagogical mathematics, participants' conceptualisation of everyday mathematics is seen through the lens of school mathematics. For example, in this extract Ross is quite open that getting his students using their measuring skills was designed to get out of the classroom. Nik draws on his lived experience of mathematics to motivate his students.

In section 5.1.3, on academic mathematics, several participants reflected on whether the mathematics they encounter in their in-field discipline can be defined as mathematics. Ben and Pete rejected the idea that the mathematics involved in their sports science and music degrees was academic mathematics. Pete's conclusion that 'it is just life' points towards everyday mathematics. He talks about 'busking' with mathematics, and Nik talks about taking an intuitive approach, both of which are commensurate with the definition used in this thesis of everyday mathematics as informally structured. Elsewhere the mathematics used in participants' own discipline is formally structured:

> Is physics the theory behind maths? But there …. Hmmm… [*pause*] right [*laughter*] Oh, that hurts my brain. I guess with physics it's the theory. The mathematical part is more the application of the theory…. When you break it down, you're using your mathematical skills to demonstrate the theory. (Liz)

Finally, an interesting conceptualisation of mathematics came from comments about what mathematics is not, namely English:

> I had a lot more interest in maths than I did in English. (Lee)
>
> We've always been a kind of more mathsy kind of family than an English kind of family. (Jon)
>
> I've always preferred maths over English because there's quite often a definitive answer. (Ben)
>
> I think I got on better with maths at school because of that and because I probably preferred working with numbers than I did with letters. (Zoe)

5.2 Discussion

School mathematics is the dominant form of mathematics in participants' conceptualisation. Other forms of mathematics were viewed through the lens of school mathematics, which itself was seen through a student lens. This section begins by discussing participants' conceptualisation of school mathematics through a student lens, followed by discussion of how participants related school mathematics to other forms of mathematics.

5.2.1 School mathematics through a student lens

Participants' conceptualisations of school mathematics had three common elements. There was a focus on public examinations, on mathematics as a defined body of knowledge and as a process of posing and answering closed questions. These were all seen through a student lens.

Chapter 8 will explore public examinations as a reason for doing mathematics. This section explores other potential reasons why public examination and qualifications were central elements of their conceptualisations of mathematics. The high profile afforded to examinations and qualifications may reflect the research context. Participants were being interviewed because they teach mathematics out-of-field and so their mathematics qualifications may have been at the forefront of their mind. It may also reflect their reasons for teaching out-of-field, with several participants having been invited to teach mathematics to improve GCSE outcomes in their school (for example, Lee and Jon, section 8.1.2). All participants were working in an environment of high emphasis on academic success where GCSE results dominate school discourse, especially for Jon, Nik, Liz and Zoe as senior leaders (Greany *et al.*, 2016). From a critical perspective 'the basic goal of standardised mathematics tests is to produce winners and losers,' (Aikenhead, 2021b, p. 13), and participants appeared to want to champion the 'losers' and make as many 'winners' as possible. Participants suggested that seeing themselves in their 'students' shoes' (Emil) puts them in a 'better position' (Nik) than an in-field teachers. Clare uses language that positions in-field teachers of mathematics as Other in relation to her and learners: for example, 'they take for granted' and 'they don't realise' (Akkerman and Bakker, 2011). Out-of-field teachers are not unusual for valuing qualifications in mathematics (Brown and McNamara, 2011); it is in seeing them through a students' lens in which they perceive themselves to be different.

A conceptualisation of mathematics as a defined body of knowledge will be discussed in chapter 6. This section discusses conceptualisations of the content of school mathematics and how participants appeared to view them through a student lens. References in the data to mathematical content that is not part of the school curriculum, such as logistics or architecture, are the exception. Most of the mathematical content referred to by participants falls into the curricular categories of number, algebra, geometry, statistics and probability, and is consistent with the assertions noted in section 2.2 that for most people school mathematics is their only experience of the subject (Davis and Renert, 2014). This is the 'specific mathematics that teachers need to know' referred to by de Souza Pereira Grilo and Cerqueira Barbosa (2022, p. 136). When Lee concludes that the mathematics he identified in the Wembley image was all related to the school curriculum, and that there was also nothing on the school curriculum that could not be found in this image, I interpret this that he is seeing the school curriculum as representing all of mathematics.

Participants expressed a belief that knowing school mathematics in the form that it is presented to their students in the curriculum is appreciated by students who, in Nik's words, preferred it to 'a 100%, pure maths, conceptual approach'. They appeared to conceptualise mathematics as posing and answering closed questions. This will be explored in depth in chapter 7. Just as participants appeared to focus on qualifications and content as they believed these to be what mattered to their students, so the conceptualisation of mathematics as questions requiring a correct solution can also be interpreted as seeing school mathematics through a student lens. Boaler (2002) found evidence of learners who liked mathematics because there were only right or wrong answers but found these to be in the minority. For the majority who are not 'someone that can pick up answers really fast and … can answer questions really fast' (Darragh, 2014, p. 83), there is the pedagogy of support which Darby identified as being the signature pedagogy of mathematics (2010). The pedagogy of support includes close attention to student difficulties and establishing a non-threatening classroom environment, both of which can be interpreted as being participants' aims in comments such as Jon's 'empathy' and Ben helping his students to find the 'easiest way'.

Through their focus on public examinations and on mathematics as a defined body of knowledge and as a process of posing and answering closed questions, out-of-field teachers participating in this study conceptualised mathematics largely as school mathematics seen through their students' eyes. The literature locates the boundary that out-of-field teachers are crossing as the discontinuity between their in-field and out-of-field subjects, between different knowledge domains or 'fields' (Hobbs, 2013a), recontextualising their pedagogical knowledge in a novel field. The data discussed in this chapter could suggest that the boundary is located within mathematics, that out-of-field teachers are navigating the boundary between themselves as learners of mathematics and themselves as teachers of mathematics. They are using their own experiences as learners of mathematics as boundary objects to aid recontextualisation, which the next section explores.

5.2.2 Relating school mathematics to other forms of mathematics

Data collection tools were designed to focus on how out-of-field teachers conceptualise mathematics rather than how they teach it. Participants nevertheless talked about pedagogical mathematics: as observed by Davis and Simmt (2006), it is not possible to unpick the complex relationship between a teachers' pedagogical and other knowledges of mathematics. For these out-of-field teachers, however, pedagogical mathematics was understood in terms of its relationship to school mathematics, as were the other forms of mathematics. School mathematics was situated as the parent discipline with other forms as recontextualisations (Figure 5.2).

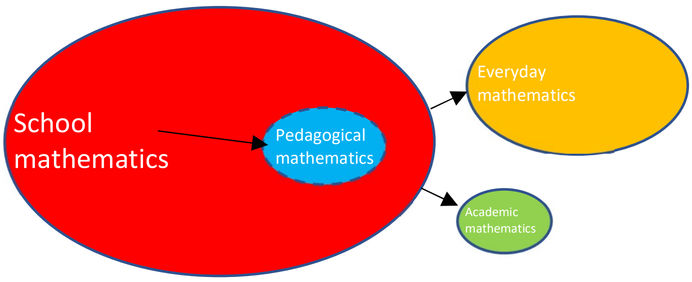


Figure 5.2 Diagrammatic representation suggesting how most participants appeared to position school mathematics in relation to other forms of mathematics

I interpret participants as viewing pedagogical mathematics as a subset of school mathematics. The two participants new to teaching out-of-field, Liz and Zoe, were the most explicit about how they were recontextualising their existing mathematical and in-field knowledge into pedagogical mathematics. Both appeared to draw on similar tools to support this recontextualisation. One was their in-field pedagogical knowledge (rather than the general pedagogical knowledge proposed by Shulman (1986)); the other was their own (and their offspring's) experience of learning mathematics. This recontextualisation reflects their epistemology of mathematics and will be explored in Chapter 7.

An implication of the proposed model of participants' positioning of school mathematics (Figure 5.2) is the direction of recontextualisation: the inverse to that implied by Shulman (1986) (Figure 2.6b). For participants, school mathematics is the starting point. Several participants expressed the idea that for a small number of learners school mathematics might progress to academic mathematics. But this was almost as an aside, a footnote for the elite, which is why academic mathematics is represented as a small domain in Figure 5.2. School mathematics also informs everyday mathematics. In some cases, such as Lee's comprehensive list of mathematics in the football

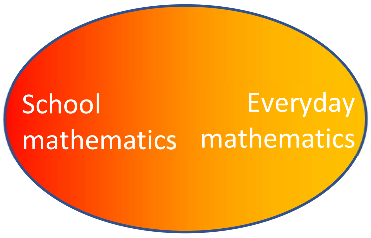


Figure 5.3 Diagrammatic representation of Lee's conceptualisation of the relationship between school and everyday mathematics

stadium photo (Appendix E), school mathematics and everyday mathematics were seen as almost identical in terms of content, shown in Figure 5.3 with weak boundary strength.

The strong collection code identified by Bernstein (2000) is still evident. An interesting manifestation of this is the conceptualisation of mathematics as 'not English' that arose several times in the data, participants appearing to construct a binary to define what mathematics is not. Where participants provided an explanation of why they perceive mathematics and English as opposites, the reasons given were:

- definitive answers vs. open endedness;
- numbers vs. words;
- abstract vs. people.

At primary age, performance is only officially measured in terms of English and mathematics (DfE, 2022). While this is broadened for key stage 4 (age 16) public performance measures to include eight subjects, English and mathematics maintain their dominance with both being double weighted and compulsory (DfE, 2023b). The binary between mathematics and English constructed by half of the participants in this study reinforces the suggestion that their conceptualisation of mathematics is rooted in *school* mathematics as an ontologically and epistemologically distinguishable subject.

5.3 Conclusion

Out-of-field teachers participating in this study appeared to conceptualise mathematics as school mathematics, which they saw through the eyes of students. This was a complex and symbiotic blend of their own students and themselves (and sometimes their offspring) as students.

I interpreted participants to be suggesting that other forms of mathematics are not as important as school mathematics. Pedagogical mathematics was not always distinguishable from school mathematics but did appear to be a subset of school mathematics with a blurred boundary. The other forms of mathematics appeared to be more strongly bounded, with the direction of recontextualisation being unidirectional from school mathematics to everyday and academic mathematics, with the latter being alien.

This chapter has focused on the forms of mathematics dimension of the mathematics conceptual framework developed for this thesis. The next three chapters take each of the philosophical elements in turn.

6. Knowing mathematics: Out-of-field teachers conceptualising the nature of mathematics

This chapter analyses out-of-field teachers' conceptualisations of the nature of mathematics. The aspects of participants' ontologies of mathematics analysed and discussed in this chapter are the complex interplay of absolutist and fallibilist ontologies at the boundaries within mathematics and with other subjects. The literature review of ontologies of mathematics (section 2.1.1) contrasted fallibilism with absolutism, the idea that mathematics exists independently of humans and is discovered by them. The literature suggests that for all forms of mathematics people's ontology tends to be a dominant absolutism with a complex mix of fallibilism. The out-of-field teachers participating in this study tended not to conceive of mathematics as a human construct although all expressed a complex ontology which encompassed both absolutist and fallibilist ideas. As in the previous chapter this was centred on school mathematics, through a student lens.

In this thesis, mathematics is defined as a discipline of patterns and connections used to model a complex world. My positionality is embedded in this definition. I understand mathematics as a creative human construct, I am a fallibilist (Ernest, 1991).

6.1 Analysis

This first section analyses participants' ontology of mathematics. It starts (section 6.1.1) by considering their absolutist and fallibilist conceptualisations, particularly with respect to school mathematics. How they view these conceptualisations through their students' eyes is explored in the second section (6.1.2). The third section (6.1.3) considers how they relate to their in-field subject(s).

6.1.1 Absolutist or fallibilist mathematics

The absolutist idea that there is a unique correct solution to any question was expressed by several participants.

> It is sort of black and white. You either get it right or you get it wrong. (Emil)

…the black and whiteness of it. That you do it, and that's the answer. (Clare)

It's right or wrong. I liked maths because I could get to an answer and then I could check whether my answer was right. (Zoe)

I just loved solving equations. A whole page and there was just an answer. (Ben)

Not only is there a unique, correct solution, but also a process that leads to it.

I looked at it almost like a piece of Lego and you just got to put the building blocks in the right place. (Nik)

They [*students*] will get a lot of confidence from that because it's procedural. (Jon)

This extract from Ross' interview appears to be saying the same thing:

I would show them my method on the board, and I would ask them if they got the right answer. (Ross)

He uses the definite article to refer to '*the* right answer', suggesting that he is conceptualising mathematics as involving a binary of right or wrong answers. He also describes demonstrating a method to his students. But instead of the definite article he uses a possessive pronoun. It is '*my* method', implying that he made an active choice to use that one amongst others. Nik commented about the data collection tool that gave five different approaches to multiplication (Figure 4.5):

I do think that's quite an interesting resource or question that you've used because I think you don't have to do it that way. Which is interesting of itself. (Nik)

That data collection tool elicited more fallibilist comments along similar lines:

We normally get told in the maths department that we've agreed upon one method and push for that method. But if a student's not going to get that, I'd be happy to learn one [*an alternative method that they are familiar with*] to help them individually. (Ben)

> If they started doing that, I would still show them my method on the board and I would ask them if they got the right answer. And I would say if you did, then keep doing that. I wouldn't try and change them. (Ross)

The four participants who taught at the same school, Jon, Ben, Ross and Nik, received training and support in teaching mathematics. This involved both attending external training and sessions with the head of department, whose

> focus was on teaching us the concept rather than the methods behind it. She was saying we've got teachers who teach methods rather than the concepts. And she explained, she said 'right, in averages you work with the mode, mean, median and range. For the mean the method is you would add all of the numbers up and divide by the number there is. But the actual concept is that you are sharing out.' (Ross)

Ross notes that he did not entirely follow this advice.

> She [*head of department*] started trying to teach us the concept, but I think I went more to the method now, so I've lost my way a little bit. (Ross)

Jon said something similar and justified his approach, saying,

> I'm quite process driven. I think there's times and all this might make me sort of flinch, but there are times when I teach confidence through process. So in terms of maths, I'll teach confidence in the child: I say, 'do this and this and this'. And then I'll go back and unpick the maths behind, so that they get the confidence of success first. That's how I teach myself as well. (Jon)
>
> I know for a fact and because it's the way that I think kids are, they know that they are going to be examined at the end of the day…. They will get a lot of confidence from that because it's procedural. (Jon)

Nik shared similar thoughts and went on to suggest that taking a procedural rather than conceptual approach benefited his students, saying that, a lot of our students have made comments that if they get me or Jon we might explain something in more steps than the head of department would who's coming from a… conceptual approach. (Nik)

Emil and Zoe talk about how the procedural (performing statistical calculations) is used to aid conceptual understanding in their in-field subjects.

When I've taught Spearman's rank before, I've kind of just gone 'I don't need to understand how it works, I just need to be able to work it out and know what those figures mean'. I've not had to get my head… I don't need to know. It's not beneficial to know how the whole formula and all of that works. I just need to know what that formula means when it's in front of me and I need to know what the results mean, and I need to be able to explain to the kids this is what this result is and this is what this result means and what does that mean for your project. (Zoe)

I don't think that having them go through the entire process of doing a t-test by hand is very beneficial since they're not going to do it anymore in the future. A piece of software is going to do it for the future. I, however, do think it's vital that we teach them how to understand and interpret the results, which is something that I think we're still lacking. (Emil)

To draw this section to an end, this is Pete's summary of how mathematics can be seen from an absolutist or fallibilist perspective:

It's a social, sociological subject as well as a science in the sense of it's how we understand the world is as and how we interpret the world as well as the calculations we do with the world. (Pete)

6.1.2 Complex ontologies through a student lens

Chapter 5 concluded that participants appeared to see mathematics as school mathematics and through the eyes of their students. This section focuses on Emil and Pete, who used different ontologies of mathematics to the same end of meeting the needs of their students. Emil and Pete were the only two participants not teaching in mainstream maintained schools, which Pete suggested was possibly why he has a different mindset, because

> the working environment's different, and the requirements within the work environment are different. We don't have any worries about, you know, pushing people forward and you know we don't have to worry about [league] tables or anything like that. So maybe we've got more freedom to, say, this holistic idea with maths. (Pete)

Emil is teaching students who require mathematics to access higher education and finds that

> often it's their attitude towards the subject that often prevents them to want to engage. (Emil)

Both talked about mathematics anxiety amongst their students and colleagues. In Emil's explanation of the roots of mathematics anxiety he suggests an absolutist ontology of mathematics.

> I think maths is genuinely that black and white, you know…. You either get it right or you get it wrong. And there is a right answer and then incorrect answer, yes. There are some things that are still not quite clear in maths, although there are very few. But essentially that is what maths is. You can extract some visual representations from the numbers and try and present them in a more visual way. But in reality, maths is black and white. It's an equation, it's a problem that needs to be solved. It's a right answer or wrong answer. And we do, and we do tend to try, and you know, we do try to encourage, encourage students and suggest to students that, that being wrong is not the end of the world. I think that's another stigma that is associated to maths that if you're wrong, you are stupid and you can't do maths. (Emil)

Emil then goes on to talk about how he uses his knowledge of psychology to help students learn mathematics despite its absolute nature. He describes using visual imagery to illustrate the conceptual underpinning of fractions, how he has used chocolate as a reward to motivate, and how he challenges students to learn from mistakes:

> Because of maths’ nature being so binary, you're either wrong or you're right. And if you're wrong, you just feel embarrassed. You just feel stupid and that's normal. So, what I try to do is remove this stigma. That I said to them, ‘look, it's fine to be wrong. It's okay to be wrong. It's not something that will start the end of everything. Okay.’ And I try to tell them, ‘be wrong, okay, do not be afraid to be wrong’. I tried to exude that. And sometimes when I present something on the board, I go wrong. And yes, it feels a little bit embarrassing, you know, you're the teacher, you should be the one that's all-knowing supposedly. But I'm wrong. But whenever I go wrong, I correct myself, I apologise and I say, ‘okay, look, I made a mistake – did that make me stupid? Did that make me lose my job or anything like that? No. I was wrong. I went back, I saw it and learned from that. I corrected my mistake. So that's what I want you guys to do.’ I think if we can move past that, that can help us learn from our mistakes. So, we tried to teach them to learn from their mistakes. And I also have this one little thing I tell them, my best scenario policy and I tell them, I'd rather you be wrong in my class so we can go back and see where you, where the mistake came from rather than you being wrong on your assessment, panicking and me not being able to help you there. I think that's quite nice. I feel they take it very well. That liberating sentiment in there. (Emil)

Pete demonstrates the kind of liberation that Emil aims for in his students. During his interview he became engrossed in the five mathematics questions. At one point he was audibly calculating the square root of 132.5 and estimated it to be between 12 and 13. I recorded in my field notes that on hearing that it was actually 11.5

> *Pete immediately wants to work out why he made the error. Confident that he has identified its source, he returns to where he was. And this also reassures him because it’s a ‘nicer number’.* (Field notes)

His comment about it being a *nicer number* refers to his assertion about the first question that he

> had a confidence that if you give me a question like that, you're probably giving me something that's possible to factorise.' (Pete)

His confidence appears to derive from an understanding that these questions are under human control. People control mathematics. In contrast to Emil's approach of teaching his students how to avoid being beaten by the binary of mathematics, Pete works to present mathematics positively, using words like 'nice', 'lovely', and 'beautiful' to describe mathematics questions. He describes mathematics variously as 'being alive', 'everything' and 'the language of the universe'. These could all point to a naturally occurring, absolutist ontology of mathematics. However, his explanation of how he employs his ontology with his students and colleagues suggests a human, fallibilist ontology of mathematics:

> My line in school is that when people say, 'oh, in maths it's right or wrong,' I say it's no more right or wrong than football in the sense that if you're doing penalties, you either get it in the goal or you don't get it in the goal. But doing penalties is not playing football. It's the skills we do in order to prepare us for football. Playing football is actually the game itself. And then there's a whole million different things that could be right or wrong, there's a whole series of commentators that will tell you it, you know, disagree about what's been done well or not been done well. So my sort of line is you know that we do skills and stuff that's right or wrong, but actually, you know, when you're in the big wide world and having a conversation about tax for example, then that's a dialogue that will have multiple different right and wrong answers and you're having a conversation to say, this is what I think and someone else that HMRC person will say this is what I think and you kind of you know, this is the information I've got this information you've got… it's how we understand the world and how we interpret the world as well as the calculations we do with the world. (Pete)

Where Emil uses his background in psychology and own struggles with mathematics to help his students to overcome absolutist mathematics, Pete actively seeks to challenge absolutist ontologies so that his colleagues and students understand that they have agency.

6.1.3 Ontological relationship with own subject

Several participants explicitly referred to the mathematics in their own subject, for example, statistics in geography (Zoe and Nik) and psychology (Emil), biomechanics in sports science (Ben), and calculating hertz in music (Pete). Liz and Clare made sophisticated ontological comparisons of their own discipline with mathematics. Liz's well-developed ontology of physics relied on a less developed ontology of mathematics, while Clare's background in theology manifested itself in a deeper conceptualisation of mathematics than her knowledge of mathematical content. For Liz, mathematics was a tool to understand physics deeply, while for Clare theology was a tool to understand mathematics deeply.

In exploring the similarities and differences between mathematics and physics, Liz raised the counterfactual question of whether if there was no physics, mathematics would still exist. The extract below occurred as Liz reflected on whether physics could exist without mathematics. She had been talking about the story of Newton and the apple and I asked whether Newton invented the Newtonian equations, or whether they already existed.

> They would have been there, but just not [*pause*] … because it's the principles behind why it drops, isn't it? They're there, but nobody knows about them until somebody starts making the observations and looking for the patterns and going 'Oh yes, if you do this then this happens.' [*pause*] Ooh, I don't know. I think they have to start existing the moment you've got somebody who looks at something and says why? And at that point, it's that moving from just accepting something to asking 'why is that happening? What's the pattern?' And that's where I guess the equation, the maths part of it comes in. [*pause*] But then we've also

> got… maths as the simple baking ingredient. So, people will have made recipes before, and they will have looked at quantities and combining things as well. (Liz)

There is complexity to Liz's exploration of the ontology of Newtonian physics. Her argument is fallibilist: that physics is a human construct created to describe the physical world. She characterises physicists as observing, identifying, and describing patterns, actively constructing knowledge. And then,

> the maths part of it comes in. (Liz)

It is the 'simple ingredient'. Her final sentence gives people some agency with the 'recipes' of mathematics, but the discourse in this last sentence is of working within the parameters of something with a bounded existence.

Liz sees her discipline as being closely related to mathematics. Clare does not. Liz enjoys mathematics. Clare does not.

> [Mathematics] is not interesting, because for me the subjects that were interesting were about people: English – that was about people because it's about feelings and things and responding to things; RS – obviously; history – lessons from the past, what people did; geography – I love the human element of geography. (Clare)

By implication, for Clare mathematics is not about people. This may suggest an absolutist ontology of mathematics, denying any human agency in its existence. However, very early in her interview Clare stated:

> I've got this really interesting question – I'm sure you've done this before. I went to this philosophy thing – I can remember talking to gifted and talented kids about philosophy that is maths natural or is it man made, and I'm really interested in the philosophy of it. (Clare)

She attempted to explore the natural or human-construct question from the perspective of pure or applied a couple of times but strayed onto axiological questions of the purpose of mathematics. I nudged her back to ontology.

> Oh gosh, golden ratio and all that stuff that they do. I think there is a natural part of maths, absolutely. I think there's a number structure of maths in the universe. I think that what then happens is that human beings have learned to apply that. So, I think it's both, I've come to the conclusion. (Clare)

Clare then hits another barrier, this time realising that she does not know what academic mathematicians do.

> What else is there to discover and to do? How? But then I think, well, you could say the same thing about religion. (Clare)

Following this she is once again distracted by axiological arguments about the utility of mathematics. Through her own disciplinary lens, she can glimpse the fallibilist philosophy of mathematics, but does not feel that she has the mathematical disciplinary knowledge to develop it. This implicit fallibilism which she struggles to articulate is also evident in Clare's interactions with mathematical objects (the five questions (Figure 4.4) and representations of multiplication (Figure 4.5)). She recognises human agency in the development of the questions ('why has the person used *long thin*?') and mathematical processes. Of the unfamiliar halving method, she wonders

> how they decided that it would actually get the correct answer. (Clare)

Her use of 'they decided' suggests she thinks that the method is a human construct. She keeps questioning until she understands how the process works and its provenance, before concluding

> that blows my head, (Clare)

and conceding that this is because it is so different to the ways she was taught. This is not the first time that she acknowledges cultural influences on mathematics – she spoke earlier in the interview about culturally biased questions. Despite her lack of mathematical content knowledge Clare's own disciplinary background provides her with the tools to explore the ontology of mathematics and arrive at a sophisticated, complex conceptualisation.

6.2 Discussion

Participants' ontology of mathematics was a complex mix of dominant absolutism, with elements of fallibilism. This extract from Table 2.3 suggests that this ontology of mathematics is common whatever form of mathematics is under consideration. In this section I suggest that, as in chapter 5, the form of mathematics for which participants hold this ontology is school mathematics through a student lens. I then discuss their recontextualisation work at the boundaries within and beyond mathematics.

Table 6.1 Extract from Table 2.3 (Knowing)

		Knowing (ontology) *Teachers' beliefs about the nature of mathematics*
School mathematics	*The mathematics learnt at school* (Golding, 2017)	Complex mix of dominant absolutism, with elements of fallibilism.
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)	Complex mix of dominant absolutism, with elements of fallibilism.

Participants expressed their dominant absolutist ontology of mathematics by conceptualising it as a received body of knowledge, involving the employment of procedures that lead to a pre-ordained universally agreed single correct solution: 'black and white' or 'right or wrong' were phrases used by participants. Conceptualising mathematics in this way is a reflection of the cultural context (Xenofontos, 2018). Xenofontos found a strong influence of Ancient Greek mathematical thinking on the beliefs of twenty-first century Greek-Cypriot teachers due to the historical and sociological context in which they were teaching (*ibid*.). Participants in my study reflected school mathematics in a cultural context which privileges mathematics as the purest example of scientific rationality, a unidimensional, universal single way of seeing the world (Kincheloe, 2008). This is reflected in their absolutist ontologies.

Another aspect of the cultural context in which participants in this study are working is the strong collection code of the English education system in the 2010s (Whitty, 2017), which participants will have experienced in their own schooling. A strong collection code features distinct boundaries between different areas of the curriculum which leads to specialisation and consequently constructs the concept of *non*-specialisation. Out-of-field teachers participating in this study work within a cultural context of subject specialisation (Brooks, 2016), which determines the nature of their boundary work as they recontextualise existing knowledge to teach mathematics. The paragraphs that follow consider the various boundary zones in which ontological recontextualisation takes place, beginning with recontextualisation of existing forms of mathematics knowledge before considering the recontextualisation of academic and pedagogical knowledge from their in-field subjects.

The relationship between participants' conceptualisations of the different forms of mathematics was represented diagrammatically in Figure 5.2, with school mathematics as the dominant form of mathematics. Figure 6. 1 makes a minor adaptation to this model to represent elements of fallibilism in Pete's ontology of mathematics: recontextualisation between school and other forms of

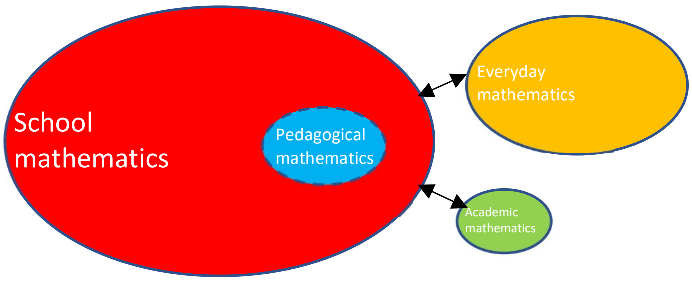


Figure 6. 1 Pete's complex ontology of mathematics

mathematics becoming bi-directional. In Ernest's (1991) binary, Pete's conceptualisation of mathematics is nevertheless absolutist. This ignores the complexity and sophistication of his ontology. Pete employed a generic pedagogical tool of analogy (Gentner and Smith, 2012) to use everyday knowledge where fallibilism is a cultural norm to argue that mathematics is fallible. Elsewhere in his interview Pete reflected that his experience of studying degree level mathematics led him to an understanding of mathematics as ‘problem solving or modelling,’ not following algorithms, and stated that ‘everything is maths’. In his football analogy Pete is not directly using everyday mathematics to illustrate the fallibilism of mathematics, but rather is using everyday knowledge which he would describe as mathematical. All four forms are recontextualised in his football analogy: his *pedagogical* mathematics is used to identify colleagues’ and students' absolutist *school* mathematics, which he responds to by using his *academic* conceptualisation of mathematics as modelling and his reflective experiences as an informal user of *everyday* mathematics. Pete is recontextualising his fallibilist conceptualisation of mathematics for the social context in which he is working (Lerman and Zevenbergen, 2004). In this way, his work is also critical: Lerman and Zevenbergen (*ibid.*) use a Bernsteinian theoretical framework to argue that the discursive practices in school mathematics can be restrictive for students unable to participate in the hegemonic masculine image of mathematics (Wiliam, Bartholomew and Reay, 2004).

Critical social justice themes were raised as participants recontextualised school mathematics into pedagogical mathematics. Only Pete used a fallibilist ontology of mathematics to make it more accessible. Emil was not the only participant who conceptualised mathematics as absolutist and sought to protect his learners from it, employing the signature pedagogy of mathematics identified by Darby (2010), the pedagogy of support. Where some identified human agency in the creation of mathematics questions, Emil located human agency in responding to mathematics questions. His approach appeared to assume questions to be incontestable and so taught his students to overcome them. Several participants consciously adopt an instrumentalist pedagogy (Skemp, 1978) to the same ends.

Ben, Ross, Nik and Jon’s school prided itself on supporting staff and students and all four teachers had had access to professional development for out-of-field teachers and ongoing departmental support. Consequently, they were all aware of Skemp's differentiation between instrumentalist ('rules without reason' *ibid*. p. 21) and relational ('knowing both what to do and why' *ibid*. p. 21) and therefore the importance of teaching conceptual as well as procedural understanding (Hiebert, 2013). All four colleagues apologetically confessed to teaching mathematics procedurally. Consistent with Belbase's (2019) traditional teachers, they all wanted to make mathematics accessible to their students, and they all wanted their students to experience success both immediately and in public examinations.

Participants' conceptualisation of mathematics through a student lens meant that they privileged students over mathematics. For example, Jon talked about a respected former colleague whose answer to the question of 'what do you teach?' was always 'children'. This is supported by my anecdotal experience of out-of-field teachers saying, 'I teach children, not a subject'. Except for Pete, these out-of-field teachers appeared to be using the student lens to recontextualise their absolutist school mathematics into an absolutist, student-focused pedagogical mathematics. As with Pete, there is complexity. The boundary between school and pedagogical mathematics is blurred, as demonstrated by Ben's oscillation between pedagogical and subject knowledge perspectives when considering how he would approach a mathematics question (section 5.1.2). It is bi-directional, with participants using their own experiences as a learner of mathematics and as a teacher to construct a student lens through which to conceptualise school mathematics and recontextualise it into pedagogical mathematics (Figure 6.2).

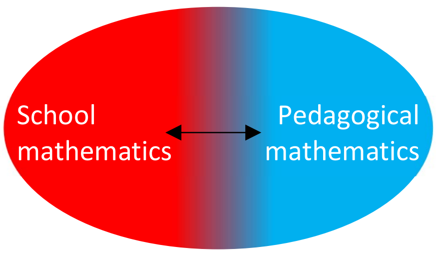


Figure 6.2 Boundary between school and pedagogical mathematics

Out-of-field teachers' recontextualisation of their professional knowledge has been widely explored (for example, Crisan and Hobbs, 2019; Rochette, 2022). This literature focuses on the recontextualisation of in-field into out-of-field pedagogical content knowledge. The focus of this study is subject knowledge, and so is interested in the relationship between and recontextualisation of in- and out-of-field disciplinary knowledge. As in the recontextualisation activities already discussed, there is complexity in participants' work at the boundary between the disciplines. Liz appeared to locate her existing mathematical knowledge within her physics knowledge, a process of recognition, not recontextualisation. Her conceptualisation of mathematics as a fixed tool with strong boundaries used to construct physics (which she saw as having porous boundaries) did not change. In contrast, Clare used her disciplinary knowledge of religious education to challenge her conceptualisation of mathematics. Religious education is concerned with exploring ontological questions (Wright, 2015), and Clare used these disciplinary behaviours when talking about mathematics.

Liz was not the only participant to conceptualise her in-field discipline as fallibilist and out-of-field as absolutist. Emil and Zoe both reflected on statistical tools (t-tests and Spearman's rank respectively) and how they related differently to mathematics as they did to their own disciplines (psychology and geography). Like Liz, they identified them as tools to use in their in-field discipline. They noted that in their in-field discipline these tools aided interpretation of empirical data, and that it was only the interpretation that interested them. Mathematics facilitated a fallibilist ontology of their subject. While expressing no desire to do so themselves, both Emil and Zoe suggested that a mathematician would look at the same tools from a different perspective and consider how they worked. They thought the mathematician's interest would be in understanding how the process works. They did not position the mathematician as creating or constructing the mathematics.

Liz perceived her in-field subject, physics, to be a good match for her out-of-field subject, mathematics. This is also why Zoe volunteered to teach mathematics out-of-field, and intuitively it appears to make sense to assign out-of-field teachers to a subject with similar philosophical roots to their own. All participants were working in a context with a strong collection code (Whitty, 2017), but did not perceive all subjects as being equally discrete or with similar boundaries between one another. In section 5.2.2 participants' tendency to see English and mathematics as binaries was discussed and reference made to the potential ontological roots of this distinction. The four participants whose in-field discipline is often perceived as being closest to mathematics ontologically (Young and Muller, 2016) held some of the most absolutist conceptualisations of mathematics. Clare and Pete, the two whose in-field disciplines (RS and music) are considered to be open-ended, creative and people-oriented were those who developed the more fallibilist ontologies of mathematics. It is possible that a teacher's ontology of their in-field subject influences their ontology of their out-of-field subject.

6.3 Conclusion

The mathematics ontology held by out-of-field teachers participating in this study appeared to be a complex mix of dominant absolutism and fallibilism, mirroring what was found in the literature in section 2.1.1. As in chapter 5, participants' conceptualised mathematics as school mathematics through a student lens.

This chapter makes two important and apparently contradictory contributions. The first is the consistency between ontologies of mathematics in the mathematics and mathematics education literature explored in section 2.1.1 and those of out-of-field teachers of mathematics. It should offer reassurance to the deficit model that raises concerns about the subject knowledge of out-of-field teachers. With this small, non-generalisable sample, there does not appear to be a wide ontological gap. The second is the nature of the differences in out-of-field teachers' ontologies of mathematics: how these are possibly recontextualisations of their ontologies of their in-field subjects, and how their focus is on school mathematics through a student lens. The theoretical framework employed by this thesis and concept of bricolage allows this contradiction to exist, allows complexity.

In this chapter there are already hints of how the ontology of out-of-field teachers participating in this study may inform their epistemology of how we learn, or come to know, mathematics. For example, fixed questions and solutions and a pedagogy of support are interrelated with notions of fixed ability and the emotional dimension of learning mathematics. These will be explored in the next chapter.

7. Learning mathematics: Out-of-field teachers conceptualising coming to know mathematics

Participants conceptualised mathematics as the mathematics learned in school, a form of school mathematics seen through the eyes of the learner (chapter 5), holding a complex mix of a dominant absolutist ontology with elements of fallibilism (chapter 6). In this chapter I analyse the data about how out-of-field teachers participating in this study understood how they and their students come to know mathematics, their epistemology.

7.1 Analysis

In the mathematics education and teaching mathematics out-of-field literature on the epistemology of mathematics (section 2.1.2), key themes were notions of ability (and the privileging of speed, accuracy and competition) and the centrality of emotion in the learning of mathematics. These were also key themes in the data and are used to structure this section.

7.1.1 Notions of ability

The absolutist idea that mathematical ability is fixed was expressed at least once by every participant (Table 7.1).

Table 7.1 An example from each participant of ideas of fixed mathematical ability

Clare	I'm so pleased that somebody's really good at maths and can do it, but I can’t.
Emil	I think that's another stigma that is associated to maths that if you're wrong, you are stupid and you can't do maths.
Ross	I was one of the lowest achieving students in top set. I sort of just hid at the back and just get on with things. But I was always good with my times tables.
Zoe	But to someone else they just can’t get their head around how it works.
Pete	I felt as though I was quite good at maths when I started A-level, so I didn't feel like I really needed to pay attention in the lessons.
Nik	He’s got a maths degree and he’s a maths genius.
Lee	For maybe well over a year I was in top set maths and ended up just copying my friend next to me and struggled all the way through so I just thought at some point I've got to go and seek help and say I shouldn't be in this class.
Liz	I’ve had some lovely conversations with some lovely really clever kids who’ve zoomed through it.
Ben	I was in top sets for maths.

Jon	And I actually tell the kids that. I say, if I'm stuck, I go and ask Mr Michaels, he's the head of department. And I say he's brilliant at it.

Clare proposes a binary: you can do mathematics, or you can't. Zoe identifies a similar binary in her own subject of geography:

> I think with a geography teacher you're either a physical geographer or you're a human geographer. And even though a lot of the sort of Spearman's [*pause*] well, no, a lot of the kind of things like, [*pause*] I see it as airy-fairy geography, the human, whereas, [*pause*] … I like the measuring of pebbles and the angles of the beach and that kind of thing, which without a maths brain is very difficult for them to understand, particularly as you're getting higher up the school. (Zoe)

I asked her to expand on the idea of a maths brain.

> You've got kids, haven't you, where it all slots into place. You've got kids that haven't. And there just seems to be, there doesn't seem to be anything in between. A kid's either really good at the maths questions in geography, or they're really bad at them. And my encounter with teachers has been the same. (Zoe)

The provenance of mathematical ability was variously attributed to genes, gender or mystery. In terms of genes, participants talked about having inherited their mathematical ability,

> we've always been a kind of more mathsy kind of family (Jon)

or lack thereof,

> my dad … was stupendous even when he got dementia … he was very good with maths, but it wasn't my natural forte. (Clare)

> I absolutely loathed the subject. That's very ironic for me because both of my parents are electrical engineers. (Emil)

Clare was not the only participant to refer to her father when considering their own mathematical ability. All three female participants made references to gender, with one saying

> I've always been told I've got a bloke's brain. (Zoe)

Liz explores both her father's role and the supposedly masculine nature of her brain in early childhood.

> My dad pushed me with maths. He was an engineer and I wanted to be an engineer. All I ever wanted in life. I wanted to work in power stations, just like him, from about the age of seven. … So, I think because of the way I was, according to my parents, I was very much into building things and how things work. I wasn't a very traditional girl. I didn't do dolls, but I did bricks, and I did building, and I did making things. And I think when you're building things and you're looking at brick structures, I think there is a mathematical element, because you're looking at a wall and you're looking at how many bricks do I need? So, I think when I was very young and did all that, I enjoyed maths, we did those sorts of things. I remember enjoying it at primary school and feeling I could do it. (Liz)

As well as attributing tangible sources of mathematical ability such as genes and gender, some participants ventured into the mythical, almost magical. Lee, for example, cannot explain why he understood mathematics, saying,

> it just clicked. (Lee)

Two participants working at the same school described their head of department, referred to here by a pseudonym, as having 'this amazing maths brain' (Nik), being 'brilliant' at mathematics (Jon), and 'a genius' (Nik). No explanation is given for why the head of department is this way, although Nik reports sharing with his students a portrait of him as Other, building up the image of the eccentric genius.

> I will say to my students, Mr Michaels, he's got a calculator at the side of his bed. And they'll say 'does he? Does he?' And he'll come in and I'll say 'Sir, I've just been telling them at the side of your bed you've got an alarm clock, a lamp, a Rubik's cube, and a calculator'. … We'll talk about how many decimal places he can recite pi to, how he can do a Rubik's cube in under 25 seconds. … And I will say to my students, 'I can't do that.' (Nik)

Mathematical ability was often identified by participants through comparison with others. Although they were not asked about it, nearly every participant at some point volunteered how their mathematical ability had been recognised when they were at school both internally (for example, which set they were in) and externally by their grade or performance at GCSE or O-Level (Table 7.2) The only participant missing from Table 7.2 is Emil, the only participant whose secondary education was not in England.

Table 7.2 Information provided by participants about their ability grouping and GCSE performance

	Reference to school's recognition of ability	Performance at GCSE or O-Level
Pete	Secondary modern top set	B
Clare	Top set	*
Nik	Early entry	B
Zoe	Top of the class	B
Lee	Top set, then set 4	B
Liz	Grammar school, then set 3 at comprehensive	C
Ross	Always in the top set	C
Ben	Top set	Not stated, but did A-Level
Jon	Top set	Not stated, but did A-Level

* Clare does not give a grade, but recounts being entered for both O-Level and CSE and being unsuccessful at the latter

Sometimes participants compared themselves with another student, in each case one identified as being of higher mathematical ability than them. Pseudonyms have been introduced for the friends mentioned by Jon and Clare. These are the two oldest participants, in their 50s and 60s respectively, and they instantly recall the name of someone they sat next to in mathematics lessons several decades previously.

> I spent a lot of time sitting next to my friend Sarah who could do maths really well. I couldn't do maths really well. … My friend Sarah would be moving on and she said to me 'do this, this and this' and I just copied her. She seemed to understand what she was doing whereas I didn't get what I was doing. (Clare)

> I sat next to Paul in maths lessons and that helped me because he was clever. (Jon)

> I think I became a bit competitive with the maths geek as well. I think we all did. (Zoe)

> For maybe well over a year I was in top set maths and ended up just copying my friend next to me and struggled all the way. (Lee)

Other measures of ability mentioned by participants include accuracy and the importance of getting answers correct, usually framed within an absolutist ontology of mathematics as right or wrong. Terminology relating to speed was used regularly by participants when referring to themselves or others solving mathematical problems.

> I like it when there's a colleague and I'm like can you work this out? Sure. They'll do it quicker. (Ross)

> You'd be able to do that with a child who wasn't particularly good. Conceptually, they would have to be taken through that a step at a time, whereas a bright kid would do that really quickly. (Clare)

> I remember doing mental quick maths at the start of every lesson. Erm, and then I don't know, I think I just like, [*pause*] like I said I just did alright at it. (Zoe)

> They'd already done it and they whizzed through the next two ones (Liz)

Although his experience of schooling in eastern Europe was different to that of the other participants, Emil's reflections on why he grew to 'loathe the subject' cover all the aspects of ability identified in this section: fixedness, gender, inheritance, speed and accuracy.

> Because I found myself very often very frustrated and crying in front of a maths problem every evening with my dad, who was often just kind of screaming or growling at me that it was so easy and he couldn't comprehend why I didn't understand it. … So yeah, my maths education was, in the psychological sense, was that it was probably okay for students who were grasping the concept in a relatively quickly and relatively easily. But for me, I think it took a little bit more time. I still got the idea, but it's just the performance as well, the actual practical of it that kind of stumped me. Because another factor is that we were often asked, in fact, we were always asked to perform our calculations by hand. We were not allowed calculators until we got to trigonometry. And then we were only allowed to bring calculators in so they can help us out with that sort of thing. Everything else we had to do by hand. So I guess there was a stigma there too, that if you couldn't do mental maths, you were stupid. … I think it might be partially because they wanted us to develop this mental arithmetic skill, which is nice to have, I don't doubt that, I don't, I don't think that is bad. I think everybody should have basic arithmetic skills in their minds. How quickly maybe multiplying, divide or subtract and add. That's great. But I think in the way was it to taught to us was, you know, it wasn't the greatest, with the attitude of you can do it great, you can't do it, you're stupid. And that's what determines it. (Emil)

Emil is determined that his students do not have the same negative experience of mathematics as he feels he had. By seeing mathematics through their learners’ eyes, Emil and other out-of-field teachers of mathematics participating in this study appear to position themselves as co-creators of mathematical knowledge alongside their students and talk about wanting to project an image of flexibility rather than the perceived rigidity of their own teachers. For example, Ross was talking about how if he comes across mathematics content that he was not confident with he prefers to consult colleagues rather than videos or textbooks. Other content in his interview suggests two reasons for this. One is because his focus is on how a particular group of students will learn the mathematics, and so a local source has greater contextual knowledge. The other is because it allows him to understand and question the approach so that he can adapt it. Ross states that this flexibility, this re-creation of the mathematics explained to him by in-field colleagues, helps his students to learn.

> So, I would just want to know how they [*colleagues in mathematics department*] work that out and then they'd show me the method. Then I'd check that and then I might think, well, I will do it a slightly different way. For example, they were trying to factorise quadratics, and they might show me a way of finding the factors of the quadratic expression. I know I would do it differently. And it's just the way I teach my kids, my foundation kids because I think that they learn a little bit easier from me. And I think the reason they learn is that I am the same. They get things from me because I'm not a specialist. Also, I see it from their point of view sometimes. But yeah, if I was to ask for any help, I would just say show me, can you look at it? I'll ask a few questions. And then I'll just pick an easier way. (Ross)

Liz echoes this on several occasions, feeling that there are better ways to help students learn. At one point, having described an interaction between her and an in-field mathematics teacher, she exclaims,

> it would be far simpler to do it a different way! (Liz)

Both Zoe and Liz were clear that what they are enjoying about teaching mathematics out-of-field is the opportunity to work one-to-one with individual learners. Throughout Liz's second interview she talks about how she is struggling. The only part of the interview where she appears to be positive about teaching out-of-field is recounting enjoyable conversations she has had with individual year seven students about the mathematics they are doing. For Zoe individual interaction with students is also the part she reports enjoying most amidst all the frustrations of teaching out-of-field.

7.1.2 Learning mathematics as emotional

Participants appeared to express emotions involved in learning mathematics through relationships with others. Most participants were asked to sketch a timeline of their confidence with mathematics. As participants narrated their timelines, some talked about their experiences of learning mathematics and how they saw their emotions as being mediated by people, not the mathematics. The role of family and its complex relationship with notions of ability were explored in the previous section. In this section I present data reflecting how participants revealed an emotional connection with mathematics through their relationship with their own teachers and their own students. Finally, I focus on one-to-one connections where participants identified a particularly strong and important emotional bond both as learner and teacher of mathematics.

Participants shared memories of teachers who had positive and negative impacts on their learning of mathematics. Ben's experiences of learning mathematics was

> mixed, depending on the teacher. I was very good for some teachers and not for others because there was very inconsistent teaching at my school. (Ben)

His school closed and transitioned into a new school, and he reported beginning to enjoy mathematics. I asked why.

> I think the teachers that came in you could tell that they loved their subject a lot more and were enthusiastic about the teaching whereas the prior teachers didn't really give off that vibe, and I think as a student I can see it …. We were a top set and a lot of us were underachieving. And then new staff members come in and it was totally different. Much better. I think that was a turning point in enjoying maths a lot more as well. (Ben)

When participants talked positively about their own teachers, they use terms such as 'connected with' (Jon), and Zoe says that she liked her A-Level teachers because 'they were human' (Zoe). Both are about a personal connection with individual teachers – Jon reported that his teacher shared his love of cricket and Zoe's teacher shared her sense of humour. Negative experiences of mathematics teachers were used by participants to explain why they moved away from mathematics.

> Our maths teachers… they tend to be quite strict for some reason, very very authoritative, very very strict for some reason. So, I gained this rather negative motivation towards mathematics. (Emil)

> I didn't really get on with the maths teachers. They were very much 'this is how you do it, get on'. I had an amazing biology teacher and really lovely chemistry teacher and a nice physics teacher. (Liz)

For Lee, his ability to form emotional connections with students is why he was asked to teach mathematics out-of-field along with

> two other guys from the PE Department, to try and bring something to the maths team a little bit different. (Lee)

While all participants talked about the emotional connection they make with their mathematics students, the four PE specialists all expressed the feeling that they had something special to offer in this respect.

> It felt like my job within the department was just sort of to take these behavioural students. (Ross)

> We seem to have quite good relationships with students as PE teachers. I don't know whether it's because of our character, whether we're team players, whether we might be a bit more understanding. (Lee)

> I can see it even now. I can tell when a student has switched off. (Ben)

> I think you learn [*as a PE teacher*] a certain toughness, but you also learn an empathy with people, particularly in team sports. (Jon)

Some go as far as to suggest their students get a better experience because of this emotional connection, building relationships with learners, such as this statement from Lee:

> I think I had 16 students in the class and managed to get 2C's and 2D's out of the group. And that was the best results they'd had out of those lower sets before…. And I went back to the staff and said, 'look, if I'm a non-specialist and I can do that, then the teachers in the maths department need to start doing that as well.' It's about building relationships with the children, really. And getting them on board. (Lee)

Lee and Zoe both talk about the importance of one-to-one relationships in connecting with mathematics. Lee talks about struggling with learning mathematics until he had a tutor while studying for GCSEs, when he says,

> it just seemed to work and it was a lot clearer for me. Being one-to-one, it just happened that we had more time to go over things. (Lee)

This experience influences how he teaches.

> I'll go over it. I'll repeat. I'll repeat it again because I can see on some of their faces that they're not quite there. And then I'll always offer the opportunity for them to come back at lunchtime, or after school. And if they if they want that help, it's always there for them…. I've been more than happy to sit with them and help them that little bit more. (Lee)

Through successive interviews Zoe expresses increasing frustration with teaching out-of-field. However, she is keen to explain that the issues are to do with management within the department, and that the only element that she does enjoy is being in the mathematics classroom, specifically having the opportunity to help individual students. After she had mentioned this for a second time, I asked why she enjoys that.

> Oh God, I don't know, I just feel like I'm doing my job, I think. I feel like I'm actually doing something and actually helping them. Whereas in geography and history you spend so much

> time ploughing through all the content that I don't think you, erm, you don't have the time to get round the classes and have those conversations. Not in the same way, [*pause*] like I might get round the class and have conversations, but in maths I'm pulling up a chair and sitting next to a kid and helping them do something. (Zoe)

Participants also expressed emotions when doing mathematics. The five questions data collection tool (Figure 4.4) was introduced to most one-off participants about three-quarters of the way through the semi-structured interview. On several occasions I recorded in my fieldnotes that this section had been an 'emotional rollercoaster'. Participants were generally comfortable and confident by this stage in the interview. There was often a palpable change when I showed them the five questions. Participants often laughed on first seeing the questions, and the words accompanying the laugh such as,

> Oh, please don't ask me to answer any. [*laughs*] (Lee)

> Is this a test? [*laughs*] (Pete)

After handing them the questions I then asked participants which they would do first. There were often long pauses at this point, including from participants who had communicated fluently until then, followed by questions or statements which suggest an emotional response.

> [*pause*] I think question one. I've got to say why now, haven't I? (Nik)

> Erm [*pause*] I feel like there's one you want me to pick. (Ross)

> [*reads under breath*] [*pause*] that is [*pause*] no [*pause*] I'm drawing brackets there. Am I doing this right? Is that what you're after? (Ben)

Ben and Clare both began by talking about the questions through students' eyes, with Clare expressing students' anger and frustration about some of the questions.

> A lot of the lower end students switch off with very text heavy questions. (Ben)

> So one of the things I notice right away is why has the person put ‘long thin’ in? A child who is not particularly good at reading or getting the concept that doesn't process that very well would not realise why that's significant to working out the rest of them. I could do that question, I’d struggle, but I could do it…. There's one, that word ‘dimensions’. Why’s it not saying to the child ‘what's its length and what's its height’? That's a barrier. Because some children could do the maths, but the actual words would be the problematic thing for them, not the actual maths. (Clare)

Participants explained their choice of which question to do first in terms of emotional connection with the question.

> That one, because it’s … it’s accessible to me. Because I know where it belongs. (Jon)

> I really don’t like question 5. (Ross)

> I really don’t like question two, because I find them boring. (Nik)

Pete became very engaged with the questions. The rich script of Pete working through the questions can be found in Appendix F. Below are snippets from this where he uses language that suggests an emotional engagement with the mathematics and process of solving the questions.

> And I enjoyed doing that because it had a word and an equation. (Pete)

> I'm going to look at 5. And I'm starting to regret it 'cause that looks a bit more complicated than question one. I thought I was doing quite well on question one. (Pete)

> I have got a feeling about 50 and two 'cause it's got 54 in the middle. And there's a two, and I can now I can double 2. So I've got a feeling about that. (Pete)

> I want to add them, so I reckon… oh dear, that doesn't work. Oh no, they’re negatives, aren’t they? (Pete)

> I was expecting it to be a nicer number than 26. (Pete)

Thanks for question three. And then question 4. This is a nice one. (Pete)

And so sometimes when I when I'm talking about the left-hand set of brackets and the right-hand set of brackets, I'm sure people in the little maths club … erm … I'm sorry, I don't mean that. I was just being facetious, not facetious, just silly. But you know there is a more comfortable way. Actually it's things like that when I think that I'm giving away that I don't quite know what I'm doing. Not that I mind, but… so yes, so I think I've got that one right. (Pete)

There was a twist designed into the five questions data collection tool: they were all the same question. All five questions relied on the same underlying mathematical structure of $x^2 - 27x + 50 = 0$. On discovering this, all participants expressed emotions in their response. Half of the participants noticed that there was some connection between the questions, including Pete. He notes that two solutions are 'remarkably similar' and then works through the rest, questioning me where he is not sure. As he does this, he expresses both positive and negative emotions as this series of snippets shows.

That's really nice to have those two questions together. It's lovely. (Pete)

Oh, oh, I love it when this happens. (Pete)

They're beautiful. They're lovely. Lovely questions. (Pete)

Isn't that nice? (Pete)

I'd have to look it up and I'm quite embarrassed about that, actually. (Pete)

Oh, that is beautiful. (Pete)

Oh, isn't that wonderful? I do like it when things come together. (Pete)

Oh, I could have just halved the lot. Yeah, yeah, oh that's irritating. (Pete)

Most participants shared Pete's tenacious desire to revisit each question and understand how they were linked and expressed an emotional response. Two other participants had strong emotional responses, with one repeatedly saying,

> wow, that's blown my mind. (Clare)

The other strong emotional response came from Nik who had been engaging with the mathematics, but as soon as he realised there was a mathematical connection that he didn't spot, his emotional focus returned to relationships as he wondered whether his colleague Jon would notice the connection. Pete's interview stands out for his emotional connection to the mathematics. In most cases emotional connections were mediated through human relationships.

7.2 Discussion

Participants' epistemologies of mathematics suggest notions of fixed ability which privilege accuracy and speed, and which involve a complex emotional element with learning mediated through human relationships. As with their ontologies, participants' epistemologies appeared to relate to school mathematics seen through the lens of the learner (Table 7.3). This appeared to be more complex with respect to their epistemology, looking through the lens of themselves as learners, as well as of their students and other people. Other people appear to be significant in participants' epistemology of mathematics, with human relationships playing an important role in the recontextualisation work between school and pedagogical mathematics. This discussion begins by relating participants' notions of ability and the role of emotions in learning mathematics to the literature explored in section 2.1.2. It then discusses the forms of mathematics and the boundary zones where they undertook their recontextualisation work.

Table 7.3 Extract from Table 2.3 (Learning)

		Learning (epistemology) *Understanding of how we come to know mathematics*
School mathematics	*The mathematics learnt at school* (Golding, 2017)	Learn according to fixed ability, privileging of accuracy and speed. Complex emotional element.
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)	Intuition important and recognised as emotional work.
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)	Learners require support because of challenge and emotional element.
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)	Mathematics used as tool appropriate to context. Individual and societal emotional element.

Ideas of fixed ability were evident in participants' contributions about themselves as learners and about their own students. The data suggested that these out-of-field teachers regard ability in mathematics to be inherited (Ernest, 1991), gendered (Walkerdine, 1988; Mendick, 2006; Alderton, 2020) and confined to an elite (Davis, 2014). That an individual's ability can be identified by the speed and accuracy of their mathematics was also evident (Belbase, 2019). Participants talked about competitiveness being a feature of learning mathematics (Mendick, 2006), and Zoe's comment about competing with 'the geek' resonating with Mendick and Francis' (2012) work on boffin and geek identities. These ideas of fixed ability and privileging of speed and accuracy were largely expressed by participants within the context of school mathematics, and through the lens of learners. The learners' lens was not just that of their students, but also their personal experiences as learners. There appear to be similarities between participants' conceptualisations of mathematics as being dependent on fixed ability and measured by speed and those identified by researchers of in-field mathematics teachers. However, the out-of-field teachers participating in this study do not all consider themselves to have that ability or speed.

Participants expressed feelings of being Othered by mathematics, such as Pete's comment about people in the 'little maths club', possibly meaning people like Nik's eccentric genius head of department. With their dominant absolutist ontology of mathematics being either right or wrong, these emotions are consistent with the Othering presented by Walkerdine (1988) and the exclusionary masculine nature of Western mathematics described by Ernest (2004). Participants were further Othered as non-specialists (Alderton, 2020). In section 2.1.2 I used the literature to show how a categorisation of emotional responses to mathematics as positive or negative was insufficiently complex. Participants' position as out-of-field teacher and self-perception as a non-specialist added an additional dimension to their emotions. As in the opportunity model literature that considered the positive effects of failure (Bibby, 2002; Lutovac, 2019, 2020), several participants asserted that their emotional struggles as learners of mathematics made them more empathetic and better teachers than in-field mathematics teachers. Emotional engagement in mathematics is complex not only because it involves positive and negative emotions, but because the emotions are often about other human beings rather than about the mathematics. Pete was the only participant whose interview produced any data that could be interpreted as direct emotional engagement with mathematics.

Participants rarely talked about learning with respect to everyday or academic forms of mathematics. While nearly every participant made an emotional response when I showed them the five questions data collection tool (Figure 4.4), I interpret their responses as being emotions about me as researcher and perceived mathematics expert, rather than as a response to the mathematics itself. Ben, for example, appeared to be confident throughout his interview apart from at this point, when he repeatedly checked whether he was doing what I wanted. Pete's sustained, rich and emotional response to the five questions (Appendix F) reinforces the point that emotional engagement with mathematics is complex. He became excited and embarrassed, he experienced pleasure and frustration. Like the research mathematicians interviewed by Burton (2001), he was comfortable to experience mathematics as emotional. Pete also hints that like Burton's participants, he considered this to be inappropriate in the mainstream mathematics classrooms, commenting that working in a special education setting freed him from such constraints. As the only participant to have studied any academic mathematics, Pete's thoughts about emotions in mathematics appear to be consistent with those in the literature. In Nik's everyday mathematics anecdote about his colleague Jon buying paint, the main emotion, shame, is about human relationships, about how others perceive mathematical abilities. Participants' epistemology of mathematics, their thoughts about learning school, academic and everyday mathematics, appear to be largely consistent with those found in the literature and summarised in Table 7.3. My discussion so far has suggested that when participants talked about learning mathematics, they were talking about school mathematics through a learner's lens. In the next paragraphs I suggest that this affects their pedagogy of mathematics and their recontextualisation work.

It was in the boundary zone between school and pedagogical mathematics that participants appeared to be undertaking their recontextualisation work with respect to learning mathematics. The importance of emotions and participants' awareness of their students' potential to be Othered or excluded by notions of fixed ability, competitiveness, speed and accuracy led to the perceived need for a pedagogy of support (Darby, 2010). Several participants stated this explicitly and as they talked about how their teaching of mathematics was informed by their own experience as learners of mathematics, I experienced a glimpse into their recontextualisation work at this boundary. Ross' strategy of asking an in-field mathematics teacher to show him how to do some mathematics, and then choosing to do it 'an easier way', is one example of this. Emil's pedagogy was founded on his perception of having been in the same position as his students and was designed to protect them. These are examples of a pedagogy of care (Watson, 2021). Some participants positioned themselves as sharing the experience of learning mathematics with their students. They were co-learners. In the case of the two participants teaching mathematics for the first time, Liz and Zoe, this meant literally sitting alongside their students working with them. Participants did not position themselves as the authority in the classroom – as Nik said, 'I am 100% not the expert' – what Byun (2019) describes as decentred epistemic authority. Several participants expressed the opinion that they understood their students better than an in-field teacher of mathematics would. The validity of this statement is less important than the implication that participants were consciously aware of the value-laden emotional nature of mathematics education. The emotions they identified in learning mathematics were almost exclusively located in relationships, rather than mathematical content, a fallibilist perspective.

To represent participants' recontextualisation work in the learning of mathematics, only school and pedagogical mathematics are included in the diagram (Figure 7.1). Academic and everyday mathematics rarely featured. The boundary between school and pedagogical mathematics is blurred, and this was particularly the case with respect to the emotional dimension of learning mathematics as participants appeared to express emotions about themselves as a learner and teacher of mathematics. The recontextualisation is represented in this diagram as bidirectional. While the direction appeared to be mainly of recontextualising participants' own experiences as learners of school mathematics into pedagogical mathematics, both forms of mathematics informed each other. This symbiosis occurred because participants appeared to conceptualise the learning of mathematics through human relationships, and so they used human relationships as tools for recontextualisation. The human relationships that they employed included those with their in-field mathematics colleagues, their students, their own offspring, and memories of relationships with parents, schoolfriends and their own teachers.

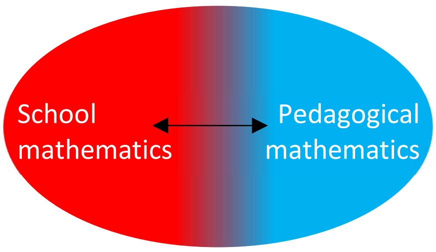


Figure 7.1 Participants recontextualisation of epistemological knowledge

I end this discussion by considering the influence of participants' in-field subject on their epistemology of mathematics, their recontextualisation work at the boundary between the pedagogy of their in-field subject and pedagogical mathematics. As noted in section 3.2.2 and discussed in section 6.2 I have often heard the idea expressed anecdotally that out-of-field teaching is possible because professionals 'teach children, not a subject'. The implication is that out-of-field teachers' boundary crossing work is between the pedagogy of their in- and out-of-field subjects. Some of the data could be interpreted as being recontextualisation between their in- and out-of-field subjects. For example, Ross stated that he taught lessons where his students went around the school measuring objects because he liked active lessons, not to meet mathematical learning objectives. While this is an example of recontextualisation of in-field pedagogy, it is mediated through Ross' own experiences of learning mathematics.

7.3 Conclusion

The epistemologies of mathematics expressed by the out-of-field teachers participating in this study involve a complex emotional element with learning being mediated through human relationships. As in chapters 5 and 6, participants' conceptualisations of mathematics are related to school mathematics seen through the lens of the learner, but in the case of how they conceptualise learning mathematics, the lens is often that of themselves as learner of school mathematics.

School mathematics is the most common form of mathematics that the out-of-field teachers participating in this study have consciously experienced as learners. They have also not had a formal educational experience to recontextualise their mathematics knowledge into pedagogical mathematics. The finding in this and the two preceding chapters that participants conceptualise mathematics as school mathematics through a student lens is therefore not surprising. What is noteworthy is the complexity of the student lens with respect to how participants conceptualise mathematics epistemologically. This complexity arises as the epistemology formed by their reflection on their own experiences of learning mathematics is recontextualised when they find themselves as a teacher of mathematics. The process and outcome of this recontextualisation appears to be that out-of-field teachers of mathematics position themselves alongside their learners. Their talk about learning can be interpreted as a fallibilist co-construction of knowledge with their students. Yet absolutism is also evident in how they talk about the binary nature of mathematics questions and solutions, in notions of fixed ability and the privileging of speed and accuracy. This tension between absolutism and fallibilism is illustrated in how participants portrayed competition as an inevitable, sometimes helpful, sometimes unhelpful element of mathematics.

This chapter makes the contribution that out-of-field teachers of mathematics participating in this study appear to privilege human relationships in the learning of mathematics. Participants expressed emotions more when talking about their relationships with others that accompanied the learning of mathematics than about mathematics itself. The next chapter considers what this means with respect to out-of-field teachers' axiology of mathematics, why they value doing mathematics.

8. Doing mathematics: Out-of-field teachers conceptualising the purpose of mathematics

The previous three chapters suggest that the form of mathematics conceptualised by the out-of-field teachers participating in this study is school mathematics through a student lens. It was previously (chapter 7) suggested that participants privilege emotions and human relationships over mathematics. This may provide an idea of why they value doing mathematics. Three purposes for doing mathematics were identified in the DfE (2013) programmes of study and explored in the literature review (2.1.3): functional, intrinsic and gatekeeper. This chapter also considers a possible new category: mathematics for personal growth.

8.1 Analysis

The first part of the analysis (8.1.1) considers reasons for doing mathematics, subdivided into mathematics as functional, for its own sake, as gatekeeper and for personal growth.

Although most were not asked about their reasons for teaching mathematics out-of-field, all participants talked about it. This data is included in the analysis (section 8.1.2) to provide a rich layer in understanding out-of-field teachers' reasons for doing mathematics.

8.1.1 Reasons for doing mathematics

Mathematics as functional

Several participants stated explicitly that when teaching they like to relate mathematics to real life contexts, with one attributing his rapport with students to his ability to

> bring a bit more to real life situations. (Lee)

Here participants talk about how they relate mathematics to professional and personal contexts in the classroom:

> In what jobs might you use decimals? This was a bottom set, and they were really good at coming up with ideas of jobs that you would actually use. Obviously, accountants to begin

with because you're working with money. But then they said things like painter and decorator for measuring and things like that, that sort of jobs coming up with, so that was really interesting. I liked the fact that we're doing this and then think about how this might apply to jobs. (Clare)

We talked about a cup of tea. If you put milk in it, you've got something hot and cold. What happens to the temperature then? Temperature goes down. So, we were talking about you're adding a negative, something cold to something hot. It goes down and they kind of saw that, which is quite good. (Liz)

When asked the email follow-up question about what he would put into the curriculum, Ben's response was functional:

I would love to see different option routes for certain target groups within the two tiers. For example, each tier could have a group most appropriate for those going into manual labour, one for data analysists, one for higher education and so on. For those who haven't made their mind up yet, there may still be a more general option, similar to what is currently in place. In my opinion, this may give the individual more of a head start in their chosen career. (Ben)

All who responded to the follow-up email question focused on mathematics 'that would be useful in everyday life' at key stage 3 and 'mostly practical application' for foundation GCSE (Jon). Topics mentioned included:

the 'world of finance'. Students need a clear understanding of how money works, such as debt, credit, loans, mortgages and salaries. The affordability of living. (Lee)

Creating a budget, understanding and using £ per kg; £ per 100g, £ per 0.1kg; flexibly, as seen in the supermarket. (Pete)

These extracts suggest an axiology of mathematics that values it solely as a practical tool for life.

However, while participants led with functional content in email and interview responses, they moved beyond it. Table 8.1 suggests some other reasons participants gave for emphasising applications of mathematics.

Table 8.1 Reasons for emphasising applications of mathematics

Emphasising applications of mathematics can help to…	Example extract
… motivate students	If you can see a purpose in something, at least it makes your learning seem a little but more worthwhile. (Liz)
… capture their interest	I think that's quite easy to link to real life… so that kind of question does interest people. (Nik)
… prepare for the future	we start to work out what we're good at and what we like and what we are going to use. (Clare)
… aid understanding	I tend to [*bring in real life situations*] so they can picture it a little bit easier. (Lee)

There was disagreement between participants about the place of conceptual understanding with respect to the functional value of mathematics. At one point Nik apologised

> that's probably just because I try and teach not so much the concept, but how it links to the real world, (Nik)

suggesting they are mutually exclusive. On the other hand, Clare appears to conflate conceptual understanding with utility.

> My niece was talking to me about further maths, the application of maths and engineering and stuff and somebody who's going to go on to be a future engineer is going to have those concepts all the way through, aren't they? (Clare)

Emil, Zoe and Liz consider the relationship to be more complex. In each case they did this while reflecting on the relationship of mathematics to their own subject (psychology, geography and physics respectively). While reflecting on the teaching of statistical tests, both Emil and Zoe note that of three elements (mathematical concepts behind the tests, the algorithms for applying the tests and the need to understand and interpret the results of the tests), mathematics teaching ignores the last and their own disciplines ignore the first of these (Figure 8.1). Liz explored the relationship in depth, leading me to ask whether mathematics is the tool that allows conceptual explanations in physics,

> Hmmm. [*pause*] Right. [*laughter*] Oh that hurts my brain. I guess with physics it's the theory and the application. The mathematical part is more the application of the theory… so … oh … erm … yes …. Then they're kind of intertwined, aren't they as well? [*pause*] I don't think I can answer that. [*pause*] Because I think they're too intertwined. When you break it down, you're using your mathematical skills to demonstrate the theory. (Liz)

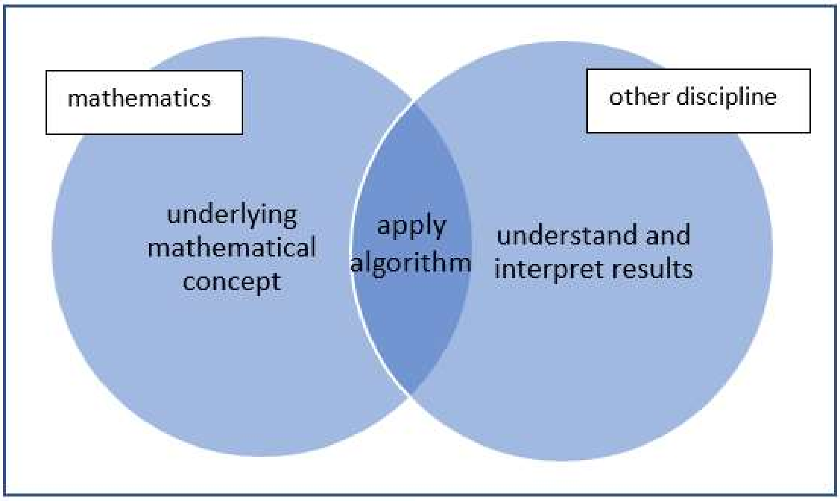


Figure 8.1 Relationship between mathematical concept and function

Several participants commented on topics that could be argued to have been surpassed by technology. In two cases this was related to their own discipline – maps on phones in geography (Zoe) and pre-wired sockets in physics (Liz). In both cases the in-field teacher attempted to justify the retention of the content. Nik asked why there was non-calculator paper at GCSE when we always have access to a calculator on our phone, and Emil noted that,

> every single scientific degree, including psychology, including sports science, in order to do statistics, they use a piece of software like SPSS, (Emil)

yet he is required to teach students to carry out the t-test by hand. These two out-of-field teachers could not understand why students are required to learn mathematics that technology can do for us.

Mathematics for its own sake

Participants rarely explicitly stated that mathematics has its own intrinsic value, and when they did it was often following on from an assertion of the utility of mathematics.

> I would like to see more maths that would be useful in everyday life, although there would also need to be content which would prepare students for KS4 and beyond. We cannot lose sight of that. I don't buy into the 'when will we ever need trigonometry' whinge, because I believe that some things in life are just worth doing. (Jon)

> I do think that maths is vital. I think some aspects of maths is more important than others in a utilitarian way. I don't think any of it is wasteful. (Clare)

Implicit expressions of the intrinsic value of mathematics are inevitably my interpretation, such as the excitement when participants discovered that all five questions were the same. This was part of consideration of the emotional nature of learning mathematics, the expression of which often revealed an appreciation of the value of mathematics for its own sake, for example:

> I think the teachers that came in, you could tell that they loved their subject and were enthusiastic about it … and I think that was a turning point in enjoying maths a lot more as well. (Ben)

Two participants used the phrase 'maths is everywhere' and spoke of mathematics in what I noted in my fieldnotes as terms of awe. Talking about the photo of the Alhambra, Emil said

> It's quite beautiful, and especially because this Arabic style, that's where we get most of modern maths with numerals, with algebra, with that sort of thing. I do try and implement some history in my maths…. So, I would say it [*mathematics*] is everywhere. It's in the architecture, it's the wonderful texture the tiles have … it's everywhere, and to me it's absolutely beautiful, in multiple ways. (Emil)

When talking with colleagues, Pete describes mathematics as

> the language of the universe. (Pete)

By doing this he hopes to communicate

> this idea that we're doing [*mathematics*] all the time and it's not separate from our being. If we're alive and we're breathing and we're computing volume all of the time without realising, and if we can do that without realising if we can breathe and intake oxygen and there's nothing to stop us then or doing Pythagoras afterwards, you know. (Pete)

Mathematics as gatekeeper

Every participant related their own learning of mathematics to public examinations at age of 16 (Table 5.1). I did not ask about GCSEs, but the majority stated their grade (Table 7.2). The value attributed to examinations in learning mathematics was stated explicitly with respect to participants' own students. Lee expressed pride in the C grades achieved by his first mathematics group, and Jon stated bluntly,

> you've always got in the back of your mind the time pressure and the exams. (Jon)

Participants expressed the value they attributed to mathematics in comments they made about how their students learn. When expressing surprise that her mathematics students did not appear to be expected to show their working out, Liz appeared to be valuing conceptual understanding. As she continued it seemed that she was not wanting an insight into what students were thinking, but explicitly states that the point is preparing them to pass their GCSE:

> it's really different because the expectation with year 7 is not always writing down their working out, which to me sounds very foreign and very odd because we spend our time going, 'show your working out, show your working out.' And I also know that at GCSE they have to show their working out. So, I quite often go in and say 'I want you to write your working out when you're doing this'. And then I go round because their other teacher doesn't insist on it and they're not doing it. But I'm going 'but you need to show it because

> when you get to GCSE, blah blah blah,' but they're going 'but I can do it, it's right.' That's not the point. You need to show it at GCSE. (Liz)

When asked to pick which question he would do first, Jon apologised for selecting the straightforward algebraic equation. I asked why he felt the need to apologise. He explained that

> it takes away the exploration of maths, (Jon)

but that the reason he'd pick that one is that his students would also pick that one because

> I know for a fact, and because it's the way that I think kids are, that they know that they are going to be examined at the end of the day, and I know that my students in my class would love to see that on the paper. Because they'd go straight in and say, right, I've got three options here. I'm either going double brackets, formula, or completing the square. (Jon)

All participants appeared to perceived mathematics as a gate*way*. Clare was the only participant to address ideas of mathematics as gate*keeper*.

> You've got what the government said recently about the fact that you can't go in and do a degree unless you've got much in GCSE or wherever. A bit topsy-turvy. Very artistic people, for example, you know, it's going to be a barrier to them. (Clare)

She immediately seeks to justify why such a barrier may be necessary, saying

> But then, why do you need a maths GCSE? Because it's a certain way of training your brain. The actual thing, even if you're never going to use it again, it's using your brain in a certain way, I think it's going to be something that's important to life. (Clare)

I put Ernest's (2020) comparison of mathematics to classics to her. While she reflected that mathematics could be perceived as elitist, she returned to her theme of training the brain in useful ways.

Mathematics for personal growth

This subsection includes extracts which I interpreted as being about participants' views on the purpose of doing mathematics and is additional to the reasons for mathematics given by the programme of study (DfE, 2021). In the discussion (section 8.2) I will use them to explore the idea that these represent an alternative purpose, which I am calling mathematics for personal growth. It was noted previously that all participants – unsolicited – shared their GCSE (or equivalent) outcome. Some of these were shared with pride, others with shame, but all with an emotional engagement which could be interpreted to suggest they derived or lost personal worth from learning and being examined in mathematics.

> I could see how it worked and I was like 'oh wow'. There's that little moment where you actually fully understand something, and I came out with a B in maths which was the top grade in that intermediate paper. (Lee)
>
> We were a top set and a lot of us were underachieving. And then new staff members come in and it was totally different. Much better. I think that was a turning point in enjoying maths a lot more as well…. Lessons were more engaging, there was more content …. it was more challenging (Ben)
>
> And it was a long time ago… I don't care now. I didn't even care then. (Pete)
>
> I wasn't confident in my ability at the time. I think that's probably why I didn't go down the maths route. I think A-level really sort of made me think I was probably not as good as I could be and as others around me. [*laughs*] (Jon)

Liz and Zoe both expressed enjoyment of supporting students one-to-one and commented on how observing them make progress was what was keeping them going. Clare twice gave examples of individual learners who she described as feeling proud when they overcame barriers. She appeared to share their pride, using the same phrase both times,

> I played a part in that. (Clare)

Ross expressed enjoying learning mathematics more when he was able to vicariously share in his students' sense of achievement:

> I enjoyed learning bits of maths and I thought all right, okay, I do enjoy parts of it. I enjoyed it when certain students did well. I had, like a couple of set threes with bright kids in the class, clever kids. And I thought I am getting something out of this. I felt like I was getting something. (Ross)

Several participants talked about how they deliberately teach to promote that feeling of success (Jon), and support young people (Pete), formalising mathematics for personal growth as a reason for studying mathematics.

> I think that that the doing and the success coming first…, they need that, because they're probably coming from sort of, well, earlier experiences in maths where it'd be 'I don't like maths, I can't do maths, maths is too hard'. So, I think sometimes getting in success first, go back to the actual mathematics behind it works quite well for me as well. (Jon)

> My goal is to support, you know, vulnerable pupils and I think maths is quite a good way of doing that. (Pete)

8.1.2 Reasons for teaching mathematics out-of-field

I rarely asked participants about their reasons for teaching mathematics out-of-field, yet all talked about it. It has already been noted that this is an area requiring further research (see section 3.1.2). Participants' reasons for teaching mathematics out-of-field are presented here to add a layer of richness to understanding how they value doing mathematics. Reasons participants gave for teaching mathematics out-of-field fall into two overlapping categories: to teach in their chosen institution, and because they considered themselves to be the best option the school had.

Clare, Pete, Zoe and Emil all chose to work in an institution knowing that it would mean teaching out-of-field. Their reasons for doing so are diverse. Clare took early retirement for personal reasons but found that she missed working with young people. She returned to her former employer to be ‘in my comfort zone’, even though she knew she would have no choice over what subjects she taught. Zoe and Emil accepted jobs in which they knew they would be teaching out-of-field to work in an institution which they believed would develop their careers. Zoe accepted that working in a challenging environment with potential for promotion and personal impact on school improvement would involve teaching out-of-field. Emil wanted to launch an academic career and approached someone he knew in the university for any job that would provide him with opportunities to

> move on to psychology because that’s my passion. (Emil)

Pete was focused on a single institution (pseudonymised):

> I taught a bit of music and then was a youth worker for a long time and worked with disadvantaged young people. I came back into teaching thinking ‘I want to work in special. I want to work alongside vulnerable young people and support them.’ At the time I wanted to work at Manor. So I applied for Manor without really knowing what I was applying for, just sort of, you know, I fancy working for you if you have got a job and they invited me to an interview. At the end they said, ‘brilliant, you've got the job.’ And I said, ‘what job?’ And they said, ‘maths teacher.’ And I said, ‘OK.’ So I sort of landed in maths kind of by chance, really. (Pete)

Pete does not report what he felt about mathematics before this, but followed the extract above with

> I really enjoyed teaching it. (Pete)

Emil knew that he was entering into a job where he would be required to teach a subject he did not want to teach. When he made contact with the university he was told,

> ‘we're looking for maths tutors.’ And that was a red flag for me because as a student, I absolutely loathed the subject. It was a big struggle for me because I love physics, I love

> chemistry, I love biology. I love all of those things that involve maths, but I hated maths itself. So I very reluctantly accepted, just to try it out at the start. (Emil)

On the other hand, Zoe knew she would have to teach out-of-field and actively sought mathematics teaching:

> I mentioned that if I had to teach another subject out of humanities, I'd quite like to have a go at some maths…. I think both I and the other geography teacher said that if we had to teach outside geography then we'd quite like to give maths a go…. So I've always said I'd quite like a go at it. (Zoe)

Ben, Nik and Ross also spoke about how they chose to teach at their school knowing that they would be teaching mathematics out-of-field. For Ben it meant job security.

> It was an opportunity to get onto the teaching pay-scale and actually get the teachers' pension and everything like that, so I thought I would go for it. (Ben)

Ross could see potential for career development,

> I thought it was a little bit of a steppingstone, two strings to my bow, sort of thing. (Ross)

Ben, Nik and Ross, along with their colleague Jon, also fall into the second category of considering themselves to be an appropriate choice by the school, with Nik summing this up as follows:

> The principal at the time offered me the position of maths teacher because they didn't have maths staff and she thought I was a good teacher of anything. (Nik)

Lee explains why his senior leadership team asked him to teach mathematics, and how he believes they were correct to do so:

> they asked me and two other guys from the PE Department to try and bring something to the maths team a little bit different. And so I was given the low set. Quite challenging students. The head teacher said, 'right they're yours. Mould them. Do what you want with

> them so that they get the grade.' And I think I had 16 students in the class and managed to get 2C's and two D's out of the group. And that was the best results they've had out of those lower sets before. I was super happy with that and confident with what I taught them. And then I went back to the maths staff and said, 'look if I'm a non-specialist and I can do that, then the teachers in the maths department need to start doing that as well.' (Lee)

Liz also expressed the opinion that the school was right to invite her to teach mathematics out-of-field. At first she asserts that this is to do with the proximity of her in-field subject, physics, to mathematics:

> They obviously looked at the physicists as they must be able to do it. They said, 'you can do A-Level physics, so you must be able to do maths.' (Liz)

I pushed on why it was her rather than any of the other physics teachers who was asked. She responded:

> I have four of us in our department. I know that we are very different in styles and we probably all could deliver maths to an extent. But the way we approach things and the way that we look at things is different. I will make sure that I understand how you do a thing first. I will go and I'll put the effort in to know for my own sake how I work something out because if I don't understand it, I can't help students. So I've got to find out and I need to work out. I do work with people who would quite easily say, 'well, I've given them a thing, and the answer's in the back. Just let them get on with it, mark it at the end.' They would literally just let them get on with it and then mark them at the end. But I want to be able to understand the bits behind it. (Liz)

8.2 Discussion

Participants' axiological perspectives appear on the surface to value mathematics as functional knowledge, providing humans with useful tools. This would align their reasons for doing

mathematics with those presented in Table 8.2 as everyday mathematics. However, in this section I will explain that I interpret their assertions about real-life applications of mathematics as suggesting that their main reason for doing mathematics is personal fulfilment, especially through gatekeeping qualifications. This aligns their reasons for doing mathematics with school mathematics.
Another aspect of doing mathematics for personal fulfilment suggested by the data is the sense of achievement or satisfaction that people derive from learning or helping someone to learn mathematics. I propose this as a further reason for doing mathematics and refer to it as mathematics for personal growth. Following discussion of these axiologies within the framework in Table 8.2, I will consider the boundaries within and beyond this framework at which participants are undertaking recontextualisation work. The theme of mathematics for personal growth continues in the discussion about recontextualisation and connects to some final thoughts on participants' reasons for agreeing to teach mathematics out of field.

Table 8.2 Extract from table 2.3 (Doing)

		Doing (axiology)
		Conceptualisations of purposes for doing mathematics
School mathematics	*The mathematics learnt at school* (Golding, 2017)	Acts as gateway/gatekeeper, qualifications valued.
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)	Mathematics valued intrinsically.
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)	Acts as gateway/ gatekeeper, qualifications valued.
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)	Mathematics valued as a tool to other ends.

All participants talked about mathematics as being functional, the 'relevance imperative' for learning mathematics identified by Darby-Hobbs (2011). Darby-Hobbs uses data collected in Australian schools to build a categorisation of the relevance imperative in mathematics and science teaching, basing her work on Newton's (1988) theoretical framework of relevance. Newton's identification of external and internal relevance provides a good model for how participants in this study spoke about making mathematics relevant to their students. Newton defines external relevance as making subject content relevant to learners' lives, like Clare did when asking students to think about decimals being used in everyday life. Internal relevance is used to make content more attractive and easier to learn as reflected in Table 8.1. Darby-Hobbs (*Op. Cit*.) finds that teachers adopted different approaches when using real-life contexts to make meaning for their learners, but that whatever their approach, they sought to connect students' lives to the subject with the purpose of enhancing engagement and supporting the learning process. Participants were thus using real world context not because the content itself was relevant, but because it aids teaching and learning. They were not using everyday mathematics, but pedagogical mathematics: mathematical knowledge structured for teaching. Darby-Hobbs' framework is helpful in suggesting that participants may have emphasised real world applications for mathematics for reasons other than because of an underlying axiology of functional, everyday mathematics. It is not so helpful in suggesting what their axiology was. Throughout the data, participants appear to value qualifications in mathematics, the GCSE in particular. This is consistent with their ontological and epistemological conceptualisations of mathematics as school mathematics through the lens of the learner. In section 2.1.3 I identified that in both school and pedagogical mathematics, mathematics acts as a gateway or gatekeeper, with qualifications in mathematics being valued. The reasons why participants valued qualifications are complex, involving emotional engagement and a big-picture perspective which is focused on their needs and the needs of their students and schools. I want to consider this complexity from two perspectives, which I am calling mathematics for personal fulfilment:

- out-of-field teachers appear to want their students to feel success in learning mathematics,
- and out-of-field teachers express that they want to feel that they have made a difference.

Before this, I will briefly consider intrinsic value as a reason for doing mathematics, an axiology I associated with academic mathematics (section 2.1.3).

It is not the case that participants did not value mathematics for its own sake, contra to Brooks' (2016) argument that out-of-field teachers don't see the intrinsic value of their subject. Pete and Emil both spoke about the beauty of mathematics and Clare repeatedly engaged in philosophical debate about the nature of mathematics. Participants' acknowledgement of the intrinsic value of mathematics appeared to be subordinate to the need to use mathematics for personal fulfilment. For example, Ben, Nik, Ross and Jon eschewed conceptual learning in favour of exam success (albeit apologetically). Maybe the best illustration of this is Pete's lament, echoing Lockhart's (2009), that he is only able to help his colleagues and students to experience the beauty and excitement of mathematics because in their special school setting they are not driven by examination results. I got the feeling that it was not the intrinsic value of mathematics that Pete was thinking about. When he says, 'maths is the language of the universe' he explains that he wants people to understand that mathematics 'it is not separate from our being'. I interpret this as wanting his colleagues to value themselves as mathematical beings. It's about people, about their personal fulfilment, not about the mathematics. Pete wants his colleagues to have positive experiences with mathematics. All participants expressed the importance of students having a positive emotional experience of learning mathematics, consistent with a pedagogy of support identified by Darby (2010) as the signature pedagogy of mathematics.

This discussion of participants' axiology of mathematics suggests that it is pedagogical mathematics that is the dominant form they are conceptualising, unlike in chapters 6 and 7 where school mathematics dominated. Their recontextualisation work takes their own experiences of school mathematics, their own ontological and epistemological conceptualisation of mathematics, and recontextualises it in their new role as teacher of mathematics. Zoe had positive experiences as a learner of mathematics, and reported enjoyment of teaching mathematics out-of-field when she was able to help her students to experience success, saying 'I feel like I'm actually doing something and actually helping them.' Emil's school experiences were not positive, and he states that he puts himself in his learners' shoes to help them avoid feeling stupid or embarrassed. Nik speaks explicitly about recontextualisation from his own experiences of school mathematics to pedagogical mathematics, saying that it's always in the back of his mind how he was taught mathematics as he considers how he can engage his own students.

To further explore the complexities of the work participants are undertaking as they recontextualise their own experiences of knowing and learning school mathematics into pedagogical mathematics so that their learners value the subject, I will use the context of reasons participants had for agreeing to teach mathematics out-of-field. In section 8.1.2 reasons given by participants for teaching mathematics out-of-field were loosely categorised as being because they chose the institution in which they wanted to work, or because they considered themselves to be the best option available to the school. In Barańska and Zambrowska's (2022) self-determination theory study, out-of-field teachers gave similar reasons for teaching mathematics, which they characterise as the extrinsic motivations of the desire to retain or gain employment and their personal desire to teach mathematics relating to personal interest, preference or sense of competence. These interact with the loose categorisation of personal fulfilment as reasons for doing mathematics discussed in this section: out-of-field teachers wanting students to feel good about learning mathematics and wanting to feel that they themselves have made a difference. Liz and Zoe, teaching mathematics out-of-field for the first time, both reported that it was 'making a difference' that kept them going. Zoe was teaching out-of-field so that she could work in her school of choice, and Liz felt she was the best option when the school found themselves short of in-field mathematics teachers. Lee also fell into the latter category of reasons for teaching mathematics out of field, and reported having told his in-field mathematics colleagues that they should try to emulate the difference he made to his students' levels of achievement. Pete and Emil both chose their institution rather than the subject they would teach there. Extracts in section 7.1.2 illustrate how they both sought to help their students to feel good about learning mathematics, albeit from opposing perspectives of sharing its beauty (Pete) and protecting them from being harmed by it (Emil). Table 8.3 places these examples in a two-way table to illustrate how they begin to provide some coverage of the complexities of reasons for teaching mathematics out-of-field considering both the mathematics and out-of-field elements of what participants appear to be valuing.

Table 8.3 Interactions between reasons for doing mathematics and reasons for teaching mathematics out-of-field

		Reasons for agreeing to teach mathematics out-of-field	
		Chose institution, not subject	Best person available
Valuing mathematics for personal fulfilment	Want students to feel good	Emil, Pete	Lee
	Want to feel good about self	Zoe	Liz

Gaining qualifications in mathematics as an aspect of personal fulfilment can be identified in the data. Lee spoke about students' achievement in their GCSEs, Liz spoke about feeling that she had brought something to her students' potential in GCSE examinations by her focus on showing working out. As I worked with the data I began to wonder whether, regardless of examination outcome, participants were seeing mathematics as important for personal growth, both theirs and their students. While I was immersing myself in my data, I had a conversation with a colleague about the value of learning mathematics. She asserted that learning multiplication tables was valuable in its own right. I argued against this. She listened to my argument, tentatively agreed, and then said, 'but knowing my tables just makes me feel good about myself.' Hearing this flooded light on what I had heard from participants. Personal growth is an important reason for learning mathematics.

By personal growth I am not referring to the 'foundation for understanding the world' referred to in the mathematics programmes of study (DfE, 2021, p. 3) or transferrable skills (Bertrand and Namukasa, 2020). Participants did refer to transferrable skills. Liz and Clare, for example, talked about how mathematics trains your brain and develops thought processes. And an element of the ideal curriculum described by Lee included the opportunity for students to 'think outside the box' so that they could regain the creativity that he suggested they had lost. Mathematics for personal growth is not about how the skills will be used, but how they make an individual feel about themselves.

The idea of mathematics for personal growth is also not synonymous with the emotional element of learning mathematics, although possibly a subset. It is not just that learning mathematics is emotional. More than that: to experience certain emotions is a reason for doing mathematics. How mathematics makes an individual feel about themselves, learning mathematics as emotional, was considered in the last chapter (7.1.2), and the data and literature agreed that emotions are an important element in learning mathematics (Boylan, 2009). Participants spoke about wanting to help their students to experience success as a motivator to the extent that student success became a motivator for them – Ross and Clare are quoted in the previous section sharing the vicarious feeling of success. I tentatively propose mathematics for personal growth as a purpose of learning mathematics, experiencing personal struggles as a learner of school mathematics and recontextualising them as a positive experience for learners, and their teacher.

8.3 Conclusion

The data analysed and discussed in this chapter suggests that the out-of-field teachers participating in this study valued the learning of mathematics for reasons of personal fulfilment, their own and their students. Personal fulfilment can be manifested in qualifications in mathematics and in personal growth, that is, doing mathematics to experience certain emotions. Participants talked about the functional and intrinsic value of mathematics, but much less often than they talked about the purpose of personal fulfilment.

Chapter 7 suggested that the student lens was a complex mix of their own experiences as learners of mathematics as well as their students'. This complex relationship between self and student is developed in this chapter through personal fulfilment referring both to the purpose of learning mathematics so that learners will feel good about themselves, and the feeling of having made a difference by helping learners. The empathy that participants expressed for their students as learners of mathematics was interpreted in chapter 7 as being derived from their own experiences as learners and is also consistent with the signature pedagogy of support in mathematics. Identifying qualifications and personal growth as elements of personal fulfilment further illuminates its complexity. These are not distinct. Participants expressed pride or disappointment in their own mathematics qualifications, in their capacity to answer mathematics questions, and in their students' achievements.

This chapter builds on the contribution made by chapter 7 that the out-of-field teachers participating in this study appeared to privilege human relationships over mathematics. While participants did value mathematics intrinsically and for its utility, it was the human element, mathematics for personal fulfilment, that characterises their axiology of mathematics. As in the preceding chapters, it is school mathematics viewed through a learner lens, both themselves and their students, that forms the basis for their conceptualisation of mathematics.

9. Out-of-field teachers knowing, learning and doing mathematics: Conclusion

9.1 Research Focus: contribution

The out-of-field teachers participating in this study appeared to conceptualise mathematics as school mathematics through a complex student lens of their own and their students' experiences of mathematics. Ontologically (their beliefs about the nature of mathematics) their conceptualisation was a complex mix of predominantly absolutist mathematics (existing independently of the human mind) with elements of fallibilism (an evolving human construct), consistent with the mathematics and mathematics education literature. Epistemologically (understanding how people come to know mathematics), human relationships were privileged in the learning of mathematics over mathematical content. This was developed in participants' axiologies (reasons for doing mathematics), which suggested that personal fulfilment is the purpose for learning mathematics, both through public examination and personal growth, that is, doing mathematics in order to experience certain emotions.

The theories of recontextualization (Bernstein, 1990) and boundary-crossing (Akkerman and Bakker, 2011) were employed to understand how participants developed their conceptualisation of mathematics. Participants appeared to be drawing mostly on their experiences of school mathematics as learners, recontextualising this knowledge into pedagogical knowledge. Most of their boundary-crossing work appeared to occur at the boundary between school and pedagogical mathematics. This boundary was weak and the direction of recontextualisation from school to pedagogical mathematics. Academic mathematics was mostly irrelevant to participants in this study, and everyday mathematics conceptualised as recontextualised school mathematics.

This was not a comparative study, nor does the bricolage as methodology allow any generalisable claims (Kincheloe and Berry, 2004). The contribution this study makes is to suggest that out-of-field teachers may bring something different with them to the subject they are teaching, but not too different. This sameness and difference is illuminated in Table 9.1. A secondary table has been added to the conceptual framework that was developed in chapter 2, summarised in Table 2.3 and used throughout chapters 5 – 8. It is not an additional row as it is not an additional form of mathematics. It is aligned with the conceptual framework so that out-of-field teachers' conceptualisations of mathematics discussed in this research can be understood in the context of conceptualisations of mathematics in the mathematics and mathematics education literature.

Table 9.1 Out-of-field participants' conceptualisations of mathematics

		Knowing (ontology)	Learning (epistemology)	Doing (axiology)
		Teachers' beliefs about the nature of mathematics	*Understanding of how we come to know mathematics*	*Conceptualisations of purposes for doing mathematics*
School mathematics	*The mathematics learnt at school* (Golding, 2017)	Complex mix of dominant absolutism, with elements of fallibilism.	Learn according to fixed ability, privileging of accuracy and speed. Complex emotional element.	Acts as gateway/ gatekeeper, qualifications valued.
Academic mathematics	*The activities that advance mathematical knowledge* (Watson, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.	Intuition important and recognised as emotional work.	Mathematics valued intrinsically.
Pedagogical mathematics	*Mathematical knowledge for teaching* (Ball *et al.*, 2008)	Complex mix of dominant absolutism, with elements of fallibilism.	Learners require support because of challenge and emotional element.	Acts as gateway/ gatekeeper, qualifications valued.
Everyday mathematics	*Mathematical knowledge informally or contextually structured as it is used* (Davis and Renert, 2014)	Complex mix of dominant absolutism, with elements of fallibilism.	Mathematics used as tool appropriate to context.	Mathematics valued as a tool to other ends.

Out-of-field participants' conceptualis	*One who has taught for six or more years and has a degree and*	Complex mix of dominant absolutism, with	Human relationships central to learning	Mathematics is for personal fulfilment as qualifications and

ations of mathematics	*teaching qualification in a subject other than mathematics.*	elements of fallibilism, based on school mathematics through a student lens.	school mathematics, through lens of self as learner and own students.	personal growth. School mathematics through a student lens.

The recurrent interpretation of the data in all dimensions of the conceptual framework is that participants conceptualise mathematics as school mathematics through a student lens. The original conceptualisation of school mathematics presented in Table 2.3 and developed through engaging with the mathematics and mathematics education literature in chapter 2 does not exclude the student lens. What is different in the way that out-of-field participants' conceptualisation of school mathematics has been interpreted throughout this study lies in the human dimensions of the epistemology and axiology of participants. In Nik's words, avoiding the 'very, very dry' teaching he experienced in order to interest his learners with his 'not-a-maths-specialist point of view'. From a critical perspective, the power that determines culturally and societally informed conceptualisations of mathematics is perceived by the mathematics and mathematics education literature to lie in mathematics itself and the needs of society. Out-of-field teachers participating in this study appeared to perceive school mathematics from a different perspective as the mathematics needed by students: Nik explains that students appreciate that he is 'from a similar starting point' to them. In a complex relationship, students' needs are determined by the needs of society. The difference is not so much the location of the power, as the direction from which it is viewed. None of the participants in this study questioned the value of mathematics, but all privileged its personal and individual value over its societal, economic or cultural value.

The gap in the knowledge that this research sought to address was how out-of-field teachers of mathematics conceptualise mathematics. Although data collection tools were designed to focus on knowing, learning and doing mathematics, not teaching mathematics, pedagogical mathematics was inevitably and inextricably entwined in the data collected, as in Ben's oscillation while solving a mathematics question: 'I always try and teach the kids … [*reads under breath*] … that is … no … I'm drawing brackets there … so I am thinking about forming an equation from the worded question there.' The data analysis and discussion in chapters 5 to 8 seeks to privilege participants' ideas about mathematics rather than its teaching. As this extract from Ben's interview shows, this was challenging at times. In section 1.1.1 I argue that other researchers who experienced the same challenge have tended to respond by focusing on pedagogical knowledge. In this thesis I have responded to Hobbs (née Darby)'s call to research the knowledge and beliefs teachers bring with them (Darby, 2009b, 2009a).

The mathematics conceptual framework (Table 2.3) was developed for the purpose of structuring this thesis. An unplanned contribution of this thesis is this mathematics conceptual framework which challenges and disrupts homogenous, fixed, universal conceptualisations of mathematics (Davis and Sumara, 2006; Kincheloe, 2008). I noted in section 1.3.1 that it was my intention that this thesis be accessible to academics outside of the fields of mathematics education and teaching out-of-field as well as to a lay audience. I believe that the mathematics conceptual framework contributes to achieving this. I hope that it will also be a useful tool for researchers of mathematics and mathematics education.

The framework used in this thesis to conceptualise out-of-field teachers (Hobbs *et al.*, 2020) similarly challenges and disrupts homogenous, fixed conceptualisations: in this case conceptualisations of out-of-field teachers. It does this by demonstrating variety in the knowledge, experience and practice of out-of-field teachers, as shown in Figure 4.2 which positioned participants in this study in Hobbs *et al.*'s framework (*ibid.*). Using this conceptual framework in this way has potential value to professionals assigning and supporting out-of-field teachers such as senior leaders in schools and departmental leaders. Senior leaders could use the measurable criteria when considering potential out-of-field teachers, for example recognising that a teacher with near misalignment of specialism (dimension 1.2) with a low partial current teaching proportion (dimension 2.1) might be more appropriate where only one or two lessons need covering, while using an experienced teacher (dimension 3.2) with a high longitudinal proportion (dimension 2.2) and stability (dimension 2.3) for a long-term vacancy in spite of technical misalignment (dimension 1.1). This thesis suggests that out-of-field bring valuable knowledge with them: the self-report criteria could help school and department leaders to ensure that the right opportunities are available. For example, they could consider the school context (dimension 5.1) and support culture (dimension 5.2), ensuring that out-of-field teachers have adequate access to opportunities, resources and support. Out-of-field teachers bring something with them, and Hobbs *et al*.'s framework (*ibid*.) can help school leaders to identify the knowledge and qualities they bring and what support to provide to enable them to flourish.

This is not the first study to use the bricolage to explore learners' and teachers' (including out-of-field teachers') conceptualisations (for example, Luitel and Taylor, 2007; Gine, 2011; Howley, 2013; Armstrong, 2017). Its flexibility, the way that it responds to the context of the study, means that every study employing the bricolage is unique and therefore makes a methodological contribution. I hope that the responsive use of creative elicitation tools within the context of semi-structured interviews is a fresh approach. One example of this is the twist in the five mathematics questions which all had the same underlying mathematical structure of $x^2 - 27x + 50 = 0$. The revelation of the twist elicited data so rich that it gave insight into all areas of the study: the insight it allowed into the human agency involved in the presentation of mathematics led to exploration of ideas of fallibilism; it brought out emotions and competitiveness; it revealed a tenacious hunger to understand the mathematics. The use of creative elicitation tools within the methodological context of the bricolage is a contribution made by this thesis.

A key contribution of this thesis is its use of an opportunity model to explore the complexity of out-of-field teaching and to illuminate the agency of the out-of-field teacher. Many out-of-field teachers and leaders in schools recognise the complexity of their work, and this has been previously recognised in the literature. Existing research has recognised the complexity of out-of-field teachers' work as they recontextualise pedagogical knowledge of their in-field to out-of-field subjects. An original feature of this study was to focus on out-of-field teachers' recontextualisation work at boundaries *within* the subject they are teaching out of field. This broadens and deepens understanding of the complexity of their recontextualisation work. This thesis draws attention to the schooling and lived-experience in the subject that they are teaching out-of-field that teachers bring with them. It argues that out-of-field teachers have complex conceptualisations of the subject they are teaching out-of-field, sometimes similar to those of in-field teachers, sometimes more similar to those of their students, sometimes very different. Lee's rich and wonderfully varied analysis of the mathematics present in Wembley Stadium is one example of this complexity. Another is the full range of emotions expressed by Pete while working through some mathematics questions, from embarrassment and frustration to sheer delight. And Emil wished to protect his learners from experiencing mathematics in the same negative way he had as a learner.

Some conceptualisations of mathematics expressed by out-of-field teachers of mathematics were in tension with my personal conceptualisations of mathematics and those I teach to beginning teachers. I personally do not share Zoe's absolutist conceptualisation that mathematics is either 'right or wrong', that she 'liked maths because I could get to an answer and then I could check whether my answer was right,' or her assertion that there is a 'maths brain' which you either have or do not have. It would be wrong to argue that the conceptualisations of mathematics expressed by participants in this study are unproblematic in the classroom context, indeed it illustrates the risk associate with out-of-field teaching in relation to mathematics education, in this example ideas of fixed ability and mindsets (Boaler and Dweck, 2015) and the fixed universality of mathematics knowledge (Davis and Sumara, 2006, Kincheloe, 2008). But it would also be wrong to believe that out-of-field teachers bring nothing with them. The complexity of the recontextualisation work undertaken by out-of-field teachers within the subject they are teaching out of field appears to focus on learners' conceptualisations of the nature of the subject, how it is learnt and reasons for doing mathematics. The data is rich with their creativity (such as Pete's busking with mathematics to demonstrate to his colleagues that it is not all about right or wrong), their insights (like when Lee talks about the mathematics of emergency evacuation of a sports stadium), their empathy (Emil has walked in his students' shoes, the emotional connection that Jon makes).

I hope that this thesis will serve to demonstrate to out-of-field teachers and school leaders that alongside the well-documents risk associated with out-of-field-teaching (Ingersoll, 1998), out-of-field teachers have something to bring to the subject they are teaching out-of-field and to their students. While there are risks associated with out-of-field teaching, it is less of a risk than the literature usually portrays, and the risk is lessened if the voice of these teachers is listened to and their knowledge built upon as the starting point for their personal and professional development. Out-of-field teaching will never be eradicated and is unlikely to be a better alternative to in-field teaching. But it can be embraced and viewed as an opportunity, an opportunity for in- and out-of-field teachers, their students and the school community. I give two participants the last words:

> wow, that's blown my mind. (Clare)

> Oh, isn't that wonderful? I do like it when things come together. (Pete)

9.2 Limitations

The critical complex approach of this thesis, with its use of the bricolage, facilitated an opportunity model. That the bricolage makes no claims of generalisability is not a limitation within a critical complex theoretical framework. It allows the research to focus on and acknowledge the complexity of the phenomenon of teaching out-of-field, understanding that no two experiences are the same, and that the same experience will be interpreted differently depending on the interpreter and their historical, social context. For this reason, it will never be possible to hear the unmediated voice of participants. What we hear is influenced by the context and method of data collection and the positionality of the researcher who collects and analyses it. Rather than being a limitation of the bricolage, its embracing of the researcher's positionality and its role in outcomes is an advantage. Embracing my positionality in all aspects from my choice of research focus onwards has enabled me to tap rich resources of data, including working with two participants at the school where I learnt mathematics.

Working with participants at the school where I was a student occurred because finding participants was more challenging than anticipated. The challenges I encountered when recruiting participants may suggest that a selection bias occurred. Recruiting participants is often challenging, but I had been confident I would not encounter this problem because the literature, government data, the media and anecdotal evidence all pointed towards a prevalence of out-of-field teaching of mathematics. On several occasions I followed leads which did not result in the recruitment of participants. In each case the circumstances suggested to me that potential participants and gatekeepers held a deficit model of teaching out-of-field, views not in alignment with the focus of my study, and so decided not to take part. It may be that my study only attracted participants and schools that had an opportunity model of teaching out-of-field. An anonymous and largescale questionnaire might mitigate against this limitation but would not yield such rich data.

I found that the out-of-field teachers participating in this study tended to conceptualise mathematics as school mathematics, with academic and everyday mathematics secondary. The smaller role played by academic mathematics is not surprising in those with little or no experience of it. I had thought that everyday mathematics might feature more prominently in the conceptualisations of experienced teachers whose most recent mathematical engagement would have been with everyday mathematics throughout their adult lives, but everyday mathematics was also secondary to the school form. Pedagogical mathematics was evident throughout, and I have commented on how it was often impossible to disentangle from school mathematics. In each of chapters 5 – 7, however, I concluded that school mathematics through a student lens was the dominant form of mathematics. At the beginning of each interview I explained that my focus was on knowing, learning and doing mathematics, not teaching. The dominance of school mathematics over pedagogical mathematics may therefore be a product of research design rather than reflecting out-of-field teachers' conceptualisations of mathematics.

The mathematics conceptual framework I developed has provided the structure for this thesis and facilitated broad and deep analysis of the subject. In inception its purpose was to challenge ideas of mathematics as homogenous. I believe that it has done that, but that it could have gone further. I found that the form of everyday mathematics might have been too broadly defined. Future versions of the conceptual framework should further break everyday mathematics down. I suggest 'economic everyday mathematics' and 'folk or common-sense everyday mathematics' as two further subdivisions that would have been helpful in the analysis of some of the data and in understanding recontextualisation work at the boundaries between forms of mathematics. It is possible that further subdivision is desirable.

9.3 Future directions for research

Perhaps inevitably in an opportunity model of research, I identified many future directions of research while carrying out this study. Sometimes it was a challenge to hold back from diving into enticing new areas. Potential directions for future research can be loosely categorised as in-field, out-of-field and comparative studies.

This thesis developed a mathematics conceptual framework that challenges assumptions about the homogeneity of mathematics. The reason for doing this was to help the researcher, the researched and users of the research understand how the researcher was conceptualising mathematics, and to understand that conceptualisations of others may vary. For the framework to serve this purpose, it requires further development, particularly development of the concept of everyday mathematics, as noted in section 9.2. It was noted in section 1.1.1 that Beswick (2012) argues that teachers' beliefs about the nature of mathematics and own experiences of learning mathematics can be overlooked in the literature, and White-Fredette (2009) called for research focusing on how teachers view the mathematics they teach. This thesis does this in the context of teaching mathematics out-of-field, but more research is required into conceptualisations of mathematics of in-field teachers.

This was not designed as a comparative study to keep its focus narrow and deep, to allow a more nuanced exploration of conceptualisations of mathematics of out-of-field teachers. Throughout the collection and analysis of data I wondered whether I would hear and interpret the same conceptualisations if I were talking to people with different relationships to mathematics. The research called for in the previous paragraph into the conceptualisations of in-field teachers of mathematics, would make an interesting comparative study that would also support the further identification of what it is that is different that out-of-field teachers bring with them, and what could be used to enhance the experience and learning of students. The out-of-field participants in this study appeared to view mathematics through the lens of their students, or more accurately through their perception of how their students conceptualise mathematics. Studies into how students' conceptualisations of mathematics compare with those of their teachers are required to further understand this element of the out-of-field teachers' work. Comparative studies into the relationship between in-field teachers, out-of-field teachers and students of mathematics would support an opportunity model, searching for commonalities and what they bring. A further comparative study of those with no connection to mathematics teaching (teachers and nonteachers) may raise further questions about the nature of knowing, learning and doing mathematics.

Participants were not asked about why they came to be teaching mathematics out-of-field, but all shared their story. This is an important area requiring further research. All participants expressed agency in the decision to teach out-of-field. It was noted in the previous section that there may have been selection bias in recruitment of participants. So further research should seek participants with a range of attitudes towards teaching out-of-field and work from the premise that these are professional adults who are at liberty to choose their employment. Research is needed that explores why teachers agree to teaching out-of-field. More challenging, but potentially more informative would be to seek people who were asked to teach out-of-field but chose not to, asking what the decision looked like and what the consequences were.

This study focuses on boundaries within the subject that participants were teaching out-of-field. Further research should consider the boundaries between participants' in- and out-of-field subjects, touched on here but without the scope to develop it. Mathematics was the out-of-field subject chosen for this thesis because it is a common out-of-field subject, because it is the one that I have experience of teaching in- and out-of-field, and because it is a subject that I am passionate about. Repeating this study but for history, a subject that I have only taught in-field, or for geography, that I have only taught out-of-field, would build the body of knowledge and also develop understanding of the role of my positionality in data collection and analysis. This research should also be replicated by other researchers with various positionalities to build a broad body of knowledge to deepen understanding of what out-of-field teachers bring with them. Replications of this research with more focus on the social and cultural context of the out-of-field teacher would also deepen understanding of the opportunities provided by out-of-field teaching, focusing on schools in economically deprived areas where there is a higher incidence of teaching out-of-field.

9.4 Reflexive Conclusion

This thesis opened by introducing my research focus with sections on the gap in out-of-field knowledge (section 1.1.1) and my reflexive positionality (section 1.1.2). This chapter has so far concluded by considering how this thesis has contributed to the field, how it has addressed the gap in the out-of-field knowledge. This final part of the concluding chapter is personal, exploring reflexively what I am taking from this thesis.

Like mathematics, like my participants, I am not simple or homogenous. My understandings flex with time and space. Competing voices, competing versions of me have driven this study, and I began to recognise these voices in terms of versions of me at different stages of my life. Seventeen-year-old Fiona is at the heart of this study. Still clinging to the excitement of exploration, the amazing discovery that I could count to infinity, playing number games on car journeys, my seventeen-year-old self was confronted with an unyielding, inflexible absolutism. I felt excluded by an alien conceptualisation of mathematics. At seventeen I lacked the vocabulary to express this or the confidence and opportunity to explore how else mathematics can be conceptualised. That opportunity has been realised in this study.

Fast forward ten years. Twenty-seven-year-old Fiona is head of mathematics. My decade in exile with history is now over. I have the confidence and authority to understand that mathematics can be conceptualised in different ways. I am no longer Othered by mathematics. But I am frustrated by the label 'non-specialist'. At the age of twenty-seven I was leading a mathematics department of around twenty teachers. The only two who were technically in-field were the only two whose teaching Ofsted judged to be inadequate. The majority of my team, those creatively driving the improvement of our department, were labelled as non-specialists.

For seven years through my thirties and forties I studied for a mathematics degree, sticking with it through significant life changes and personal challenge. This love for learning has been another driver in the writing of this thesis. However, even with a degree in mathematics and no longer being considered an outsider, people still reflect that they are surprised that someone can enjoy and succeed in both history and mathematics. Early in this research I came to understand that this surprise is derived from the strong boundaries between subjects in our culture's conceptualisation of knowledge. As I worked with the literature and participants' data, I realised that there are boundaries within subjects too. These are social and cultural, and many of us are not aware these boundaries exist. We tend to assume that when someone else thinks of mathematics that they are thinking of the same thing that we are. I have dedicated my work as a teacher and leader of school mathematics and as a university lecturer to challenging conceptualisations of mathematics. I strive to make the boundaries within mathematics visible to others and help them to cross the boundaries into areas of mathematics that are creative, social, beautiful, surprising and messy. I have been determined to share my positive experiences of out-of-field teaching – not just my personal experiences, but as a leader and educator of out-of-field teachers.

This thesis puts my positive experiences of mathematics and as an out-of-field teacher of mathematics into writing. It recontextualises the personal thoughts and feelings of my younger self into academic prose. It roots my understanding of mathematics within a conceptual framework and analyses the diverse conceptualisations of ten other people who have experienced teaching mathematics out-of-field. I want seventeen-year-old Fiona to know that mathematics is beautiful, complex and social, and that there are as many different ways of conceptualising mathematics as there are people. I want twenty-seven-year-old Fiona to know that teaching out-of-field is complex work, and that those engaging in that work bring with them diverse, challenging and interesting conceptualisations of mathematics. This is why this thesis is dedicated to seventeen- and twenty-seven-year-old Fiona.

Word count: 49899

References

Adamson, F. and Darling-Hammond, L. (2012) 'Funding disparities and the inequitable distribution of teachers: Evaluating sources and solutions', *Education Policy Analysis Archives*, 20(37), pp. 1–46.

Adler, J., Hossain, S., Stevenson, M., Clarke, J., Archer, R., Grantham, B. (2013) 'Mathematics for teaching and deep subject knowledge: voices of Mathematics Enhancement Course students in England', *Journal of Mathematics Teacher Education*, 17(2), pp. 129–148.

Aguirre, J. (2009) 'Teacher domain specific beliefs and their impact on mathematics education reform', in MaaB, J. and Schloglmann, W. (eds) *Belief and Attitude in Mathematics Education: New Research Results*. Rotterdam: Sense Publications, pp. 45–58.

Aikenhead, G. (2021a) 'Resolving Conflicting Subcultures Within School Mathematics: Towards A Humanistic School Mathematics', *Canadian Journal of Science, Mathematics and Technology Education*, 21(2), pp. 475–492.

Aikenhead, G. (2021b) 'School Mathematics: Towards Ending Its Cycle of Myths', *Philosophy of Mathematics Education Journal*, 37, pp. 1–26.

Akkerman, S.F. and Bakker, A. (2011) 'Boundary crossing and boundary objects', *Review of Educational Research*, pp. 132–169.

Alderton, J. (2020) 'Kelly's story: transformative identity work in primary mathematics teacher education', *Gender and Education*, 32(2), pp. 145–160.

Alexander, R. (2017) Towards dialogic teaching: Rethinking classroom talk. 5th edn. Dialogos.

Allen, R. and Sims, S. (2018a) 'Do pupils from low-income families get low-quality teachers? Indirect evidence from English schools', *Oxford Review of Education*, 44(4), pp. 441–458.

Allen, R. and Sims, S. (2018b) *How do shortages of maths teachers affect the within-school allocation of maths teachers to pupils?* Available at: www.nuffieldfoundation.org (Accessed: 23 September 2023).

Andersson, A. and Wagner, D. (2019) 'Respond or Dismiss: Interactions That May Support Loving, Bullying and Solitude in Mathematics', *Journal of the Canadian Association for Curriculum Studies*, 17(1), pp. 47–74.

Andrade-Molina, M. and Valero, P. (2015) 'The Sightless eyes of reason: Scientific objectivism and school geometry', in *CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education*. Prague, pp. 1551–1557.

Andrews, P. (2007) 'The curricular importance of mathematics: a comparison of English and Hungarian teachers' espoused beliefs', *Journal of Curriculum Studies*, 39(3), pp. 317–338. A

Andrews, P. and Hatch, G. (1999) 'A new look at secondary teachers' conceptions of mathematics and its teaching', *British Educational Research Journal*, 25(2), pp. 203–223.

Apple, M.W. (1982) *Education and power*. Oxford: Ark.

AQA (2021) 'GCSE Mathematics Foundation Tier Paper 2 Calculator 8300/2F'. Available at: https://filestore.aqa.org.uk/sample-papers-and-mark-schemes/2021/november/AQA-83002F-QP-NOV21.PDF (Accessed: 23 September 2023)

Armstrong, A. (2017) 'Bricolage in middle years school mathematics', *For the Learning of Mathematics*, 37(2), pp. 19–24.

Arzi, H.J. and White, R.T. (2008) 'Change in teachers' knowledge of subject matter: A 17-year longitudinal study', *Science Education*, pp. 221–251.

Askew, M., Rhodes, V., Brown, M., Wiliam, D., Johnson, D. (1997) *Effective teachers of numeracy: Report of a Study Carried out for the Teacher Training Agency*. London: King's College, University of London.

Atkinson, P. (1995) 'From structuralism to discourse: Bernstein's structuralism', in Sandovnik, A. (ed.) *Knowledge and Pedagogy. The Sociology of Basil Bernstein*. Norwood: Ablex, pp. 83–95.

Ausman, T.A. (2018) *Contested Subjectivities: Loving, Hating, and Learning Mathematics*. University of Ottawa, Canada.

Ball, D.L., Thames, M., and Phelps, G. (2008) 'Content knowledge for teaching: What makes it special?', *Journal of Teacher Education*, 59(5), pp. 389–407.

Bamber, S. (2015). Learning to teach mathematics: Navigating the landscape of teacher education. (Doctoral dissertation). University of Chester, United Kingdom. Available at https://chesterrep.openrepository.com/bitstr eam/handle/10034/596533/Sally%20Bamber%20Thesis%20July%202015%20Hard%20Bound.pdf?sequence=1&isAllowed=y (Accessed 15/3/24)

Barańska, B. and Zambrowska, M. (2022) 'Neither Fully "In", Nor Completely "Out" of the Field: The Case of Teaching Mathematics as a Second Subject in Poland', in Hobbs, L. and Porsch, R. (eds) *Out-of-Field Teaching Across Teaching Disciplines and Contexts*. Springer Nature Singapore, pp. 285–307.

Bartholomew, H., Darragh, L., Ell, F., Saunders, J. (2011) '"I'm a natural and I do it for love!": exploring students' accounts of studying mathematics', *International Journal of Mathematical Education in Science and Technology*, 42(7), pp. 915–924.

Becker, H.J. (2000) 'Secondary Teachers of Mixed Academic Subjects: "Out-of-Field" Problem or Constructivist Innovators', in *2000 Meetings of the American Educational Research Association*. Available at: https://escholarship.org/content/qt2pn6n6h5/qt2pn6n6h5.pdf (Accessed: 23 September 2023).

Begle, E. (1979) *Critical Variables in Mathematics Education: Findings from a Survey of the Empirical Literature.* Washington DC: Mathematical Association of America and the National Council of Teachers of Mathematics.

Beisiegel, M. and Simmt, E. (2012) 'Formation of mathematics graduate students' mathematician-as-teacher identity', *For the Learning of Mathematics*, 32(1), pp. 34–39.

Belbase, S. (2019) 'Meanings, Dimensions, and Categories of Mathematics Teacher Beliefs: A Navigation through the Literature', *International Journal on Emerging Mathematics Education*, 3(1), pp. 1–26.

BERA (2018) *Ethical Guidelines for Educational Research*. 4th edn. London: British Educational Research Association.

Bernales, M., and Powell, A. B. (2018) 'Decolonizing Ethnomathematics', *Ensimon Em Revista*, *3*, pp. 565–587.

Bernstein, B. (1973) 'On the classification and framing of educational knowledge', in Brown, R. (ed.) *Knowledge, Education, and Cultural Change: Papers in the Sociology of Education*. London: Routledge, pp. 363–392.

Bernstein, B. (1990) *Class, codes and control: The structuring of pedagogic discourse*. Abingdon: Routledge.

Bernstein, B. (2000) *Pedagogy, symbolic control, and identity: Theory, research, critique*. Revised Ed. Lanham, Maryland: Rowman & Littlefield.

Berry, K. and Kincheloe, J.L. (2004) *Rigour and Complexity in Educational Research: Conceptualising the bricolage*. Maidenhead: Open University Press.

Bertrand, M.G. and Namukasa, I.K. (2020) 'STEAM education: student learning and transferable skills', *Journal of Research in Innovative Teaching & Learning*, 13(1), pp. 43–56.

Beswick, K. (2012) 'Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice', *Educational Studies in Mathematics*, 79, pp. 127–147.

Bibby, T. (2002) 'Shame: An emotional response to doing mathematics as an adult and a teacher', *British Educational Research Journal*, 28(5), pp. 705–721.

Billingsley, B. (2017). Teaching and learning about epistemic insight. *School science review*.

Bjälkebring, P. and Peters, E. (2021) 'Money matters (especially if you are good at math): Numeracy, verbal intelligence, education, and income in satisfaction judgments', *PLOS ONE*, 16(11), p. e0259331.

Blanton, M.L., Westbrook, S. and Carter, G. (2005) 'Using Valsiner's zone theory to interpret teaching practices in mathematics and science classrooms', *Journal of Mathematics Teacher Education*, 8(1), pp. 5–33.

Bloor, D. (1973) 'Wittgenstein and Mannheim on the sociology of mathematics', *Studies in History and Philosophy of Science*, 4(2), pp. 173–191.

Boaler, J. (1994) 'When Do Girls Prefer Football to Fashion? An analysis of female underachievement in relation to "realistic" mathematic contexts', *British Educational Research Journal*, 20(5), pp. 551–564.

Boaler, J. (2002) 'The Development of Disciplinary Relationships: Knowledge, Practice, and Identity in Mathematics Classrooms', *For the learning of mathematics*, *22*(1), 42-47.

Boaler, J. (2009) *The elephant in the classroom: Helping children learn and love maths*. London: Souvenir Press.

Boaler, J. and Dweck, C. (2015) *Mathematical mindsets: unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco: Wiley.

Boaler, J. and Greeno, J.G. (2000) ‘Identity, agency, and knowing in mathematics worlds’, in Boaler, J. (ed.) *Multiple perspectives on the teaching and learning of Mathematics*. Westport: Ablex Publishing, pp. 171–200.

Bolander, B. and Watts, R.J. (2009) ‘Re-reading and rehabilitating Basil Bernstein’, *Multilingua*, 28(2–3), pp. 143–173.

Borko, H. *et al.* (1992) ‘Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily?’, *Journal for Research in Mathematics Education*, 23(3), pp. 194–222.

Bosse, M. and Törner, G. (2013) ‘Out-of-field Teaching Mathematics Teachers and the Ambivalent Role of Beliefs - A First Report from Interviews’, in *Current state of research on mathematical beliefs XVIII. Proceedings of the MAVI-18 Conference*. Helsinki, 12-15 September 2012, pp. 341–355.

Bosse, M. and Törner, G. (2015) ‘Teacher Identity as a Theoretical Framework for Researching Out-of-Field Teaching Mathematics Teachers’, in Bernack-Schüler, C., Leuders, R.E., and Eichler, A. (eds) *Views and Beliefs in Mathematics Education*. Wiesbaden: Springer Spektrum, pp. 1–13.

Boylan, M. (2009) ‘Engaging with issues of emotionality in mathematics teacher education for social justice’, *Journal of Mathematics Teacher Education*, 12(6), pp. 427–443.

Boylan, M. (2016) ‘Ethical dimensions of mathematics education’, *Educational Studies in Mathematics*, 92, pp. 395–409.

Boylan, M. and Povey, H. (2009) ‘Telling Stories About Mathematics’, in Black, L., Mendick, H., and Solomon, Y. (eds) *Mathematical Relationships in Education: Identities and Participation*. New York: Routledge, pp. 47–58.

Boylan, M. and Yackel, E. (2009) ‘Book Reviews: Masculinities in mathematics’, *Research in Mathematics Education*, 11(1), pp. 85–94.

Brooks, C. (2016) *Teacher Subject Identity in Professional Practice*, *Teacher Subject Identity in Professional Practice*. Abingdon: Routledge.

Brown, A. and Danaher, P.A. (2019) 'CHE Principles: facilitating authentic and dialogical semi-structured interviews in educational research', *International Journal of Research and Method in Education*, 42(1), pp. 76–90.

Brown, M., Brown, P. and Bibby, T. (2008) '"I would rather die": Reasons given by 16-year-olds for not continuing their study of mathematics', *Research in Mathematics Education*, 10(1), pp. 3–18.

Brown, T. and McNamara, O. (2011) *Becoming a mathematics teacher: identity and identifications*. Dordrecht: Springer.

Bruner, J., (1996) *The Culture of Education*. Harvard University Press.

Bryk, A. (2010) 'Make math a gateway, not a gatekeeper', *The Chronicle of Higher Education*, *56*(32), B19-B20.

Buerk, D. (1982) 'An experience with some able women who avoid mathematics', *For the Learning of Mathematics*, 3(2), pp. 19–24.

Bugwark, E. (2021) 'Travails of Out-of-Field Teachers: A Qualitative Inquiry', *Journal of World Englishes and Educational Practices*, 3(2), pp. 36–57.

Burton, L. (2001) 'Research mathematicians as learners - And what mathematics education can learn from them', *British Educational Research Journal*, 27(5), pp. 589–599.

Burton, L. (2004) *Mathematicians as enquirers: Learning about learning mathematics*. New York: Springer.

Bütüner, S.Ö. and Baki, A. (2020) 'The Use of History of Mathematics in the Mathematics Classroom: An Action Study', *International Journal of Education in Mathematics, Science and Technology*, 8(2), pp. 92–117.

Byun, S. (2019) ‘Against the Epistemic Horizon toward Powerful Discourse’, in *North American Chapter of the International Group for the Psychology of Mathematics Education*, pp. 1649–1658.

Campbell, C., Porsch, R. and Hobbs, L. (2019) ‘Initial teacher education: Roles and possibilities for preparing capable teachers’, in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of ‘Teaching Out-of-field?’: International Perspectives on Teaching as a Non-specialist*, pp. 243–267.

Campbell, L. (2018) ‘Pedagogical bricolage and teacher agency: Towards a culture of creative professionalism’, *Educational Philosophy and Theory*, 51(1), pp. 31–40.

Chan, K.K.H. and Yung, B.H.W. (2018) ‘Developing Pedagogical Content Knowledge for Teaching a New Topic: More Than Teaching Experience and Subject Matter Knowledge’, *Research in Science Education*, 48(2), pp. 233–265.

Clarke, V., Braun, V. and Hayfield, N. (2015) ‘Thematic Analysis’, in J. Smith (ed.) *Qualitative Psychology: A Practical Guide to Research Methods*. 3rd edn. London: SAGE.

Claxton, G. (2021). *The future of teaching and the myths that hold it back*. Routledge.

Cohen, L., Manion, L. and Morrison, K. (2017) *Research Methods in Education*. 8th edn. Abingdon: Routledge.

Colucci-Gray, L., Burnard, P., Cooke, C., Davies, R., (2017) *BERA Research Commission Reviewing the potential and challenges of developing STEAM education through creative pedagogies for 21st learning: how can school curricula be broadened towards a more responsive, dynamic, and inclusive form of education?* Available at: https://doi.org/10.13140/RG.2.2.22452.76161 (Accessed: 23 September 2023).

Cooper, B. (2002) ‘Social class and ’real-life’ mathematics assessments’, in P. Gates (ed.) *Issues in Mathematics Teaching*. London: Routledge Falmer, pp. 245–258.

Costa, R., de Britto, A. and Waltenberg, F. (2018) *Does out-of-field teaching affects the quality of student learning? An analysis with panel data from the School Census*, *Invetigaciones de Economia de la Educacion*. Available at: https://repec.economicsofeducation.com/2018barcelona/13-05.pdf (Accessed: 6 February 2022).

Crisan, C. (2022). From a personal to a pedagogically powerful understanding of school mathematics. *International perspectives on knowledge and quality: Implications for innovation in teacher education policy and practice*, p. 205-224.

Crisan, C. (2021) 'Mathematics', in Pettet, M., Cuthbert, A.S., and Standish, A. (eds) *What Should Schools Teach?: Disciplines, subjects and the pursuit of truth*. London: UCL Press, pp. 234–250.

Crisan, C. and Hobbs, L. (2019) 'Subject-specific demands of teaching: Implications for out-of-field teachers', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*. Singapore: Springer Nature, pp. 151–178.

Crisan, C. and Rodd, M. (2011) 'Teachers of mathematics to mathematics teachers: a TDA Mathematics Development Programme for Teachers', in Smith, C. (ed.) *Proceedings of the British Society for Research into Learning Mathematics*, pp. 29–34.

Crotty, M. (1998) *The foundations of social research: Meaning and perspective in the research process*. London: SAGE.

de Cruz, H., Neth, H. and Schlimm, D. (2010) 'The cognitive basis of arithmetic', in Lowe, B. and Mueller, T. (eds) *Phimsamp. Philosophy of Mathematics: Sociological Apsects and Mathematical Practice: Philosophy of Mathematics. Sociological Aspects and Mathematical Practice*. College Publications, pp. 59–106. Available at: http://nbn-resolving.de/urn:nbn:de:bsz:352-284076 (Accessed: 3 September 2022).

Cuoco, A., Goldenberg, P.E. and Mark, J. (1996) 'Habits of mind: An organizing principle for mathematics curricula', *Journal of Mathematical Behavior*, 15(4), pp. 375–402.

Darby, L. (2007) ‘Negotiating mathematics and science school subject boundaries: The role of aesthetic understanding’, in *Proceedings of the 2007 Australasian Science Education Research Association Conference*, Fremantle, Australia, 11-14 July. Available at: https://www.asera.org.au//wp-content/uploads/2020/11/Conference-2007_amost_complete.pdf (Accessed: 23 September 2023)

Darby, L. (2009a) ‘Subject cultures and pedagogy: comparing mathematics and science’. Doctoral Thesis, Deakin University. Available at: https://dro.deakin.edu.au/view/DU:30027409 (Accessed: 2 May 2022).

Darby, L. (2009b) ‘Translating a “Relevance Imperative” into Junior Secondary Mathematics and Science Pedagogy’, *Eurasia Journal of Mathematics, Science and Technology Education*, 5(3), pp. 277–288.

Darby, L. (2010) ‘Characterising secondary school teacher imperatives as subject (signature) pedagogies: A Pedagogy of Support in maths and a Pedagogy of Engagement in science’, in *Proceedings of the 2010 Australian Association for Research in Education Conference*, pp. 1–14. Available at: http://dro.deakin.edu.au/view/DU:30052016 (Accessed: 7 February 2021).

Darby-Hobbs, L. (2011) ‘Responding to a Relevance Imperative in School Science and Mathematics: Humanising the Curriculum Through Story’, *Research in Science Education*, 43(1), pp. 77–97.

Darling-Hammond, L. (1999) ‘Teacher quality and student achievement: A review of state policy evidence’, *Education Policy Analysis Archives*, 8(1), pp. 1 – 44.

Darling-Hammond, L. and Ball, D.L. (1998) *Teaching for high standards: What policymakers need to know and be able to do*, CPRE Research Reports. Available at: https://repository.upenn.edu/cpre_researchreports/6/ (Accessed: 31 December 2021).

Darragh, L. (2014) ‘Recognising “good at mathematics”: using a performative lens for identity’, *Mathematics Education Research Journal*, 27(1), pp. 83–102.

Davis, B. (2008) ‘Complexity and Education: Vital simultaneities’, *Educational Philosophy and Theory*, 40(1), pp. 50–65.

Davis, B. (2014) ‘Toward a more power-full school mathematics’, *For the Learning of Mathematics*, 34(1), pp. 12–17.

Davis, B. and Renert, M. (2009) ‘Mathematics-for-teaching as shared dynamic participation’, *For the Learning of Mathematics*, 29(3), pp. 37–43.

Davis, B. and Renert, M. (2014) *The math teachers know: profound understanding of emergent mathematics*. New York: Routledge.

Davis, B. and Sengupta, P. (2020) ‘Complexity in Mathematics Education’, in S. Lerman (ed.) *Encyclopedia of Mathematics Education*. Springer, pp. 113–117.

Davis, B. and Simmt, E. (2006) ‘Mathematics-for-teaching: an ongoing investigation of the mathematics that teachers (need to) know’, *Educational Studies in Mathematics*, 61, pp. 293–319.

Davis, B. and Sumara, D. (2006) *Complexity and Education: Inquiries into Learning, Teaching and Research*. Abingdon: Routledge.

Davis, P., Williams, J., Black, L., Hernandez-Martinez, P., Pampaka, M., Nicholson, N., Wake, G. (2008) ‘Students’ mathematical identity and its relation to classroom mathematics social practice.’, Paper presented at the British Educational Research Association Conference, Institute of Education, London. [Nonpublished document]

Day, C. (2004) *A passion for teaching*, *A Passion for Teaching*. London: Routledge Falmer.

Dee, T. and Cohodes, S. (2008) ‘Out-of-field teachers and student achievement: Evidence from matched-pairs comparisons’, *Public Finance Review*, 36(1), pp. 7–32.

Denton, J. (2022) ‘Mixing Methodologies: A Sliding Continuum or an Iterative Cycle?’, in Khosrow-Pour, M. (ed.) *Research Anthology on Innovative Research Methodologies and Utilization Across Multiple Disciplines*. IGI Global, pp. 222–242.

Denzin, N. (2013) *Interpretive autoethnography*. 2nd edn. Thousand Oaks: SAGE.

Denzin, N. and Lincoln, Y.S. (2000) *The Sage handbook of qualitative research*. 2nd edn. Thousand Oaks: SAGE.

Denzin, N. and Lincoln, Y.S. (2005) *The Sage handbook of qualitative research*. 3rd edn. Thousand Oaks: SAGE.

Denzin, N. and Lincoln, Y.S. (2011) *The Sage handbook of qualitative research*. 4th edn. Thousand Oaks: SAGE.

Denzin, N. and Lincoln, Y.S. (2017) *The Sage handbook of qualitative research*. 5th edn. Thousand Oaks: SAGE.

Derrida, J. (1978) ‘Structure, sign, and play in the discourse of the human sciences’, in *Writing and Difference*. Chicago: University of Chicago Press, pp. 351–370.

Dewey, J. (1916) *Democracy and Education.* New York: The Macmillan Company.

DfE (2010) *The Case for Change*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/526946/The_case_for_change_The_importance_of_teaching.pdf (Accessed: 23 September 2023)

DfE, (2016) *‘Specialist and non-specialist’ teaching in England: extent and impact on pupil outcomes*. Available at: https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/578350/SubjectSpecialism_Report.pdf (Accessed: 23 September 2023).

DfE (2021) *National curriculum in England: mathematics programmes of study*. Available at: https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study/national-curriculum-in-england-mathematics-programmes-of-study (Accessed: 23 September 2023).

DfE (2022) *Primary school accountability in 2022: technical guide*. Department for education. Available at: https://www.gov.uk/government/publications/primary-school-accountability (Accessed: 2 June 2023).

DfE (2023a) *Initial teacher training (ITT): criteria and supporting advice*. Available at: https://www.gov.uk/government/publications/initial-teacher-training-criteria/initial-teacher-training-itt-criteria-and-supporting-advice (Accessed 23 September 2023).

DfE (2023b) *Secondary accountability measures: Guide for maintained secondary schools, academies and free schools*. Available at: https://www.gov.uk/government/publications/progress-8-school-performance-measure (Accessed: 2 June 2023).

Dimitriadis, G. and Kamberelis, G. (2006) *Theory for Education*. Abingdon: Routledge.

Doig, B., and Williams, J.(2019) 'Introduction to interdisciplinary mathematics education', in Doig, B., Williams, J., Swanson, D., Borromeo Ferri, R., Drake, P. (Eds) *Interdisciplinary Mathematics Education: The State of the Art and Beyond,* Cham: Springer.

Donitsa-Schmidt, S. and Zuzovsky, R. (2021) 'Becoming an in-field teacher in Israel: motivations and knowledge of out-of-field science teachers', *European Journal of Teacher Education*, 44(5), pp. 668–687.

Dörfler, W. (2003) 'Mathematics and mathematics education: Content and people, relation and difference', *Educational Studies in Mathematics*, 54, pp. 147–170.

Doxiadis, A. (2003) 'Embedding mathematics in the soul: Narrative as a force in mathematics education', in *Third Mediterranean Conference of Mathematics Education*. Athens, pp. 1–24.

Available at: https://apostolosdoxiadis.com/wp-content/uploads/2008/05/Embeddingmath.pdf (Accessed: 21 January 2022).

Drake, P. (2009) ‘Working for learning: Teaching assistants developing mathematics for teaching’, *Journal of Mathematics Teacher Education*, 12(1), pp. 67–82.

Drake, S. and Burns, R. (2004) *Meeting standards through integrated curriculum,* ASCD.

Drew, P. (2015) ‘Conversation Analysis’, in Smith, J. (ed.) *Qualitative Psychology: A Practical Guide to Research Methods*. 3rd edn. London: SAGE.

Dudley-Marling, C. (2015) ‘The resilience of deficit thinking.’ *Journal of Teaching and Learning*, *10*(1).

Durrant, J. (2022) ‘Creative postgraduate pedagogy for personal and professional learning: Risks and rewards’, in Saunders, L. and Trotman, D. (eds) *Creative postgraduate pedagogy for personal and professional learning: Risks and rewards." The Professional Doctorate in Education: Activism, Transformation and Practice*. Newcastle: Cambridge Scholars Publishing, pp. 67–90.

Eastaway, R. (2008) *How Many Socks Make a Pair? Surprisingly Interesting Maths*. London: Autumn Press.

Eastaway, R. and Wyndham, J. (2003) *How long is a piece of string? more hidden mathematics of everyday life*. London: Chrysalis.

Eastaway, R. and Wyndham, J. (2005) *Why Do Buses Come in Threes? The Hidden Maths of Everyday Life*. London: Portico.

Ely, M., Anzul, M., Friedman, T., Garner, D., and Steinmetz, A. (2003) *Doing qualitative research: Circles within circles*, *Doing Qualitative Research: Circles Within Circles*. London: Falmer Press.

Ernest, P. (1991) *The philosophy of mathematics education*, *The Philosophy of Mathematics Education*. London: Routledge.

Ernest, P. (1994) *Constructing mathematical knowledge: Epistemology and mathematics education*. London: Falmer Press.

Ernest, P. (2000) ‘Why teach mathematics’, in Bramall, S. and White, J. (eds) *Why learn mathematics?* London: Institute of Education, pp. 1–14.

Ernest, P. (2004) ‘Images of mathematics, values and gender: A philosophical perspective’, in Allen, B. and Johnston-Wilder, S. (eds) *Mathematics Education: Exploring the Culture of Learning*. Abingdon: Routledge Falmer.

Ernest, P. (2010) ‘Add it up: Why teach mathematics?’, *Professional Educator*, 9(2), pp. 44–47.

Ernest, P. (2015) ‘Postmodern mathematics’, in *The Proceedings of the 12th International Congress on Mathematical Education*. Seoul, pp. 605–608. Available at: https://doi.org/10.1007/978-3-319-12688-3_68 (Accessed: 26 September 2023).

Ernest, P. (2018) ‘The Ethics of Mathematics: Is Mathematics Harmful?’, in Ernest, P. (ed.) *The Philosophy of Mathematics Education Today*. Cham: Springer, pp. 187–216.

Ernest, P. (2020) ‘The dark side of mathematics: Damaging effects of the overvaluation of mathematics’, in Ineson, G. and Povey, H. (eds) *Debates in Mathematics Education*. 2nd edn. Abingdon: Routledge, pp. 29–42.

Fairclough, N. (2013) *Critical discourse analysis: The critical study of language*. London: Routledge.

Fauskanger, J. (2015) ‘Challenges in measuring teachers’ knowledge’, *Educational Studies in Mathematics*, 90, pp. 57–73.

Field, J. (2020) ‘A whole school intervention for teaching, learning and understanding Times Tables’, *Primary Mathematics*, 24(1), pp. 17–22.

Forestier, K. and Crossley, M. (2015) ‘International education policy transfer–borrowing both ways: The Hong Kong and England experience’, *Compare: A Journal of Comparative and International Education*, 45(5), pp. 664–685.

de Freitas, E. (2008) ‘Mathematics and its other: (Dis)locating the feminine’, *Gender and Education*, 20(3), pp. 281–290.

Friedman, S.J. (2000) ‘How Much of a Problem? A Reply to Ingersoll’s “The Problem of Underqualified Teachers in American Secondary Schools”’, *Educational Researcher*, 29(5), pp. 18–20.

García González, M.S. and Sierra, G.M. (2020) ‘The history of a teacher’s relief of his mathematics anxiety: the case of Diego’, *Educational Studies in Mathematics*, 103(3), pp. 273–291.

Gelman, R. and Gallistel, C.R. (1978) *The Child’s Understanding of Number*. Cambridge, MA: Harvard University Press.

Gentner, D. and Smith, L. (2012) ‘Analogical reasoning’, in *Encyclopedia of human behaviour*, pp. 130–136. Available at: https://groups.psych.northwestern.edu/gentner/papers/GentnerMaravilla_2018-Handbook.pdf (Accessed: 17 June 2023).

Gine, R. (2011) ‘The Interplay of Identity, Context, and Purpose in a Study of Mathematics Teaching and Learning’, in Kress, T.M. (ed.) *Critical Praxis Research*. Cham: Springer Open.

Goldhaber, D.D. and Brewer, D.J. (2000) ‘Does teacher certification matter? High school teacher certification status and student achievement’, *Educational Evaluation and Policy Analysis*, 22(2), pp. 129–145.

Golding, J. (2017) ‘Is it mathematics or is it school mathematics?: Presidential address to The Mathematical Association, April 2017’, *The Mathematical Gazette*, 101(552), pp. 385–400.

Goodson, I. (1983) *School Subjects and Curriculum Change*, *School Subjects and Curriculum Change*. Breckenham: Croom Helm.

Goos, M. (2006) 'Why teachers matter', *The Australian Mathematics Teacher*, 62(4), pp. 8–13.

Goos, M. (2015) 'Learning at the Boundaries.', in M. Marshman, V. Geiger, and A. Bennison (eds) *Mathematics Education Research Group of Australasia*. Available at: https://www.merga.net.au/Public/Public/Publications/Annual_Conference_Proceedings/2015_MERGA_CP.aspx (Accessed: 26 September 2023)

Goos, M. and Guerin, A. (2021) 'Insights into the Pedagogical Practices of Out-of-Field, Infield, and Upskilled Teachers of Mathematics', in Leong, Y.H. *et al.* (eds) *Excellence in Mathematics Education: Foundations and Pathways (Proceedings of the 43rd annual conference of the Mathematics Education Research Group of Australasia)*, pp. 203–210.

Gordon, M. (2019) 'Identity, culture, and pedagogical challenge: the presentation of mathematics', *Philosophy of Mathematics Education Journal*, 35, pp. 1–15.

Gov.uk (2010) *Equality Act 2010.* Available at: https://www.legislation.gov.uk/ukpga/2010/15/contents (Accessed 29 September 2023).

Gov.uk (2023) *On improving attainment in mathematics*. Available at: https://www.gov.uk/government/speeches/pm-speech-on-improving-attainment-in-mathematics-17-april-2023 (Accessed: 26 September 2023)

Greany, T. *et al.* (2016) *Trends in Maths and Science Study (TIMSS): National Report for England*. Available at: https://discovery.ucl.ac.uk/id/eprint/1529891/1/TIMSS_2015_England_Report_FINAL_for_govuk.pdf (Accessed: 29 September 2023)

Guba, E.G. and Lincoln, Y.S. (1982) 'Epistemological and methodological bases of naturalistic inquiry', *Educational Communication & Technology*, 30(4), pp. 233–252.

Hall, J. (2021) ‘Mathematical Representations in Magazine Advertisements: Have the Messages Changed in a Decade?’, *Journal of Humanistic Mathematics*, 11(1), pp. 136–165.

Hammersley, M. (2013) *What is Qualitative Research?* London: Bloomsbury.

Hannula, M. S., Leder, G. C., Morselli, F., Vollstedt, M., & Zhang, Q. (2019). Fresh perspectives on motivation, engagement, and identity: An introduction. *Affect and Mathematics Education: Fresh Perspectives on Motivation, Engagement, and Identity*, 3-14.

Hardy, G.H. (1967) *A mathematician’s apology*. Cambridge: Canto.

Hawking, S. (2006) *God Created the Integers: The mathematical breakthroughs that changed history*. London: Penguin Books.

Hekimoglu, S. (2010) ‘Mathematics and Martial Arts as Connected Art Forms’, *The Mathematics Educator*, 20(1), pp. 35–42.

Helmane, I. and Briška, I. (2017) ‘What is developing integrated or interdisciplinary or multidisciplinary or transdisciplinary education in school?’, *Signum Temporis*, 9(1), pp. 7–15.

Hiebert, J. (2013) *Conceptual and procedural knowledge: The case of mathematics*. London: Routledge.

Hillman, J. (2014) *Mathematics after 16: the state of play , challenges and ways ahead*. London. Available at: www.nuffieldfoundation.org (Accessed: 31 January 2021).

Hobbs, L., Campbell, C., Herbert, S., and Cole, M., (2012) ‘Discipline-related commitments and identities of out-of field early career teachers’, in *Contemporary Approaches to Research in Mathematics, Science, Health and Environmental Education*. Melbourne: Deakin University Press.

Hobbs, L. (2012) ‘Examining the aesthetic dimensions of teaching: Relationships between teacher knowledge, identity and passion’, *Teaching and Teacher Education*, 28(5), pp. 718–727.

Hobbs, L. (2013a) ‘Boundary crossings of out-of-field teachers locating learning possibilities amid disruption’, in J. Langan-Fox and C.L. Cooper (eds) *Boundary-Spanning in Organizations: Network, Influence, and Conflict*. New York: Routledge, pp. 7–28.

Hobbs, L. (2013b) ‘Teaching “out-of-field” as a boundary-crossing event: Factors shaping teacher identity’, *International Journal of Science and Mathematics Education*, 11(2), pp. 271–297.

Hobbs, L. (2014) ‘Removing the deficit: Possibilities for learning at the boundaries between in-field and out-of-field teaching’, in L. Hobbs and G. Törner (eds) *Taking an international perspective on ‘out-of-field’ teaching: Proceedings and Agenda for Research and Action from the 1st Teaching Across Specialisations (TAS) Collective Symposium*, pp. 30–32.

Hobbs, L., Du Plessis, A.E., *et al.* (2019) ‘Examining the complexity of the out-of-field teacher experience through multiple theoretical lenses’, in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of ‘Teaching Out-of-field?’: International Perspectives on Teaching as a Non-specialist*, pp. 87–128.

Hobbs, L. *et al.* (2020) *Defining and mapping out-of-field teaching in Victorian government schools, Prepared for the Victorian Department of Education and Training*. Available at: https://ooftas-collective.org/s/TOOF-Mapping-Report-Hobbs-et-al-2020_Definitions-only.pdf (Accessed: 6 February 2022).

Hobbs, L., Delaney, S., Campbell, C., Speldewinde, C.A., and Lai, J. (2021) ‘Developing a multifacted definition of out-of-field teaching: Distilling the literature and capturing expert insights’, in White, P. *et al.* (eds) *Methodological Approached to STEM Education Research Volume 2*. Newcastle upon Tyne: Cambridge Scholars Publishing, pp. 126–150.

Hobbs, L. and Porsch, R. (2021) ‘Teaching out-of-field: challenges for teacher education’, *European Journal of Teacher Education*, 44(5), pp. 601–610.

Hobbs, L. and Quinn, F. (2020) 'Out-of-field teachers as learners: Influences on teacher perceived capacity and enjoyment over time', *European Journal of Teacher Education*, 44(5), pp. 627–651.

Hobbs, L. and Quinn, F. (2021) 'Chicken Wings and a Deflated Football: Metaphors of the Complexity of Learning to Teach Science and Mathematics Out-of-Field', in Geelan, D., Nichols, K., and McDonald, C.V. (eds) *Complexity and Simplicity in Science Education*. Cham: Springer Nature.

Hobbs, L. and Törner, G. (2019a) 'Teaching out-of-field as a phenomenon and research problem', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*. Singapore: Springer, pp. 3–20.

Hobbs, L. and Törner, G. (2019b) 'The out-of-field phenomenon: Synthesis and taking action', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*. Singapore: Singer, pp. 309–322.

Hobbs, L., Campbell, C., Quinn, F., Vale, C., Lyons, T., Speldewinde, C., and Tytler, R. (2019) 'Teachers new to teaching across the stem subjects: How to create cultures of support, innovation and collaboration', in Hobbs, L., Campbell, C., and Xu, L. (eds) *Deakin STEM Education Conference 2016/2018 Selected works*. Warun Ponds: Deakin University, pp. 38–43. Available at: https://deakinsteme.org/wp-content/uploads/2019/06/Proceedings-2016-2018-Pub.pdf#page=38 (Accessed: 14 December 2021).

Hodgen, J. (2011) 'Knowing and Identity: A Situated Theory of Mathematics Knowledge in Teaching', in Rowland, T. and Ruthven, K. (eds) *Mathematical Knowledge in Teaching*. Dordrecht: Springer, Dordrecht, pp. 27–42.

Hodges, T.E. and Cady, J.A. (2012) 'Negotiating contexts to construct an identity as a mathematics teacher', *The Journal of Educational Research*, 105(2), pp. 112–122.

Hogg, E. (2013) *Volunteering in Older Age from a Lifecourse Perspective: situating older adults' volunteering in holistic and lifelong context*. PhD thesis, University of Northumbria. Available at:

https://search.proquest.com/openview/2aa1b34480fbc31c28935f67915af0bd/1?pq-origsite=gscholar&cbl=51922&diss=y (Accessed: 12 May 2023).

Holland, J. (2014) *Complexity: A very short introduction*. Oxford: Oxford University Press.

Horkheimer, M. (1972) *Critical theory: Selected essays*. New York: Continuum.

Horsten, L. (2022) 'Philosophy of Mathematics', *The Standford Encyclopedia of philosophy*. Available at: https://plato.stanford.edu/archives/spr2022/entries/philosophy-mathematics/ (Accessed: 4 December 2021).

Hottinger, S. (2016) *Inventing the mathematician: Gender, race, and our cultural understanding of mathematics*. Albany: State University of New York Press.

Howley, C. (2013) 'Another Way to Read "The Rural": A Bricolage of Maths Education', in B. Green and M. Corbett (eds) *Rethinking Rural Literacies*. New York: Palgrave Macmillan US, pp. 93–112.

Hughes, M. and Greenhough, P. (1998) 'We Do It A Different Way At My School', in Watson, A. and Winbourne, P. (eds) *New Directions for Situated Cognition in Mathematics Education*. New York: Springer.

Ingersoll, R. and Smith, T. (2003) 'The wrong solution to the teacher shortage', *Educational Leadership*, 60, pp. 30–33.

Ingersoll, R.M. (1998) 'The Problem of Out-of-Field Teaching', *The Phi Delta Kappan*, 79(10), pp. 773–776.

Ingersoll, R.M. (1999) 'The problem of underqualified teachers in American secondary schools', *Educational Researcher*, 28(2), pp. 26–37.

Ingersoll, R.M. (2001a) 'Rejoinder: Misunderstanding the Problem of Out-of-Field Teaching', *Educational Researcher*, 30(1), pp. 21–22.

Ingersoll, R.M. (2001b) 'The Realities of Out-of-Field Teaching.', *Educational Leadership*, 58(8), pp. 42–45.

Ingersoll, R.M. (2002) *Out-of-field teaching, educational inequality, and the organization of schools: An exploratory analysis*, *CPRE Research Reports*. Available at: https://repository.upenn.edu/cpre_researchreports/22/ (Accessed: 5 January 2022).

Ingersoll, R.M. (2003) *Out-of-field teaching and the limits of teacher policy*, *The Center for the Study of Teaching and Policy and the Consortium for Policy Research in Education*. Available at: https://repository.upenn.edu/gse_pubs/143/ (Accessed: 5 January 2022).

Ingersoll, R.M. (2019) 'Measuring out-of-field teaching', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field': International Perspectives on Teaching as a Non-specialist*. Singapore: Springer Nature, pp. 21–51.

Ingersoll, R.M. (2020) 'Misdiagnosing the Teacher Quality Problem', in Fuhrman, S.H., Cohen, D.K., and Mosher, F. (eds) *The State of Education Policy Research*. New York: Routledge, pp. 291–306.

Ingersoll, R.M. and Gruber, K. (1996) *Out-of-field teaching and educational equality*, *National Center for Education Statistics*. Available at: https://repository.upenn.edu/gse_pubs/546/ (Accessed: 5 January 2022).

Itter, D. and Meyers, N. (2017) 'Fear, Loathing and Ambivalence toward Learning and Teaching Mathematics: Preservice Teachers' Perspectives.', *Mathematics Teacher Education and Development*, 19(2), pp. 123–141.

Ivinson, G. and Duveen, G. (2006) 'Children's recontextualisations of pedagogy', in Moore, R. *et al.* (eds) *Knowledge, power and educational reform*. London: Routledge, pp. 109–127.

Jones, B. and Ball, S.J. (2023) *Neoliberalism and Education*. Taylor & Francis.

Jorgensen, R., Gates, P. and Roper, V. (2014) ‘Structural exclusion through school mathematics: using Bourdieu to understand mathematics as a social practice’, *Educational Studies in Mathematics*, 87, pp. 221–239.

Kaasila, R., Hannula, M.S. and Laine, A. (2012) ‘“My personal relationahip towards mathematics has necessarily not changed, but...”: analyzing preservice teachers’ mathematical identity talk’, *International Journal of Science and Mathematics Education*, 10(4), pp. 975–995.

Kara, H. *et al.* (2021) *Creative research methods in education: Principles and practices*. Bristol: Policy Press.

Kaspersen, E., Pepin, B. and Sikko, S.A. (2017) ‘Measuring STEM students’ mathematical identities’, *Educational Studies in Mathematics*, 95(2), pp. 163–179.

Keogh, J.J., Maguire, T. and O’Donoghue, J. (2018) *Adults, Mathematics and Work*. Leiden: BRILL.

Kessler, V. (2019) ‘Spirituality in mathematics’, *Journal for the Study of Spirituality*, 9(1), pp. 49–61.

Khalil, D., Lake, E. and Johnson, A. (2019) ‘Teachers’ classroom engagement structures: A comparative study of a novice US and an experienced UK mathematics teacher’, in M.S. Hannula, M.S., *et al*. (eds) *library.oapen.org*. Cham: Springer Open, pp. 255–282.

Kincheloe, J.L. (2001) ‘Describing the Bricolage: Conceptualizing a New Rigor in Qualitative Research’, *Qualitative Inquiry*, 7(6).

Kincheloe, J.L. (2004) ‘The Knowledges of Teacher Education: Developing a Critical Complex Epistemology’, *Teacher Education Quarterly*, pp. 49–66.

Kincheloe, J.L. (2005) ‘On to the next level: Continuing the conceptualization of the bricolage’, *Qualitative Inquiry*, 11(3), pp. 323–350.

Kincheloe, J.L. (2008) *Knowledge and Critical Pedagogy: An Introduction*. Springer.

Kincheloe, J.L. (2017) 'A Critical Complex Epistemology of Practice', *Taboo: The Journal of Culture and Education*, 10(2).

Kincheloe, J.L., McLaren, P., and Steinberg, S.R. (2017) 'Critical pedagogy and qualitative research: Advancing the bricolage', in Denzin, N. and Lincoln, Y. (eds) *The Sage handbook of qualitative research*. 5th edn. Thousands Oaks: SAGE.

Kollosche, D. (2018) 'The true purpose of mathematics education: A provocation', *The Mathematics Enthusiast*, 15(1), pp. 303–319.

Krátká, M., Eisenmann, P. and Cihlář, J. (2021) 'Four conceptions of infinity', *International Journal of Mathematics Education in Science and Technology,* 53(10), pp. 2661-2685 .

Labov, W. (1972) *Language in the inner city: Studies in the Black English vernacular*. University of Pennsylvania Press.

Lakatos, I. (1976) *Proofs and refutations: The logic of mathematical discovery*. Cambridge University Press.

Lambert, R. *et al.* (2022) '"It Is Like a Feeling": Theorizing Emotion in Mathematics through Complex Embodiment', *Mathematics*, 10(937), pp. 1–17.

Lamichhane, B.R. and Belbase, S. (2017) 'Images of Mathematics Held by Undergraduate Students', *International Journal on Emerging Mathematics Education*, 1(2), pp. 147–168.

Leder, G. (2019) 'Mathematics-related beliefs and affect', in Hannula, M. *et al.* (eds) *Affect and Mathematics Education*. Cham: Springer, pp. 15–35.

Leonard, J. (2019) *Culturally Specific Pedagogy in the Mathematics Classroom: Strategies for Teachers and Students*. 2nd edn. New York: Routledge.

Lerman, S. (1983) ‘Problem-solving or knowledge-centred: the influence of philosophy on mathematics teaching’, *International Journal of Mathematical Education in Science and Technology*, 14(1), pp. 59–66.

Lerman, S. (2000) ‘The social turn in mathematics education research’, in J. Boaler (ed.) *Multiple perspectives on the teaching and learning of Mathematics*. Westport: Ablex Publishing, pp. 19–44.

Lerman, S. and Zevenbergen, R. (2004) ‘The Socio-political Context of the Mathematics Classroom’, in Valero, P., and Zevenbergen, R. (eds) *Researching the Socio-Political Dimensions of Mathematics Education*. Boston: Kluwer Academic Publishers, pp. 27–42.

Levi-Strauss, C. (1966) *The Savage Mind*. Chicago: University of Chicago Press.

Lincoln, Y.S. (2001) ‘An Emerging New Bricoleur: Promises and Possibilities—A Reaction to Joe Kincheloe’s “Describing the Bricoleur”’, *Qualitative Inquiry*, 7(6).

Llewellyn, A. (2018) *Manufacturing the Mathematical Child: A Deconstruction of Dominant Spaces of Production and Governance*. Abingdon: Routledge.

Lockhart, P. (2009) *A Mathematician’s Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*. New York: Bellevue Literary Press.

Lu, X., Leung, F.K.S. and Li, N. (2021) ‘Teacher agency for integrating history into teaching mathematics in a performance-driven context: a case study of a beginning teacher in China’, *Educational Studies in Mathematics*, 106(1), pp. 25–44.

Luft, J.A., Hanuscin, D., Hobbs, L., and Törner, G. (2020) ‘Out-of-Field Teaching in Science: An Overlooked Problem’, *Journal of Science Teacher Education*, 31(7), pp. 719–724.

Luitel, B.C. and Taylor, P.C. (2007) ‘The shanai, the pseudosphere and other imaginings: Envisioning culturally contextualised mathematics education’, *Cultural Studies of Science Education*, 2, pp. 621–655.

Luitel, B.C. and Taylor, P.C. (2011) 'Kincheloe's Bricolage', in Hayes, K., Steinberg, S.R., and Tobin, K. (eds) *Key Works in Critical Pedagogy*. Rotterdam: Sense Publishers, pp. 191–200.

Luitel, L. (2020) 'Exploring Teachers' Experiences on the Nature of Mathematics based on their Curricular and Pedagogical Practices: A Phenomenological Inquiry', *International Electronic Journal of Mathematics Education*, 15(3), p. 613.

Lundin, S. (2012) 'Hating school, loving mathematics: On the ideological function of critique and reform in mathematics education', *Educational Studies in Mathematics*, 80(1–2), pp. 73-85.

Lutovac, S. (2019) 'Pre-service mathematics teachers' narrated failure: Stories of resilience', *International Journal of Educational Research*, 98, pp. 237–244.

Lutovac, S. (2020) 'How failure shapes teacher identities: Pre-service elementary school and mathematics teachers' narrated possible selves', *Teaching and Teacher Education*, 94, p. 103120.

Marchand, T. H. J. (2018) 'Toward an anthropology of mathematizing', *Interdisciplinary Science Reviews*, *43*(3–4), pp. 295–316. https://doi.org/10.1080/03080188.2018.1528090

Mason, J. (2003) 'On the structure of attention in the learning of mathematics', *The Australisn Mathematics Teacher*, 59(4), pp. 17–25.

Mclaren, P. (2001) 'Bricklayers and Bricoleurs: A Marxist Addendum', *Qualitative Inquiry*, 7(6), pp. 700–705.

Mea, N. *et al.* (2019) 'Out-of-field social studies teaching through sustainable culture-based pedagogy: a Filipino perspective', *International Journal of Education and Practice*, 7(3), pp. 230–241.

Mendick, H. (2006) *Masculinities in mathematics*. Maidenhead: McGraw-Hill Education.

Mendick, H. and Francis, B. (2012) 'Boffin and geek identities: abject or privileged?', *Gender and Education*, 24(1), pp. 15–24.

Mertens, D.M. (2007) 'Transformative Paradigm Mixed Methods and Social Justice', *Journal of Mixed Methods Research*, 1(3), pp. 212–225.

Mlangeni, D.S. (2019) *Experiencing mathematics as a Black female – an autoethnography*. Doctoral thesis, University of Pretoria. Available at: https://repository.up.ac.za/handle/2263/78478 (Accessed: 15 April 2022).

Moore, R., Arnot, M., Beck, J. and Daniels, H. (eds) (2006) *Knowledge, power and educational reform: applying the sociology of Basil Bernstein*. London: Routledge.

Morrison, K. (2008) 'Educational Philosophy and the Challenge of Complexity Theory, Educational Philosophy and Theory', *Educational Philosophy and Theory*, 40(1), pp. 19–34.

Moses, R.P. and Cobb, C.E. (2001) *Radical equations: civil rights from Mississippi to the algebra project*. Boston: Beacon Press.

Nardi, E. and Steward, S. (2003) 'Is mathematics T.I.R.E.D? A profile of quiet disaffection in the secondary mathematics classroom', *British Educational Research Journal*, 29(3), pp. 345–367.

NCETM (2021) *Specialist Knowledge for Teaching Mathematics Programme*. Available at: https://www.ncetm.org.uk/maths-hubs-projects/specialist-knowledge-for-teaching-mathematics-secondary-non-specialist-teachers-programme/ (Accessed: 14 May 2023).

Newton, D.P. (1988) 'Relevance and science education', *Educational Philosophy and Theory*, 20(2), pp. 7–12.

Ní Ríordáin, M., Paolucci, C. and Lyons, T. (2019) 'Teacher professional competence: What can be learned about the knowledge and practices needed for teaching?', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*, pp. 129–149.

Ocean, J., Ersozlu, Z. and Hobbs, L. (2021) 'Using Q Methodology to Construct a Questionnaire to Explore the Extent to Which Out-of-field and In Field Secondary School Maths Teachers Value "Military" Practices in Mathematics Education', in White, P. *et al.* (eds) *Methodological Approaches to STEM Education Research Volume 2*. Newcastle upon Tyne: Cambri, pp. 151–170.

Ocean, J., Sawatzki, C. and Ersozlu, Z. (2021) 'Spotting the military influence in mathematics education', *Australian Primary Mathematics Classroom*, 26(1), pp. 20–24.

Ocean, J. and Skourdoumbis, A. (2016) 'Who's counting? Legitimating measurement in the audit culture', *Discourse: studies in the cultural politics of education*, 37(3), pp. 442–456.

Osberg, D., Biesta, G., & Cilliers, P. (2008). From representation to emergence: Complexity's challenge to the epistemology of schooling. *Educational philosophy and theory*, *40*(1), 213-227.

Paradis, V.J. (2013) *Did Joe Lyons Kincheloe Discover the Golden Chalice for Knowledge Production? The Application of Critical Complex Epistemology and the Multidimensional Critical Complex Bricolage*. PhD thesis. Available at: http://joekincheloe.us/sitebuildercontent/sitebuilderfiles/2final_diss_rev_2_paradis_vanessa_multidimensional_critical_complex_bricolage.pdf (Accessed: 29 September 2023).

Part, T. (2016) 'A study into identity formation: Troubling stories of adults taming mathematics'. Doctoral thesis, London Metropolitan University. Available at: https://repository.londonmet.ac.uk/1160/1/PartTracy%20-%20PhD%20Full%20Thesis.pdf (Accessed 29 September 2023).

Pearson (2021) Mathematics Paper 3 (Calculator) Higher Tier 1MA1/3H. Available at: https://qualifications.pearson.com/content/dam/pdf/GCSE/mathematics/2015/exam-materials/1MA1_3H_que_20211109.pdf (Accessed: 14 October 2023)

Peck, F.A. (2018) 'Rejecting Platonism: Recovering Humanity in Mathematics Education', *Education Sciences 2018, Vol. 8, Page 43*, 8(2), p. 43.

Pinar, W.F. (2001) ‘The Researcher as Bricoleur: The Teacher as Public Intellectual’, *Qualitative Inquiry*, 7(6), pp. 696–700.

Plato (2007) *The Republic (Penguin Classics)*. Penguin Classics.

Du Plessis, A.E. (2013) *Understanding the out-of-field teaching experience*. PhD thesis, The University of Queensland. Available at: https://espace.library.uq.edu.au/view/UQ:330372/s4245616_phd_submission.pdf?dsi_version=751e2d5db3c65ca4d00b269e7084704d (Accessed: 31 December 2021).

du Plessis, A.E. (2015a) ‘Effective education: Conceptualising the meaning of out-of-field teaching practices for teachers, teacher quality and school leaders’, *International Journal of Educational Research*, 72, pp. 89–102.

du Plessis, A.E. (2015b) ‘Understanding the out-of-field teaching experience’, *British Journal of Sociology of Education*, 36(5), p. 815.

du Plessis, A.E., Hobbs, L., Luft, J.A., and Vale, C. (2019) ‘The out-of-field teacher in context: The impact of the school context and environment’, Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of ‘Teaching Out-of-field?’: International Perspectives on Teaching as a Non-specialist*. Singapore: Springer, pp. 217–242.

du Plessis, A.E. (2020) ‘The Lived Experience of Out-of-field STEM Teachers: a Quandary for Strategising Quality Teaching in STEM?’, *Research in Science Education*, 50(4), pp. 1465–1499.

du Plessis, A.E., Carroll, A. and Gillies, R. (2015) ‘Understanding the lived experiences of novice out-of-field teachers in relation to school leadership practices’, *Asia-Pacific Journal of Teacher Education*, 43(1), pp. 4–21.

Porsch, R. (2016) ‘Fachfremd unterrichten in Deutschland. Definition - Verbreitung - Auswirkungen.’, *Die deutsche Schule*, 108; Jg. 2(1), pp. 9–32.

Porsch, R. and Whannell, R. (2019) 'Out-of-field teaching affecting students and learning: What is known and unknown', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*, pp. 179–191.

Price, A. *et al.* (2019) 'Teaching out-of-field internationally', in Hobbs, L. and Törner, G. (eds) *Examining the Phenomenon of 'Teaching Out-of-field?': International Perspectives on Teaching as a Non-specialist*. Singapore: Springer, pp. 53–83.

Qin, L. and Bowen, D. (2019) 'The distributions of teacher qualification: A cross-national study', *International Journal of Educational Development*, 70, 10284.

Rochette, E. (2022) 'Understanding the Complexity of Science Teachers' Lived Experience as They Navigate Two Out-of-Field Areas: Implications and Possibilities', in Hobbs, L. and Porsch, R. (eds.) *Out-of-Field Teaching Across Teaching Disciplines and Contexts*, pp. 99–116.

Rogers, M. (2012) 'Contextualizing Theories and Practices of Bricolage Research', *The qualitative report*, 17(48), pp. 1–17.

Rowland, T., Huckstep, P. and Thwaites, A. (2004) 'The knowledge quartet: considering Chloe', in *Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education* (pp. 1568-1578).

Rubel, L.H. *et al.* (2022) 'Intersectional feminism to re-envision mathematical literacies & precarity', *Research in Mathematics Education*, 24(2), pp. 224–248.

Rubel, L.H. and McCloskey, A. v. (2021) 'Contextualization of mathematics: which and whose world?', *Educational Studies in Mathematics*, 107(2), pp. 383–404.

Sadovnik, A. (1995) *Knowledge and pedagogy: The sociology of Basil Bernstein*. Bloomsbury Academic.

Sani, N. and Burghes, D. (2021) ‘Longitudinal study of ’retraining’non-maths specialist teachers to become capable, confident teachers of mathematics’, *International Journal of Mathematical Education in Science and Technology* 53(9) pp. 2438-2464.

Schoenfeld, A.H. (2015) ‘What Counts, When? Reflections on Beliefs, Affect, Attitude, Orientations, Habits of Mind, Grain Size, Time Scale, Context, Theory, and Method’, in Pepin, B., Roesken-Winter, B. (eds) *From beliefs to dynamic affect systems in mathematics education. Springer, pp. 395-404.*

Shah, C., Richardson, P. and Watt, H. (2020) *Teaching ‘out of field’ in STEM subjects in Australia: Evidence from PISA 2015*. 511. Essen: Global Labor Organization (GLO). Available at: https://www.econstor.eu/bitstream/10419/215639/1/GLO-DP-0511.pdf (Accessed: 29 September 2023)

Shah, L. *et al.* (2019) ‘Diagnosing the current state of out-of-field teaching in high school science and mathematics’, *PLoS ONE*, 14(9), pp. 1–12.

Shapiro, S. (2000) *Thinking about mathematics: the philosophy of mathematics*. Oxford: Oxford University Press.

Sharp, H. (2019) ‘Bricolage research in history education as a scholarly mixed-methods design’, *History Education Research Journal*, 16(1), pp. 50–62.

Sheppard, K. *et al.* (2020) ‘Out-of-field teaching in chemistry and physics: An empirical census study’, *Journal of Science Teacher Education*, 31(7), pp. 746–767.

Sheridan, J., Chamberlain, K. and Dupuis, A. (2011) ‘Timelining: Visualizing experience’, *Qualitative Research*, 11(5), pp. 552–569.

Shirbagi, N., Saeidi, F. and Nemati, S. (2018) ‘De-professionalism of Teachers: Experiences of the Out of Field Teaching’, *Quarterly Journal of New Thoughts on Education*, 14(2), pp. 139–166.

Shulman, L.S. (1986) ‘Those Who Understand: Knowledge Growth in Teaching’, *Educational Researcher*, 15(2).

Singh, H., Luft, J.A. and Napier, J. (2021) ‘The development of ePCK of newly hired in-field and out-of-field teachers during their first three years of teaching’, *European Journal of Teacher Education*, 44(5), pp. 611–626.

Singh, S. (1997) *Fermat’s last theorem: the story of a riddle that confounded the world’s greatest minds for 358 years*. London: Fourth Estate.

Singh, S. (1999) *The code book: The secret history of codes and code-breaking*. London: Fourth Estate.

Singh, S. (2013) *The Simpsons and their mathematical secrets*. London: Bloomsbury.

Skemp, R. (1978) ‘Relational understanding and instrumental understanding’, *the Arithmetic Teacher*, 26(3), pp. 9–15.

de Souza Pereira Grilo, J. and Cerqueira Barbosa, J. (2022) ‘It is necessary to know a specific mathematics to teach mathematics’, *The Mathematics Enthusiast*, 19(1), pp. 136–157.

Stephan, M. (2020) ‘Sociomathematical norms in mathematics education’, in Lerman, S. (ed) *Encyclopedia of Mathematics Education*. Cham: Springer, pp. 802-805.

Stinson, D.W. (2004) ‘Mathematics as “gate-keeper”(?): Three theoretical perspectives that aim toward empowering all children with a key to the gate’, *The Mathematics Educator*, 14(1), pp. 8–18.

Su, F. (2020) *Mathematics for human flourishing*. Yale University Press.

Sulak, T.N., Wilson, R., Renbarger, R.L., Kaul, C.R. and O’Guinn, N. (2020) ‘The relationships between numeracy scores and soft skills in employed and unemployed Americans’, *New Horizons in Adult Education and Human Resource Development*, 32(2), pp. 19–39.

Telles, J.A. (2000) ‘Biographical connections: Experiences as sources of legitimate knowledge in qualitative research’, *International Journal of Qualitative Studies in Education*, 13(3), pp. 251–262.

Tobin, K. and Kincheloe, J.L. (2015) ‘Doing educational research in a complex world’, in Tobin, K. and Kincheloe, J.L. (eds) *Doing educational research: a handbook*. Rotterdam: Sense Publishers, pp. 3–14.

Tomas-Fulgado, J. (2020) ‘Instructional Flexibility in Out of Field Teaching’, *The URSP Research Journal*, 6(1), pp. 1–7.

Vale, C., Campbell, C. and White, P. (2021) ‘Beliefs and practices of secondary teachers crossing subject boundaries to teach mathematics out-of-field’, *Mathematics Education Research Journal*, 33, pp. 589–612.

Valério, M. and Ferrara, S. (2022) ‘Numeracy at the dawn of writing: Mesopotamia and beyond’, *Historia Mathematica*, 59, pp. 35–53.

Vandenbussche, L., Edelenbos, J. and Eshuis, J. (2019) ‘Coming to grips with life-as-experienced: Piecing together research to study stakeholders’ lived relational experiences in collaborative planning processes’, *Forum Qualitative Sozialforschung*, 20(1), p.23.

Vidal Alangui, W. (2019) ‘Beyond songs and dances: Ethnomathematics and the challenge of culture’, *Revista Latinoamericana de Etnomatemática*, *13*(3), pp. 88–107.

Walkerdine, V. (1988) *The mastery of reason: cognitive development and the production of rationality*. London: Routledge.

Watson, A. (2008). School mathematics as a special kind of mathematics. *For the Learning of Mathematics*, *28*(3), p. 3-7.

Watson, A. (2021) *Care in mathematics education: alternative educational spaces and practices*. Cham: Palgrave Macmillan.

White-Fredette, K. (2009) ‘Why not philosophy? Problematizing the philosophy of mathematics in a time of curriculum reform’, *The Mathematics Educator*, 19(2), pp. 21–31.

Whitten, D. (2018) *Understanding the beliefs and behaviours of low-skilled adults as they re-engage with mathematics*. Doctoral dissertation, The University of Waikato. Available at: https://researchcommons.waikato.ac.nz/bitstream/handle/10289/12044/thesis.pdf?sequence=4 (Accessed: 3 October 2023).

Whitty, G. (2017) ‘Taking subject knowledge out and putting it back in again? A journey in the company of Michael Young’, in Guile, D., Lambert, D., and Reiss, M.J. (eds) *Sociology, curriculum studies and professional knowledge: new perspectives on the work of Michael Young*. Abingon: Routledge, pp. 17–30.

Wiliam, D., Bartholomew, H. and Reay, D. (2004) ‘Assessment, learning and identity’, in Valero, P., Zevenbergen, R. (eds) *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology*. Boston: Springer, pp. 43–61.

Wolfram, C. (2020) *The Math(s) Fix: An Education Blueprint for the AI Age*. Wolfram Media.

Xenofontos, C. (2018) ‘Greek-Cypriot elementary teachers’ epistemological beliefs about mathematics’, *Teaching and Teacher Education*, 70, pp. 47–57.

Yardley, A. (2019) ‘Bricolage as method’, *SAGE Research Methods Foundations*. London: SAGE. Available at: http://dx.doi.org/10.4135/9781526421036 (Accessed: 3 October 2023).

Yardley, F. (2022) ‘Maths, history, God, knitting and me: A reflexive bricolage of identity’, in Hobbs, L. and Porsch, R. (eds) *Out-of-field teaching across teaching disciplines and contexts*. Singapore: Springer.

Yardley, F. (2023a) ‘Thesis or Therapy?’, in *Postgraduate research conference: Finding your voice*. Canterbury, February 2023.

Yardley, F. (2023b) ‘Using the bricolage to study bricoleurs’, in *Faculty of Arts, Humanities and Education Research Conference*. Canterbury, June 2023.

Young, M. and Muller, J. (2016) *Curriculum and the Specialization of Knowledge*. Abingdon: Routledge.

Zazkis, R. and R. Leikin, (2010), ‘Advanced Mathematical Knowledge in Teaching Practice: Perceptions of Secondary Mathematics Teachers’, Mathematical Thinking and Learning, 12 (4): 263–81.

Appendix A Glossary

The definition is how I understand this term in the context of this thesis. No reference is given where I have drawn on multiple sources to define a term. More detail is given in the body of the text where relevant. For more information about the thinking behind this glossary see section 1.3.2. The links in the third column are to where I first use or define the term.

Term	Definition	Link
Ability	The capacity of an individual to learn	2.1.2
Absolutism	The belief that mathematics exists independently of the human mind and is out there waiting to be discovered (Ernest, 1991)	2.1.1
Academic mathematics	The activities that advance mathematical knowledge (Watson, 2008)	1.2.2
Agency	Having the capacity to make choices and act on them (Lu *et al*., 2021)	1.1.1
Axiology	The ethical value people attribute to knowledge (Mertens, 2007)	1.2.2
Belief	What people hold to be true cognitively and affectively (Goldin *et al*., 2009)	1.1.1
Boundary	A socio-cultural difference leading to discontinuity in action or interaction (Akkerman and Bakker, 2011, p. 133)	1.1.1
Boundary crossing	A person's transitions and interactions across different sites (Akkerman and Bakker, 2011, p. 133)	1.1.1
Bricolage	A philosophical approach to research that is open to all methodologies, continuously and rigorously selecting and adapting methodologies and methods to suit the focus of the research	4.1
Complex	Describes phenomena and their interrelationships in context, which is understood to be dynamic and multi-dimensional	1.2.1
Conceptual knowledge	A rich connected web of relational knowledge (Hiebert, 2013)	2.1.1
Conceptualisation	What an individual believes something to be, and how they understand it (Andrews and Hatch, 1999)	2
Critical complex research	Seeks to interpret complex phenomena through a critical lens	1.2.1

Critical research	The theoretical perspective that society is unequal and seeks to reveal, understand, and challenge the imbalances of power	1.2.1
Deficit model	A perspective that focuses on what is lacking (Dudley-Marling, 2015)	1.1.1
Doing mathematics	Conceptualisation of purpose for doing mathematics	1.2.2
Empirical data	Data collected and analysed using a quantitative methodology based on an epistemology of certainty (Kincheloe 2008b)	2.1.1
Epistemology	How people acquire and communicate knowledge (Mertens, 2007)	1.2.2
Everyday mathematics	Mathematical knowledge informally structured as we use it (Davis and Renert, 2014)	1.2.2
Fact	Events that occurred (Denzin, 2013)	1.1.2
Facticity	How facts were lived and experienced (Denzin, 2013)	1.1.2
Fallibilism	The belief that mathematics is a human construct which is evolving and culturally and historically situated (Ernest, 1991)	2.1.1
Heterogenous	Of diverse forms	Abstract
Homogenous	Of similar forms	1.2.2
Identity	how people make sense of our experience of being in the world (Boylan and Povey, 2009)	1.1.1
Integer	Whole numbers (i.e. … -3, -2, -1, 0, 1, 2, 3…)	2.1.1
Interdisciplinary	Using a similar learning approach across disciplines (Helmane and Briška, 2017)	3.2.2
Knowing mathematics	Teachers' beliefs about the nature of mathematics	1.2.2
Knowledge	Our response to things that brings forth new worlds (Osberg *et al.*, 2008)	1.2.2
Learning mathematics	Understanding of how people come to know mathematics	1.2.2
Lens	A focus on something that regards it from the perspective of another	Abstract
Mathematics	a discipline of patterns and connections used to model our complex world	Abstract
Method	Techniques or procedures used to collect and analyse data (Crotty, 1998)	4.1

Methodology	The thought behind the choice and use of particular methods of data collection and analysis (Crotty, 1998)	4.1
Model	Simplify or idealise by focusing on relevant features and disregarding others	1.2.2
Multidisciplinary	Subjects treated as distinct (Helmane and Briška, 2017)	3.2.2
Other	Being different, or alien to one's identity or sense of self	2.1.2
Ontology	The nature of the phenomena being investigated (Mertens, 2007)	1.2.2
Opportunity model	An optimistic perspective that assumes capability and competence (Hobbs and Törner, 2019a)	1.1.1
Out-of-field teacher of mathematics	One who has taught for six or more years and has a degree and teaching qualification in a subject other than mathematics	3.1.1
Pedagogical mathematics	Mathematical knowledge for teaching (Ball *et al.*, 2008)	1.2.2
Philosophy/philosophical	A theory or attitude that guides actions and behaviours	4.1
Procedural knowledge	Knowledge of the language, symbols, rules and algorithms for finding solutions (Hiebert, 2013)	2.1.1
Recontextualisation	To place knowledge in a new context, crossing over boundaries	1.1.1
Research	Any systematic, critical and self-critical enquiry which aims to contribute to the advancement of knowledge	1.1
School mathematics	The mathematics learnt at school (Golding, 2017)	1.2.2
System	A phenomenon that involves the interaction of many sub-components or agents (Davis and Sumara, 2006)	1.2.1
Transdisciplinary	Making connections and integrating between disciplines (Helmane and Briška, 2017)	3.2.2
Western	Term used by Kincheloe to refer to European and Anglophone predominantly White global minority cultures (Kincheloe, 2005)	1.2.1

Appendix B Stylistic choices

Sometimes I use words for stylistic reasons while being aware of ignoring some of the nuance in their meaning. This table explains some of these stylistic choices.

Term(s)	
England	I use England, English rather than UK, Great Britain, British because of the significant differences in the education system in the nations of the UK.
Methodology	For fluency I sometimes refer to the bricolage as a methodology while defining and understanding it as a philosophy.
My faith	References to my faith may jar for some readers. I include my faith because it is part of me. My desire to be honest in this thesis is rooted in my Christian faith, so it would be inconsistent for my reflexive positionality to ignore this element.
Other	Other is capitalised when it is used to refer to its philosophical use to denote being in a context where one is different or alien to one's identity or sense of self.
Study/project/research/thesis	I sometimes treat these words as (almost) synonymous to avoid repetition of the same term within sentences and paragraphs.
Subject/discipline/field	I treat these words as (almost) synonymous to avoid repetition of the same term within sentences and paragraphs.
The bricolage	I use the definite article because it is an entity, not an action (Yardley, A. 2019).

Appendix C Ethics consent form and participant information

Canterbury Christ Church University

CONSENT FORM

Title of Project: Out-of-field teachers of mathematics

Name of Researcher: Fiona Yardley (Fiona.yardley@canterbury.ac.uk

Contact details:

Address:	Faculty of Arts, Humanities and Education Canterbury Christ Church University North Holmes Road Canterbury CT1 1QU
Tel:	07834 148527
Email:	Fiona.yardley@canterbury.ac.uk

Please initial box

1.	I confirm that I have read and understand the participant information for the above project and have had the opportunity to ask questions.	
2.	(If applicable) I confirm that I agree to any audio and/or visual recordings.	
3.	I understand that any personal information that I provide to the researchers will be kept strictly confidential and in line with the University Research Privacy Notice	
4.	I understand that my participation is voluntary and that I am free to withdraw my participation at any time, without giving a reason.	
5.	I agree to take part in the above project.	

Name of Participant:	Date:	Signature:
Name of person taking consent *(if different from researcher)*	Date:	Signature:
Researcher:	Date:	Signature:

Copies: 1 for participant
1 for researcher

Out-of-field Teachers of Mathematics

ONGOING PARTICIPANT INFORMATION

A research study is being conducted at Canterbury Christ Church University (CCCU) by me, Fiona Yardley.

Please refer to our Research Privacy Notice for more information on how we will use and store your personal data*.*

Background

In the UK a teacher teaching a subject other than the one in which they have a degree and PGCE is referred to as a *non-specialist*. This term suggests that they are lacking something and does not acknowledge the knowledge, skills and experience that they bring with them when teaching out of field. As well as their teaching skills, out-of-field teachers also bring their own experiences of learning and doing the subject they find themselves teaching. This study looks at teachers teaching maths out of field for the first time and considers what experiences as a learner, knower and doer of maths they bring with them, and how these change as they begin teaching the subject.

This research is for the award of Doctor of Education (EdD). When I first taught maths, I had a degree and PGCE in history. I have since completed a degree in maths and have extensive experience as a teacher, leader and teacher educator of mathematics.

What will you be required to do?

You will be required to meet with me, the researcher, from time to time throughout a twelvemonth period as you prepare to and then teach mathematics for the first time. Meetings will be online or in a mutually convenient location at a time that suits you. In these meetings you will have the opportunity to think and talk about mathematics and teaching mathematics. The data collected in these meetings will be analysed alongside the data of other participants and used in my doctoral thesis. In the thesis, pseudonyms will be used alongside other steps to assure the anonymity of participants.

To participate in this research you must:

- Have been asked to teach one or more lessons of mathematics on your timetable in the next academic year
- Have never taught mathematics before
- Have been teaching in England as a qualified teacher for three or more years
- Have a degree and PGCE in a subject that is not mathematics

Procedures

You will be asked to maintain contact with me, the researcher over the next twelve months.

- I will arrange meetings with you (online or face to face) from time to time to gather data. This may involve discussing your thoughts about teaching mathematics, asking about what is happening in your teaching, discussing exam questions, photos, quotes etc.
- As I am an experienced teacher and teacher educator in mathematics it is likely that opportunities will arise organically during the data collection process in which you seek support and advice. As someone who has experienced teaching mathematics out of field, I am in a position of empathy and may be able to help. Please note that this will also constitute research data.

Feedback

- A draft of any section of the written thesis that relate to you or use data collected from you will be shared with you for your reflection and feedback, as well as a final version and a summary of key findings.

Confidentiality and Data Protection

The General Data Protection Regulation (GDPR) defines personal data as that which is of a more sensitive nature, such as race, religion etc. There is no intention within the proposed scope of this research to collect and process any personal data. It is possible that during the data collection process a participant may share personal data that either they or the researcher believe to be relevant data. Where such personal data is identified, the researcher and participant (and possibly also the researcher's supervisor) will explicitly address whether it is in the public interest to collect and process this data. The personal data will only be used if there is full agreement by all parties that it is in the public interest. As with all data collected it will be anonymised.

Data can only be accessed by, or shared with:

- The researcher, Fiona Yardley, her supervisor Dr Gina Donaldson and chair Dr Judy Durrant. Data will not be transferred outside of the European Economic Area (EEA).

The identified period for the retention of personal data for this project:

- Where personal data is agreed to be relevant, it will be retained only for the duration of this project. All other data will be stored electronically under password protection for a period of five years following the completion of the project.

If you would like to obtain further information related to how your personal data is processed for this project please contact Fiona Yardley (Fiona.yardley@canterbury.ac.uk).

You can read further information regarding how the University processes your personal data for research purposes at the following link: Research Privacy Notice - https://www.canterbury.ac.uk/university-solicitors-office/data-protection/privacy-notices/privacynotices.aspx

Dissemination of results

The results of the study will be recorded as a thesis for an education doctorate (EdD) and will be published in the CCCU library.

Process for withdrawing consent to participate

You are free to withdraw your consent to participate in this research project without giving any reason. You can do this at any time up to and including when the draft of relevant sections of the thesis is shared with you. To do this email Fiona.yardley@canterbury.ac.uk stating your desire to withdraw.

You may read further information on your rights relating to your personal data at the following link: Research Privacy Notice - https://www.canterbury.ac.uk/university-solicitors-office/dataprotection/privacy-notices/privacy-notices.aspx

Any questions?

Please contact Fiona Yardley (Fiona.yardley@canterbury.ac.uk), or supervisor Gina Donaldson (gina.donaldson@canterbury.ac.uk).

Out-of-field Teachers of Mathematics

ONE-OFF PARTICIPANT INFORMATION

A research study is being conducted at Canterbury Christ Church University (CCCU) by me, Fiona Yardley.

Please refer to our Research Privacy Notice for more information on how we will use and store your personal data.

Background

In the UK a teacher teaching a subject other than the one in which they have a degree and PGCE is referred to as a *non-specialist*. This term suggests that they are lacking something and does not acknowledge the knowledge, skills and experience that they bring with them when teaching out of field. As well as their teaching skills, out-of-field teachers also bring their own experiences of learning and doing the subject they find themselves teaching. This study looks at teachers teaching maths out of field and considers what experiences as a learner, knower and doer of maths they bring with them, and how these change as they teach the subject.

This research is for the award of Doctor of Education (EdD). When I first taught maths, I had a degree and PGCE in history. I have since completed a degree in maths and have extensive experience as a teacher, leader and teacher educator of mathematics.

What will you be required to do?

You will meet with me, the researcher. Meetings will be online or in a mutually convenient location at a time that suits you. When we meet you will have the opportunity to think and talk about mathematics and teaching mathematics. The data collected when we meet will be analysed alongside the data of other participants and used in my doctoral thesis. In the thesis, pseudonyms will be used alongside other steps to assure the anonymity of participants.

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- A draft of any section of the written thesis that relate to you or use data collected from you will be shared with you for your reflection and feedback, as well as a final version and a summary of key findings.

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Dissemination of results

The results of the study will be recorded as a thesis for an education doctorate (EdD) and will be published in the CCCU library. Results may also be disseminated through journal articles, book chapters or conference presentations and proceedings.

Process for withdrawing consent to participate

You are free to withdraw your consent to participate in this research project without giving any reason. You can do this at any time up to and including when the draft of relevant sections of the thesis is shared with you. To do this email Fiona.yardley@canterbury.ac.uk stating your desire to withdraw.

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Any questions?

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Appendix D (Hobbs *et al*., 2020, p. 25-7)

Criteria Map 1. Measurable Criteria

	Standard	Dimension	Band 1	Band 2	Band 3
			TECHNICALLY IN-FIELD	Technically IN, Partially OUT	TECHNICALLY OUT-OF-FIELD
1. Qualification	Specialist Area Guidelines Suitability of qualifications for entry to ITE programs and specialist areas	1.1 Technical alignment (broad subject)	Full alignment (Discipline and methdology)	Partial alignment (Discipline or methodology)	Misalignment (Neither Discipline nor methodology)
		1.2 Specialism alignment (narrow sub discipline)	Full alignment	Near misalignment	Far misalignment
	Teaching qualification	1.3 Phase alignment	Full alignment		Misalignment
2. Workload	Teachers have a teaching workload where a proportion matches their qualifications at any one time and across the year.	2.1 Current proportion	Whole	High partial / Low partial	None
		2.2 Longitudinal proportion	Whole	High partial / Low partial	None
	Teachers have some stability in their workload that includes subjects at certain year levels or which cycles depending on circumstances.	2.3 Stability	Stable	Cyclical	Temporary
3. Capability	Teachers have the expertise needed to teach a subject. Increased expertise is related to engagement with professional learning.	3.1 Expertise	Capable (Substantial experience and development)	Practiced (Repeated experiences without development)	Beginning (No experience)
	Teachers have the capacity appropriate for their career stage to adapt to teaching new subjects.	3.2 Career stage	Experienced teacher (>6 years)	Early career teacher (1-5 years)	Graduate (<1 year)

Criteria Map 2. Self report criteria

	Standard	Dimension	Band 1	Band 2	Band 3
4. Identity	Teachers are committed and motivated to seek better ways to engage students, devote time and effort to planning and show an interest in the subject.	4.1 Commitment	Personal and professional commitment	Professional commitment	Compliance
	Teachers identify with the subject they are teaching and feel they belong.	4.2 Self-concept	Close	Peripheral	Distant
	Teachers are confident in their knowledge of the content, teaching approaches and how to support students in the subject, and to collaborate with peers.	4.3 Confidence	High	Medium	Low
5. Structures	Teachers work in school contexts that provides adequate access to opportunities despite geographical region, school size and type, and other system factors.	5.1 School context	Opportunities created	Some opportunities	Stifled opportuntiies
	Teachers have access to resources and support from colleagues, leaders, and mentors that suits their subject-specific needs.	5.2 School support culture	Fully supported	Some support	No support

Criteria Map 3. Longitudinal criteria

	Standard	Dimension	Band 1	Band 2	Band 3	
6. Pathways	Teachers have pathways for moving from out-of-field to in-field that are recognised and reduce the incidence of out-of-field teaching.	6.1 Trajectories	Qualification upgrade (reduced incidence)	Professional development concentration (no immediate reduced incidence)	Experience (no reduced incidence)	Temporary (no reduced incidence)
	Teachers accept the subject as part of their expanding role, leading to extended identities.	6.2 Role expansion	Acceptance with extended identity	Acceptance without extended identity	Non-acceptance and no extended identity	

Appendix E Lee's response to Wembley picture

Lee: Ohh yeah, it's the… it's Wembley, yeah.

FY: It is, yes. And the question is where's the maths?

Lee: Where's the maths?

FY: Where's the maths? Yeah.

Lee: Well, the maths is… in the construction of the building. And the maths is on the football pitch in terms of the lines to correct the football pitch. The measurements, so the measurements and the distances of the lines. And the capacity, so the seating. The angles to make sure that everyone is visible to see the pitch. You have to have numeracy so that you can have the same ratio of stewards to fans. The maths would be the construction of the electronics and the lighting and the scoreboards, so you'd have to be able to calculate lots of different maths within that. And... And then maths would be on the football pitch itself for the scores. And you've got tactics. So different formations on the football pitch. Then you would have the commentary, so they need to know all the statistics of the players. And… the history of the football and the footballers on the pitch. You'd also need maths for all the catering in the stadium. And the cost of things to make sure that they're making a profit, not a loss. And… there's so many other things do you want me to carry on.

FY: I do. I'm loving this. Yes, carry on.

Lee: And so... so it was the catering… bar staff... hospitality. And there's also mathematics for the emergency exits for when there's an issue, such as a fire. You've got to be able to empty the stadium in a certain amount of minutes safely and effectively. So that's obviously got to be done correctly. You could do the mathematics of the mechanics and machinery to close that roof, even though it doesn't close completely, but you can close part of it. Erm… what else? You'd need mathematics for the transport to get the timings right for people to get there on time before the football match or rugby match or whatever starts. So, there's trains that go past Wembley: they'd have to be there at a reasonable time. You'd have to have logistics for the teams to get there. You'd have to have the coaches and the car parks available for enough cars to fit in, so there's enough spaces for the people to go and watch that sport there.

FY: That's the most amazing comprehensive answer… sorry you're you're still going.

Lee: And. Uh, yeah. So it's just one more, I was going to go back to the catering. The other maths thing is they have to know that they've got enough money to give change back to the people when they’re buying all of the things there.

Appendix F Pete’s response to the five questions

Pete: I am looking at question one and I'm trying to factorise the quadratic. That’s something I teach quite a lot. Hmm… My initial feeling…. Well, I was started looking at positive 50 and looking at factors positive 50. And… Uhm…. I initially kind of went, well, it's not one and 50, 2 and 25. I had a confidence that if you give me a question like that, you're probably giving me something that's possible to factorise. So I wasn't planning on working particularly hard on it and looking at decimals or anything like that. And so I’ve got two and 25 and then I made the mistake mentally of going well, it's positive two and negative 25 because I've got a negative 27 and then I corrected myself and realised that’s stupid. It's negative two, negative 25. For that inside my factored brackets and therefore the answer’s positive two and positive 25 as a solution. Uh. And I and I enjoyed doing that because it had a word and an equation and I thought, well, that's fine. And I'm going to ignore question 2 because there's a lot of words there. And then I looked at question three. I thought I like the picture, but I'm going to ignore the words. And then I looked at question four and I thought, oh, lot of words. And then I thought question five, I'm gonna come onto that when I've done question one and then I'll go back to two, three to three and then two and four afterwards, if that makes sense. So if you wanted to know my thinking?

I'm going to look at 5. And I'm starting to regret it 'cause. That looks a bit more complicated than question one. I thought I was doing quite well on question one. Uh, and question five. I need a bit of paper for. Do you want me to answer these? [Nods] OK. Y equals negative two X squared. Now these are higher tier questions on the GCSE and I have a few higher tier pupils when they're doing this, so I generally end up with about one a year. And every now and again, I'll be honest, when I'm doing something that I don't know the answer to I might need to go away, come back to him and they always really fine about that. And you know, and at the same time I'm teaching ten other different things in the same classroom, so they quite understand it, quite used to it. Anyway, being still left to do something well, so. Whilst I go and deal with other pupils and then come back so. See, these are nicer to mark than they are to do because when I mark them, I can look at their solution and I know pretty quickly whether or not they got it right. And when I'm teaching it. Then I can explain very slowly, explain my thinking, which gives me time to work through the working out. So I'm using trial and improvement, so I'm doing a two.... Pants… positive 2X in negative X and I'm going to do on the other side. My left hand side. I'm going to do negative X and positive 2X… Oh, then I looking at factors of 10. One, two, 4, 25, 50 and 100… I know I'm looking to see what jumps out at me at something that might work, and I'm going to try. I have got a feeling about 50 and two 'cause it's got 54 in the middle. And there's a two, and I can now I can double 2. So I've got a feeling about that. Some of these start with 50 and two. And I want to add them, so I reckon… oh dear, that doesn't work. Oh no, they're negatives, aren't they? I could do a negative 50… Oh dear… So I'm going to go positive two X -. 50. Negative X… Add 2… And wing and a prayer. Minus 100… Negative two X squared. I'm regretting choosing question 5 now. Bet you question 2 is easier! Uh, and then I've got no x, no negative. Yes, 'cause 250. OK, I've got it. So negative X + 2 in one bracket and then positive two X -, 50 subtract 50 in the other bracket, for question 5. I can put it in the chat if you want. Or are you politely going to tell me I'm wrong?

FY: No. I'm working through it with you.

Pete: Oh, thank you. OK. Negative X alright, negative X + 2. And then in in the left-hand bracket and then positive two X -, 50. It's a useful exercise 'cause actually sometimes we get to things like this and I know I'm using words around. You know, I know there is a language of mathematics that I pick up. You know there is stuff that's a kind of discrete language. Where you could you read, uh, you pick it up through reading textbooks and reading other stuff. That's kind of "this is the thing to say". You know, "a factor is a number that divides exactly into another number" and over time you sort of pick up these lovely little phrases and they're useful teaching tools and you know useful way of looking like a mathematician I guess. Uh, and then there's others indiscrete ones that you can only really pick up by listening to a lot of mathematicians. Does that make sense? It's like being in that world. And so sometimes when I when I'm talking about the left hand set of brackets and the right-hand set of brackets, I'm sure people in the little maths club … erm … I'm sorry, I don't mean that. I was just being facetious, not facetious, just silly. But. But you know there is a more comfortable way. Actually it's things like that that I think that I'm giving away that I don't quite know what I'm doing. Not that I mind, but… so yes, so I think I've got that one right. *[turns to look at questions 2 and 3]* I have a square paper or I think of two numbers I them together we get 27, I'm multiplying get 50. Well. Oh, that's alright actually. Question two is just two and 25. Uhm. I wish I'd done that instead of question 5. *[now turns to question 3]* Uhm, questions three, I have a square of paper. I measured 13 and a half along two sides. You know we are and then cut away the corner piece I cut away has an area of 132.5. What's the area of the other piece? OK, cool. Well, I'm going to square. I'm not. I'm… I haven't got a calculator. I'm going to square root 132.25. Which will tell me the length of what the positive square root will tell me the length of one side. I'm going to add that too. No, I'm not 13.5. And then I'm going to square my answer. Yes, 'cause, it's square with papers. So that 132.25 is a square. And it's somewhere between 12 and 13. I'm going to guess it's about 12 and a half, so I'm going approximate my answer as 14,15. Sixteen 26. 26 squared, which? Is a big number.

FY: It's, it's exactly halfway between 11 and 12. Actually, it's 11.5. So you were very accurate. On the square root of 132.25.

Pete: Ah. Not 12.5.

FY: No, it's 11.5.

Pete: Oh, 132 is less than 144, isn't it? Yes. OK. 11.5. So it's 25. I was expecting it to be a nicer number than 26. So I did the calculation over there. So it would be 625, I think that's 25 squared. Or 425 or something like that. What's 25 squared? Yes, it's 625, right? So 625 centimetres squared. Thanks for question three. And then question 4. This is a nice one. So there's a big avenue of research into numeracy with kind of nurses and medical professionals and maths anxiety… *[reads question 4]* To find the correct dosage of an adult medication, you have to subtract 13 1/2 from a person's age and square the result. Ali's dosage is 132.5, which I happen to know is 11.5 squared. To find the correct dosage and square the result. So I'm 11.5… Odd. 13.5, which is remarkably similar to the last question. So I think Ali's, 25. That's really nice to have those two questions together. It's lovely.

FY: There's more than that. What about question one?

Pete: Oh dear. Oh, oh, I love it when this happens. Yes, we've had a 2 to 25.

FY: Question 2?

Pete: Question two and a 24. So we had two 25, two and 25. We had a 25, squared. And then we had another 25 squared and then we had two and 50, which sort of links to, oh 'cause. It would be a solution. I didn't answer question 5 properly. I bet you that I'm going to have a… I'm going to put my finger on the dart board and without doing any calculation, think that probably… It will be a coordinate of like (2,25) or something like that. Uhm. It would be zero wouldn't it? So that's not…

FY: But you would have two intercepts.

Pete: I would have said yes, yes. And so one would be. Uh, when in fact, yes. I've got a negative 25 on my right hand side and positive 2 on my left hand side. So yes positive two… And negative 25. Nothing. Because it's… it… it should…

FY: You actually had 2X - 50 as your second brackets, so it's actually positive 25.

Pete: Of course it is. Yeah. Yeah. Thank you. Yes, positive 25. And positive 2, but it's negative X yes, thank you. Uhm. They're beautiful. They're lovely. Lovely questions. I'm sure there's probably something deeper that's connected to all of them that I haven't spotted. Other than the fact there's lots of twos and 25s. And you're not going to tell me what it is?

FY: When you were answering question one, question two was almost your script.

Pete: Oh, how interesting. Yes, it was, wasn't it? Isn't that nice.

FY: So in fact, question two is just putting into words the algorithm for question one for factorising brackets. I mean it's not quite aligned because you don't have the negatives in Question 2

Pete: It is, that it's, it's just it's a factorising afterwards. I've never seen it. Like, that's lovely.

FY: Uhm, questions three and four. You solved in a problem-solving manner, but you could have solved them algebraically. Any thoughts on what process you would have used to solve them algebraically?

Pete: Mmmm… Well, it's coming. It's going to be factorising 'cause we've got, we've got some squares in there. Uhm… I'm not sure. Oh, is it something like yes, 13.5 plus X... brackets all squared… Equals… Well, it doesn't equal 132.25.

FY: It does, and it is.

Pete: It's y squared or something?

FY: And that's completing the square. So you're completing the square on both of those. So you went for factorising question one because you correctly identified that it wasn't going to be so cruel as to give you something with decimal solutions. But if you had gone at it from completing the square, you would've come across the 132.5.

Pete: Oh of course. It's at X + 7, all squared, no… no, it's the left hand side X + 5. Completing the square I guess it's something that I look up before I teach. I have done it. Uhm done it, I did MST121 OU course I think or 221. Uh. If you see 1/2 when it would have been the basic beginner stuff actually. Uh. That I found really hard. Uhm… [pause] No, I'm so I'm … I'd have to look it up and I'm quite embarrassed about that, actually. X squared. Which one would I be completing the square with? Is it? So no, it's it 'cause it's the constant afterwards, isn't it? Uhm… It's half the middle. No, no, I don't half the middle number

FY: Well, just look at look at the numbers we've got there. What is half of the middle number?

Pete: So of course it's 13 and a half. Oh, that is beautiful. It's I do half the middle number. Oh, isn't that wonderful? I do like it when things come together.

FY: And then the bottom one, the 54 might be the giveaway… as the middle number there.

Pete: Oh, I could have just halved the lot. Yeah, yeah, oh that’s irritating. Yes… Yes, of course. You know, I didn't even think about that. I'm just thinking honestly. Oh. And that's the difference between, isn't it? That's going to be a difference between the way. Somebody that's coming into this from a very heavy schooled maths background. But no, not heavy schooled. Wrong word. But kind of scholarly… And I think I probably sort of busk maths a little bit, but um, but my goal my goal is to teach maths with. My goal is to support, you know, vulnerable pupils and I think maths is quite a good way of doing that and it's, you know. So, but he's probably busking rather than playing in the orchestra.